Reference, Predication and Quantification in the Presence of Vagueness and Polysemy

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Abstract

Classical semantics assumes that one can model reference, predication and quantification with respect to a fixed domain of possible referent objects. Non-logical terms and quantification are then interpreted in relation to this domain: constant names denote unique elements of the domain, predicates are associated with subsets of the domain and quantifiers ranging over all elements of the domain. The current paper explores the wide variety of different ways in which this classical picture of precisely referring terms can be generalised to account for variability of meaning due to factors such as vagueness, context and diversity of definitions or opinions. Both predicative expressions and names can be given either multiple semantic referents or be associated with semantic referents that have some structure associated with variability. A semantic framework *Variable Reference Semantics* (VRS) will be presented that can accommodate several different modes of variability that may occur either separately or in combination. Following this general analysis of semantic variability, the phenomenon of *co-predication* will be considered. It will be found that this phenomenon is still problematic, even within the very flexible VRS framework.

Keywords

Vagueness, Polysemy, Predication, Quantification, Copredication

1. Introduction

The notion of reference and the operations of predication and quantification are fundamental to classical first-order logic. The standard semantics for this logic assumes a fixed domain of possible referent objects, with naming constants referring to unique elements of the domain, predicates being associated to subsets of the domain and quantifiers ranging over all the elements of the domain. Thus, if $\mathscr{D}$ is the domain of objects that can be referred to, then a constant name, say c, will donate an object $\delta(\mathrm{c})$, such that $\delta(c) \in \mathscr{D}$, and a predicate P will be taken to denote a set of objects $\delta(P)$, with $\delta(P) \subseteq \mathscr{D}$. Then one can straightforwardly interpret the predicating expression $P(\mathrm{c})$ as a proposition that is true if and only if $\delta(\mathrm{c}) \in \delta(P)$.

But what if we are dealing with a language that is subject to variability in the interpretation of its symbols: a name may not always refer to a unique, precisely demarcated entity; a predicate need not always correspond to a specific set of entities. Accounting for such semantic variability requires some generalisation or other modification of the classical denotational semantics.

The aim of the current paper is not to propose a single theory but rather to explore some

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representational possibilities. In the first part of this paper I consider different ways in which semantics can be given to predication in the presence of semantic variability. I shall first consider some general ideas regarding different views of vagueness, in particular the *de re* and *de dicto* accounts of this phenomenon. I shall suggest that these need not be mutually exclusive, but could be describing different aspects of semantic variability. I explore possible models of denotation, predication and quantification in the presence of such variability, first informally, with the aid of some diagrams and then in terms of a formal framework, based on *standpoint semantics* [1, 2, 3], within which variable references can be modelled. In the final section of the paper I shall consider the problem of *co-predication.* This has been the subject of considerable debate in recent years and is a phenomenon that throws up many examples that pose difficulties for different approaches and has been used to try to support/reject models of semantic variability.

1.1. Types of Vagueness

The literature on vagueness has generally assumed that the phenomena of vagueness could arise from three potential sources (see e.g. Barnes [4]):

1. indeterminacy of representation (linguistic a.k.a *de dicto* vagueness),
2. indeterminacy of things in the world (*ontic* a.k.a. *de re* vagueness),
3. limitations of knowledge (*epistemic* vagueness).

The epistemic view of vagueness has some strong advocates [5, 6, 7] and many other take the view that the logic of multiple possible interpretations takes a similar form to logics of knowledge and belief (e.g. Lawry [8], Lawry and Tang [9]). Indeed, the *standpoint semantics*, which will be used in our following analysis can be regarded as being of this form. However, the question of whether this is a deep or superficial similarity is not relevant to our current concerns; and the distinction between *de dicto* and *de re* aspects of multiple reference, would also arise within an epistemic account. So in the current paper we shall not further consider the epistemic view.

