On the Extended Preference-based Constrained Argumentation Framework

(Extended Abstract)

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Abstract

In recent years there has been an increasing interest in extending Dung's framework to facilitate the knowledge representation and reasoning process. In this paper, we discuss a recently proposed extension of abstract Argumentation Framework (AF) that allows for the representation of preferences over arguments' truth values (3-valued preferences) [1]. For instance, we can express a preference stating that extensions where argument a is false (i.e. defeated) are preferred to extensions where argument b is false. Interestingly, such a framework generalizes the well-known Preference-based AF with no additional cost in terms of computational complexity for most of the classical argumentation semantics. Then, AF is further extended by considering both (3-valued) preferences and 3-valued constraints, that is constraints of the form $\varphi \Rightarrow v$ or $v \Rightarrow \varphi$, where φ is a logical formula and v is a 3-valued truth value. We discuss the complexity of deciding acceptance of arguments in this context.

Keywords

Abstract Argumentation, Preferences, Constraints

Introduction

Recent years have witnessed intensive formal study, development, and application of Dung's abstract Argumentation Framework (AF) in various directions [2]. An AF consists of a set $\mathcal{A}$ of arguments and an attack relation $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ that specifies conflicts between arguments (if argument a attacks argument b, then b is acceptable only if a is not). We can think of an AF as a directed graph whose nodes represent arguments and edges represent attacks. The meaning of an AF is given in terms of argumentation semantics, e.g. the well-known *grounded* ($\mathtt{gr}$), *complete* ($\mathtt{co}$), *preferred* ($\mathtt{pr}$), *stable* ($\mathtt{st}$), and *semi-stable* ($\mathtt{ss}$) semantics. Intuitively, an argumentation semantics tells us the sets of arguments (called *σ-extensions*, with $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{ss}\}$) that can collectively be accepted to support a point of view in a dispute. For instance, for AF $\langle \mathcal{A}, \mathcal{R} \rangle = \langle \{\mathtt{a}, \mathtt{b}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a})\} \rangle$ having two arguments, a and b, attacking each other, there are two preferred/stable extensions, $\{\mathtt{a}\}$ and $\{\mathtt{b}\}$; neither a nor b is certainly accepted.

Several proposals have been made to extend the Dung's framework with the aim of better modeling the knowledge to be represented. These extensions include AF with constraints (CAF) [3, 4, 5] and AF with preferences [6, 7, 8, 9, 10, 11, 12, 13], among others.

As an example, consider AF $\Lambda_1 = \langle \{\mathtt{fish}, \mathtt{meat}, \mathtt{red}, \mathtt{white}\}, \{(\mathtt{fish}, \mathtt{meat}), (\mathtt{meat}, \mathtt{fish}), (\mathtt{meat}, \mathtt{white}), (\mathtt{white}, \mathtt{red}), (\mathtt{red}, \mathtt{white})\} \rangle$, shown in Figure 1(left). Intuitively, Λ_1 describes what a person is going to have for lunch. (S)he will have either fish or meat, and will drink either white wine or red wine. However, if (s)he will have meat, then (s)he will not drink white wine. Λ_1 has three preferred (stable and semi-stable) extensions $E_1 = \{\mathtt{fish}, \mathtt{white}\}$, $E_2 = \{\mathtt{fish}, \mathtt{red}\}$, and $E_3 = \{\mathtt{meat}, \mathtt{red}\}$, which represent alternative menus.

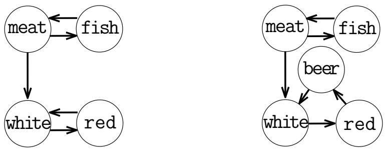

Figure 1: AF Λ_1 (left) and AF Λ_2 (right).

Assume that there is a pescetarian customer and, as a consequence, (s)he wants to discard all menus with meat by putting the constraint $\mathtt{meat} \Rightarrow \mathtt{false}$, stating that argument meat must be rejected. Thus, feasible preferred extensions are only those where meat is defeated, that is E_1 and E_2.

Assume now that there is another customer which would express the preference on menus having meat instead of fish as main dish; the preference $\mathtt{meat} > \mathtt{fish}$ can be used to encode such a desideratum. In this case no extension is discarded. Among the three above-mentioned extensions representing the alternative menus, the best one for the considered customer is selected (i.e. E_3).