1.2. *De Dicto* Vagueness

A widely held view is that all vagueness is essentially *de dicto*, and that any kind of vagueness that seems to come from another source can actually be explained in *de dicto* terms [10, 11]. A fairly typical version of such an attitude is that of Varzi who (focusing on the domain of geography, within which vagueness is pervasive) takes a strong position against ontic vagueness. Varzi's view of the relationship between vague terms and their referents is summarised in the following quotation:

> "[To] say that the referent of a geographic term is not sharply demarcated is to say that the term vaguely designates an object, not that it designates a vague object." [11]

Advocates of exclusively *de dicto* vagueness typically favour some variety of *supervaluationist* account of linguistic vagueness, within which the meanings of vague terms are explained in terms of a collection (a set or some more structured ensemble) of possible precise interpretations (often called *precisifications*). An early proposal that vagueness can be analysed in terms of multiple precise senses was made by Mehlberg [12], and a formal semantics based on a multiplicity of classical interpretations was used by van Fraassen [13] to explain 'the logic of presupposition'. This kind of formal model was subsequently applied to the analysis of vagueness by Fine [14], and thereafter has been one of the more popular approaches to the semantics of vagueness adopted by philosophers and logicians, and a somewhat similar approach was proposed by Kamp [15], which has been highly influential among linguists.

What one might call *absolutist* versions of supervaluationism are those that, following Fine, hold to a doctrine of *super-truth*.[1] This is the tenet that the truth of an assertion containing vague terminology should be equated with it being true according to all *admissible* precisifications [14, 17, 18]. In such theories the set of admissible precisifications is generally taken as primitive, just as is the set of possible worlds in Kripke semantics for modal logics. Other, supervaluation inspired, theories propose a similar semantics based on the idea that the truth of a vague assertion can only be evaluated relative to a locally determined precisification (or set of precisifications), but reject the idea that the notion of super-truth is useful. One might call such theories *relativist* supervaluation theories. Such theories include that of Shapiro [19] and my own *standpoint semantics* [1, 2].[2]

Apart from providing a general framework for specifying a *de dicto* semantics of vagueness, the supervaluationist idea is also attractive in that it can account for *penumbral connection* [14], which many believe to be an essential ingredient of an adequate theory of vagueness. This is the phenomenon whereby logical laws (such as the principle of non-contradiction) and semantic constraints (such as mutual exclusiveness of two properties — e.g. '... is red' and '.. is orange') are maintained even for statements involving vague concepts. The solution, in a nutshell, being that, even though words may have multiple different interpretations, each admissible precisification of a language makes precise all vocabulary in a way that ensures mutual coherence of the interpretation of distinct but semantically related terms.

1.3. *De Re* Vagueness

In natural language, objects are often described as vague. We commonly encounter sentences such as: 'The smoke formed a vague patch on the horizon'; 'The vaccine injection left a vague mark on my arm'; 'He saw the vague outline of a building through the fog'. In this kind of usage, 'vague' means something like 'indefinite in shape, form or extension'. Of course, 'indefinite' is almost synonymous with 'vague' so this definition is far from fully explanatory. However, if we take sentences like the aforegoing examples at face value, they seem to indicate that vagueness can be associated with an object in virtue of some characteristic of its spatial extension. One may then argue that spatial extension is an intrinsic property of an object, and that, if an object

[1] What I call 'absolutist' supervaluationism is what Williamson [16] refers to as 'traditional' supervaluationism.

[2] According to standpoint semantics, vague statements are true or false relative to a *standpoint*. My notion of standpoint is formally identified with a set of precisifications determined by a set of (consistent) judgements that are accepted as true in a given context. This idea is similar to that presented by Lewis [20], and elaborated by [21].

has a vague intrinsic property, this indicates vagueness 'of the thing'. Such vagueness may be called *de re* or *ontic*.

The idea that vagueness of objects is primarily associated with vagueness of *spatial extension* has been endorsed and examined by Tye [22], who gives the following criterion for identifying vague objects: "A concrete object *o* is vague if and only if: *o* has borderline spatio-temporal parts; and there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts, nor non-parts of *o*." (The second, rather complex condition concerns the intuition that we cannot definitely identify borderline parts of a vague object. The current paper will not consider this second-order aspect of vagueness.)

1.4. Combining *De Re* and *De Dicto* and the Idea of *De Sensu*

Rejecting the possibility of *de re* vagueness requires one to argue that the forms of language by which vagueness is apparently ascribed to objects are in fact misleading idioms, whose correct interpretation does not involve genuine ontological commitment to vague objects. However, contrary to what many proponents of one or other of the two explanations of vagueness often maintain, accepting that vagueness may be *de re* does not require one to deny that vagueness is often *de dicto*. Indeed, Tye [22] suggests that vagueness can be present both in predicates and also in objects. He argues that the vagueness of objects cannot simply be explained by saying that they are instances of vague predicates.