Considering the previous example, one could observe that the (pescetarian) user constraint could be modeled by

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modifying the AF through the addition of an (unattacked meta-) argument attacking `meat`. However, such kind of rewriting is not always easy to carry out, e.g. when constraints are defined by complex propositional formulae. In some cases, it is even not possible (e.g. under the complete semantics). In fact, the introduction of constraints and/or preferences is useful not only to separate the objective knowledge represented by the AF from the subjective restrictions and preferences added by users but also because, as it will be clear from our complexity analysis, the rewriting is not always possible.

Regarding Preference-based AF (PAF), user preferences are used to select a subset of extensions of the AF, called *best extensions* [6, 7, 8, 9, 10, 11]. There have been different proposals to define the best extensions, corresponding to different criteria for comparing pairs of extensions (e.g. democratic, elitist and KTV criteria).

A limitation of the forms of preferences proposed in the literature is that, as AF semantics may be 3-valued (arguments can be either *accepted*, *defeated*, or *undecided*) they do not allow expressing preferences referring to the status of arguments. For instance, continuing with our example, classical preferences do not allow us to express a preference for menus (i.e. extensions) containing `fish` w.r.t. menus not containing `fish` (i.e. extensions where `fish` is defeated or undecided) or to express a preference for menus surely not containing `fish` (i.e. with `fish` being defeated) w.r.t. menus surely not containing `meat` (i.e. with `meat` being defeated).

As most of the AF semantics are 3-valued, in this paper we discuss AF with *extended preferences* [1], that is preferences of the form $a^v \succ b^w$, where a and b are arguments and v and w are truth values (*true*, *false*, and *undefined*) denoting the status of associated arguments (accepted, defeated, and undecided, respectively). We also discuss the combination of extended preferences with 3-valued constraints.

We assume the reader is familiar with AF, CAF and PAF semantics. We refer the interested reader to [2] for a comprehensive overview of abstract argumentation.

AF with Extended Preferences

In this section we introduce a new form of preference for AF and extend the PAF under the KTV criterion [14].

Definition 1. *Let $\mathcal{A}$ be a set of arguments, an (extended) preference relation, denoted as $\succ$, is a strict partial order (i.e. an irreflexive, asymmetric, and transitive relation) over $\mathcal{A}^V = \{a^v \mid a \in \mathcal{A} \wedge v \in \{\mathbf{f}, \mathbf{u}, \mathbf{t}\}\}$ of the form $a^{v_1} \succ b^{v_2}$.*

Intuitively, it is allowed to define preference between pairs, where each pair consists of an argument and a truth value in $\{\mathbf{f}, \mathbf{u}, \mathbf{t}\}$, denoting *false*, *undefined*, and *true* truth values, and corresponding to the following statuses of arguments: defeated, undecided, and accepted respectively.

For instance, considering the AF Λ_2 shown in Figure 1(right), a preference $\texttt{red}^{\mathbf{t}} \succ \texttt{red}^{\mathbf{u}}$ means that we prefer menus containing red wine w.r.t. menus where red wine is undecided, whereas a preference $\texttt{fish}^{\mathbf{t}} \succ \texttt{red}^{\mathbf{f}}$ states that we prefer menus containing `fish` w.r.t. menus where `red` is false (i.e. defeated).

Definition 2. *An extended PAF (ePAF) is a triple $\langle \mathcal{A}, \mathcal{R}, \succ \rangle$ where $\langle \mathcal{A}, \mathcal{R} \rangle$ is an AF and $\succ$ is an extended preference relation.*

Definition 3. *Given an ePAF $\Delta = \langle \mathcal{A}, \mathcal{R}, \succ \rangle$ and two distinct sets of arguments $E, F \subseteq \mathcal{A}$, we have that $E \sqsupseteq F$ under* KTV *(k) criterion if $\nexists\, a^{v_1} \succ b^{v_2}$ such that $a \in v_1(F) \setminus v_1(E)$, $b \in v_2(E) \setminus v_2(F)$ holds (where $v_1, v_2 \in \{\mathbf{f}, \mathbf{u}, \mathbf{t}\}$). Moreover, $E \sqsupset F$, if $E \sqsupseteq F$ and $F \not\sqsupseteq E$.*