But the idea of vague objects in the physical world may be hard to accept. Although at the quantum level of atomic particles we may conceive of the position of an electron might be vague in a physical sense, vagueness of macroscopic physical objects seems a very odd idea. For example, in consider whether a particular twig is a part of some pile of twigs, it seems unintuitive to consider the twig pile as a vague physical object, which may or may not include certain twigs. As a palliative to this worry, I propose the existence of *de sensu* vagueness.

De sensu vagueness is a kind of indeterminism, in the form of a multi-faceted structure, that is located within the sense, that is in the semantic denotation term. In the case of names and nominal variables, this means that they can refer to semantic objects that are indeterminate with regard to any exact physical entity. Both the model of vagueness proposed in fuzzy logic [23, 24, 25] and also the idea of *dot cateogories* and *dot objects* proposed in some accounts of co-predication (e.g. [26]) can be regarded as based on a *de sensu* conception of vagueness.

2. Illustrating the Semantics of Predication

Predication involves a predicate and a nominal expression. Both the predicate and the nominal can be given a semantics that allows either *de dicto* vagueness or *de sensu* vagueness or both of these. *De dicto* vagueness is modelled within that part of the semantics that maps symbolic expressions (predicates and nominals) to their referents. If there is no *de sensu* vagueness then the referents will be precise entities, and these will normally be taken as being actual entities in the world. Thus, the reference of a nominal will be a particular precisely demarcated material entity and the reference of a property (or relation) predicate will be a determinate set (or set of pairs) of material entities. If there is *de sensu* vagueness then the references of symbols are semantic objects which can be vague in so far as their correspondence with precisely demarcated material entities is not fully determinate.

Before specifying a formal semantic framework, it may be useful to consider possible models of predication by means of diagrams. The image in Figure 1(a) depicts an aerial view of a hilly region with rocky crags. As we see, the terrain is irregular and there is no obvious unique way of dividing it into separate 'crag' objects. The name 'Arg Crag' has been given to one of the rocky outcrops. However, there may be different opinions regarding exactly which of outcrop is Arg Crag. Indeed, some people might use the name to refer to the whole of this rocky area, whereas others would consider that it refers to a more specific rock structure.

In the standard classical semantics each conceptual term denotes a fixed set of entities and each name must refer to a single precise entity. Thus, event before specifying denotations we will need to divide the craggy region into specific individual objects, to make a set of possible referents:

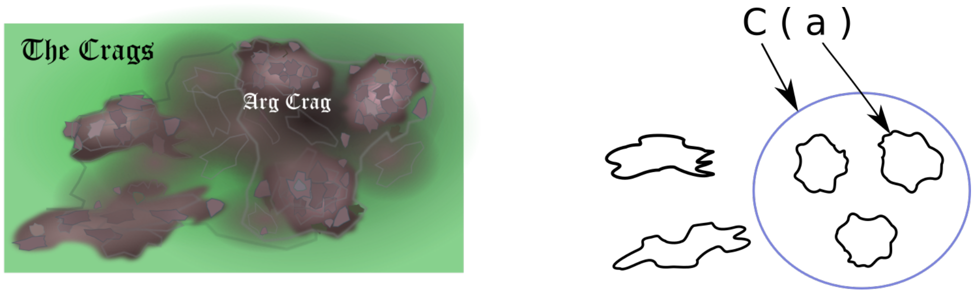

Figure 1: (a) Arg Crag and surrounding area. (b) Classical denotational semantics.

The best-known approach to modelling vague concepts that has been taken in computer science and AI is that of *fuzzy sets*. A fuzzy set is one such at rather than having a definite true or false membership condition, there are degrees of membership. Figure 2(a) depicts the standard fuzzy logic model of vagueness: each singular term denotes a single precise referent entity and each property predicate denotes a fuzzy set of precise entities. Figure 2(b) depicts the form of semantics arising for a *de dicto* theory of vagueness, where both properties and objects are precise but the predicate symbols and names of the language that are referentially indeterminate.

One might assume that the blurry concept boundary of the fuzzy logic model provides a model of predicate denotation that is radically different from that given by the supervaluationist multi-denotation model. But it can be argued that this difference is not greatly significant because we can consider the fuzzy set as essentially equivalent to a densely nested structure of classical set denotations (one for each degree of membership value). However, the semantics for evaluating a predicating expression, $C(\mathrm{a})$ is significantly different, since in fuzzy logic the truth value of $C(\mathrm{a})$ would be the degree of membership of object denoted by a with respect to the fuzzy set denoted by C; whereas, in the multiple denotation model we simply get different truth values for different choices and then (as we shall see in detail below) can determine truth in relation to a 'standpoint' regarding what choices we consider admissible.

A limitation of both the standard fuzzy model and the classical multiple reference model is that

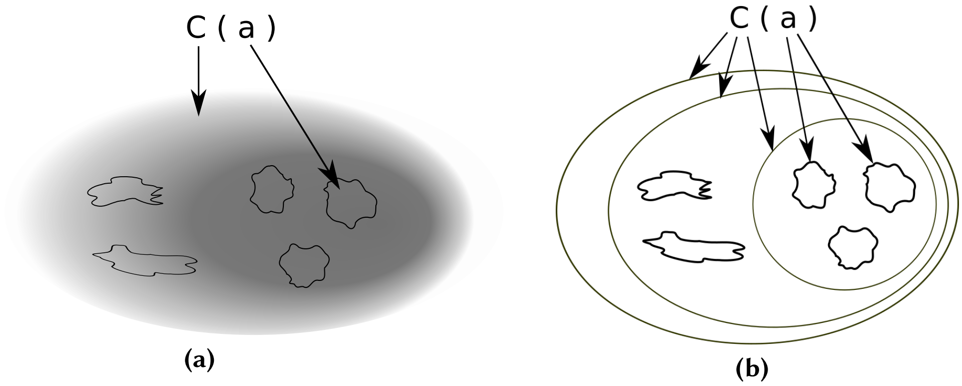

Figure 2: (a) Fuzzy model of a vague predicate. (b) Multiple denotation model, such as supervaluation semantics.

only variability in the terms is modelled, not variability in the object referred to. However, both approaches can be modified to incorporate vague referents, such as objects with indeterminate physical boundaries. For instance, objects with fuzzy extensions can be modelled as fuzzy sets of points. And within non-fuzzy multiple-reference semantics, one could also associate a set of different extensions for different precise versions of an object (or perhaps maximal and minimal extensions, as in the 'egg-yolk' representation of Cohn and Gotts [27]).

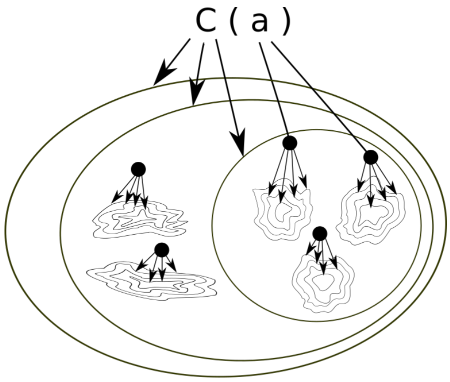

Figure 3: Possible multiplicity of reference when predicates, names and objects are all indeterminate.

Figure 3 depicts an extended form of multi-reference semantics (*The Full Multi*). Here we see not only that the predicates and names can have variable reference, but also there can be multiple possible precise versions of each reference object. This variability in the objects could correspond to *de re* vagueness, but I prefer to think of it as a form of *de sensu* vagueness, with the objects being vague semantic objects (visualised as the small black discs) that correspond to multiple precise physical objects.

It is worth noting that Figure 3 still represents a considerable simplification of the potential semantic variability that might arise. In particular, I have assumed that the global set of vague objects, together with their associations to precise entities, remains fixed, even though the subset associated with predicate C may vary. In other words C only varies in how it *classifies* objects, not in how these objects are *individuated.* The VRS semantics presented below is more general. It allows different senses of sortal predicates to be associated with different ways of individuating objects (for instance under some interpretations of 'Crag', all three of the roundish craggy objects within the innermost circle of the diagram, might be considered as parts of a single large crag.

3. Semantic Analysis of Variable Reference

We now consider what kind of semantics can account for the general form of variable denotation illustrated in Figure 3. *Standpoint Semantics* provides a quite general framework within which polysemy can be modelled in terms of the symbols of a formal language having multiple possible denotations. We shall start by specifying a simple propositional standpoint logic and then elaborate this to a first-order formalism, *Variable Reference Semantics*, within which we can model predication and quantification. But first we explain the basic idea of standpoint semantics.