Let $\sigma(\langle \mathcal{A}, \mathcal{R} \rangle)$ be the set of σ-extensions for AF $\langle \mathcal{A}, \mathcal{R} \rangle$. Given an ePAF $\Delta = \langle \mathcal{A}, \mathcal{R}, \succ \rangle$ and $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{st}, \texttt{ss}\}$, an extension $E \in \sigma(\langle \mathcal{A}, \mathcal{R} \rangle)$ is a best extension for Δ if there is no extension $F \in \sigma(\langle \mathcal{A}, \mathcal{R} \rangle)$ such that $F \sqsupset E$. The set of best σ-extensions for an ePAF Δ under KTV criterion is denoted by $\sigma_k(\Delta)$.

Considering the AF Λ_1, there are six complete extensions: $E_0 = \emptyset$, $E_1 = \{\texttt{fish}, \texttt{white}\}$, $E_2 = \{\texttt{fish}, \texttt{red}\}$, $E_3 = \{\texttt{meat}, \texttt{red}\}$, $E_4 = \{\texttt{fish}\}$ (with `white` and `red` undecided), and $E_5 = \{\texttt{red}\}$ (with `fish` and `meat` undecided). When assuming the following preferences: $x^{\mathbf{t}} \succ x^{\mathbf{u}}$ and $x^{\mathbf{t}} \succ x^{\mathbf{f}}$, for every argument x, the best complete extensions are E_1, E_2 and E_3 (which are the preferred ones). If we also have the preference $\texttt{fish}^{\mathbf{t}} \succ \texttt{meat}^{\mathbf{t}}$, then the best complete extensions are E_1 and E_2.

Notice that ePAF generalizes PAF with KTV criterion. Indeed, let $\Delta = \langle \mathcal{A}, \mathcal{R}, \succ \rangle$ be an ePAF and $\Delta' = \langle \mathcal{A}, \mathcal{R}, > \rangle$ be a PAF such that $\succ\, = \{a^{\mathbf{t}} \succ b^{\mathbf{t}} \mid a > b \text{ in } \Delta'\}$ and $> = \{a > b \mid a^{\mathbf{t}} \succ b^{\mathbf{t}} \text{ in } \Delta\}$, where $>$ is a strict partial order over arguments, then it holds that $\sigma_k(\Delta) = \sigma_k(\Delta')$ for $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{st}, \texttt{ss}\}$.

Combining Preferences with Constraints

Extended preferences and constraints have been combined so that the resulting framework, called *extended Preference-based Constrained Argumentation Framework*, other than offering a compact and easier representation of both preferences and constraints, is also more expressive than both CAF and PAF and allows to express several kinds of desiderata among extensions.

Definition 4. *An* extended Preference-based Constrained Argumentation Framework *(ePCAF) is a tuple $\Delta =$*

	AF			CAF			PAF			ePAF / ePCAF		
σ	Ver_σ	CA_σ	SA_σ	Ver_σ	CA_σ	SA_σ	Ver_{σ_k}	CA_{σ_k}	SA_{σ_k}	Ver_{σ_k}	CA_{σ_k}	SA_{σ_k}
`co`	P	NP-c	P	P	NP-c	coNP-c	coNP-c	Σ_2^p-c	P	coNP-c	Σ_2^p-c	Π_2^p-c
`st`	P	NP-c	coNP-c	P	NP-c	coNP-c	coNP-c	Σ_2^p-c	Π_2^p-c	coNP-c	Σ_2^p-c	Π_2^p-c
`pr`	coNP-c	NP-c	Π_2^p-c	coNP-c	Σ_2^p-c	Π_2^p-c	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p
`ss`	coNP-c	Σ_2^p-c	Π_2^p-c	coNP-c	Σ_2^p-c	Π_2^p-c	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p

Table 1
Complexity of the verification (Ver) and credulous (CA) and skeptical (SA) acceptance problems under complete (`co`), stable (`st`), preferred (`pr`), and semi-stable (`ss`) semantics. For any complexity class C, C-c (resp., C-h) means C-complete (resp., C-hard). An interval C-h, C' means C-hard and in C'.

$\langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \succ \rangle$, *where* $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ *is a CAF and* $\succ$ *is an (extended) preference relation (cf. Definition 1).*

The semantics of an ePCAF is given by the best extensions selected among those that satisfy the constraints.

Definition 5. *Given an ePCAF* $\Delta = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \succ \rangle$ *and a semantics* $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{st}, \texttt{ss}\}$*, a* σ*-extension* E *for* $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ *is a best* σ*-extension for* Δ *under* KTV *criterion if there is no* σ*-extension* F *for* $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ *such that* $F \sqsupset E$*.*

Continuing with our running example, consider the ePCAF $\Delta_1 = \langle \mathcal{A}_1, \mathcal{R}_1, \{\texttt{white} \Rightarrow \texttt{f}\}, \{\texttt{meat}^\texttt{t} \succ \texttt{fish}^\texttt{t}\} \rangle$, The preferred extensions for AF $\Lambda_1 = \langle \mathcal{A}_1, \mathcal{R}_1 \rangle$ are $E_1 = \{\texttt{fish}, \texttt{white}\}$, $E_2 = \{\texttt{fish}, \texttt{red}\}$ and $E_3 = \{\texttt{meat}, \texttt{red}\}$. As `white` must be false, there are only two preferred extensions satisfying the constraint: E_2 and E_3. Then, the only best preferred extension is E_3.

It is worth noting that, the best extensions would have been different if the ePCAF $\Delta = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \succ \rangle$ has been defined as an ePAF $\langle \mathcal{A}, \mathcal{R}, \succ \rangle$ with a set of constraints $\mathcal{C}$. Indeed, in such a case, the σ-extensions for Δ would have been as the best σ-extensions of $\langle \mathcal{A}, \mathcal{R}, \succ \rangle$ satisfying constraints $\mathcal{C}$, that is constraints would have been applied after preferences.

Complexity

Given an eP(C)AF Δ and a set S of arguments, the *verification* problem under KTV criterion (denoted as Ver_{σ_k}) is deciding whether S belongs to the set of best σ_k-extensions of Δ. Moreover, given an argument g, the *credulous* and *skeptical acceptance* problems (denoted as CA_{σ_k} and SA_{σ_k}) are the problems of deciding whether g belongs to any/every σ_k-extension of Δ, respectively.

As stated by the complexity results reported in Table 1, that also summarizes known results for AF, CAF and PAF, the complexity bounds of verification, credulous acceptance and skeptical acceptance for ePAF do not increase w.r.t. those of PAF under KTV semantics, except for skeptical acceptance under complete semantics that becomes Π_2^p-complete. Although the form of preference introduced is more flexible than that of PAF, the complexity does not increase in most of the cases.

We observe that ePAF is used to express preferences not allowed in PAF. As an example, consider the AF Λ_2 shown in Figure 1 (right). The PAF preference $\texttt{red} > \texttt{white}$ does not allow to restrict the set of extensions and all complete (resp. preferred) extensions are also the best ones. However, the ePAF preference $\texttt{red}^\texttt{t} \succ \texttt{red}^\texttt{u}$ allow us to select as best complete (resp. preferred) extension E_2 only.

Finally, ePCAF is generally more expressive than CAF, particularly if we consider the verification problem whose complexities increase of one level in the polynomial hierarchy for all considered semantics. Also, it turns out that ePCAF has the same complexity bounds as PAF, except for the $SA_{\texttt{co}_k}$ problem, similarly to what we have observed for ePAF.

Conclusions and Future Work

Extended preferences and (3-valued) constraints as well as the complexity results for the novel frameworks (ePAF and ePCAF) can carry over to other AF-based frameworks [15, 16, 17, 18, 19]. Indeed, as these frameworks can be rewritten into AF [20], their extended Preference-based Constrained forms could be rewritten in ePCAF, obtaining upper bounds on their complexity from ePCAF results. Lower bounds also follow if those frameworks generalize ePCAF.

As future work, we plan to investigate preferences and constraints in other frameworks extending AF [21, 22, 23, 24, 25, 26, 27, 28, 29], as well as other forms of constraints such as weak and epistemic constraints [5, 30, 31, 32].

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