Standpoint Semantics is based on a formal structure that models semantic variability in terms of the following two closely connected aspects:

- A *precisification* is a precise version of a vague language. It is used as an index to assign precise denotations to vague terms. The model is based on having a set of precisification, each corresponding to a consistent precise interpretation of the language.
- A *standpoint* is modelled as a set of *precisifications.* Each standpoint corresponds to a range of possible precise interpretations that are compatible with a particular context of language understanding. It could capture explicit specifications of terminology given by some organisation or implied constraints on meanings that arise whenever some vague statement is made in conversation (e.g. "That rock formation is a crag, but this is a boulder").

3.1. Propositional Standpoint Logic

3.1.1. Syntax

The language of *propositional standpoint logic* $\mathbb{S}_0$ is based on a non-empty finite set of propositional variables $\mathscr{P} = \{P_1, \ldots, P_n\}$. It extends the usual syntax of propositional logic by adding a set of standpoint operators $\Sigma = \{\Box_{s_1}, \ldots, \Box_{s_n}, \Box_*\}$, where $*$ is used to designate the *universal standpoint.*[3] So the language of $\mathbb{S}_0$ is the smallest set S of formulae such that $\mathscr{P} \subseteq S$ and all formulae of the forms $\{\neg\phi,\ (\phi \wedge \psi),\ \Box_\sigma\phi\} \subseteq S$ for each $\sigma \in \Sigma$ and every $\phi, \psi \in S$. One can easily extend the language by defining additional connectives (e.g. $\vee$, $\rightarrow$) and the dual operators $(\Diamond_{s_1} \ldots, \Diamond_{s_n}, \Diamond_*)$ in the usual way.

[3]The language is often augmented by also specifying a (partial) order relation over the standpoint operators, which indicates that one standpoint may be more specific or more general than another. For the purposes of the current paper, we do not consider this elaboration the basic system.

3.1.2. Semantics

In order to characterise the semantics of *standpoint logic* $\mathbb{S}_0$, we specify a class $\mathfrak{M}_{\mathbb{S}_0}$ of structures of the form $\langle \Pi, S, \delta \rangle$ where:

- $\Pi = \{\ldots, \pi_i, \ldots\}$ is a set of precisifications (which are analogous to possible worlds),
- $S =$ is a set $\{s_1, \ldots, s_n, *\}$ of subsets of Π,
- $\delta : \mathcal{P} \rightarrow \wp(\Pi)$ is a function mapping each propositional variable $p \in \mathcal{P}$ to the set $\delta(p) \subseteq \Pi$ of precisifications at which it is true. (Here, $\wp(\Pi)$ denotes the powerset of Π.)

The most distinctive elements of the model are the s_i, which model the notion of standpoint via a set of precisifications that are *admissible* for that standpoint. Thus, if $\pi \in s_i$ then all propositions that are unequivocally true according to standpoint s_i are true at precisification π.

For a model structure $\mathcal{M} \in \mathfrak{M}_{\mathbb{S}_0}$, we write $(\mathcal{M}, \pi) \models \phi$ to mean that formula ϕ is true at a precisification $\pi \in \Pi$ in $\mathcal{M}$. For a model $\mathcal{M} = \langle \Pi, S, \delta \rangle$, this relationship is defined by:

- $(\mathcal{M}, \pi) \models P$ if and only if $\pi \in \delta(P)$,
- $(\mathcal{M}, \pi) \models \neg\alpha$ if and only if $(\mathcal{M}, \pi) \not\models \alpha$,
- $(\mathcal{M}, \pi) \models \alpha \wedge \beta$ if and only if $(\mathcal{M}, \pi) \models \alpha$ and $(\mathcal{M}, \pi) \models \beta$,
- $(\mathcal{M}, \pi) \models \Box_{s_i}\alpha$ if and only if $(\mathcal{M}, \pi') \models \alpha$ for all $\pi' \in s_i$.
- $(\mathcal{M}, \pi) \models \Box_{*}\alpha$ if and only if $(\mathcal{M}, \pi') \models \alpha$ for all $\pi' \in \Pi$.

A formula ϕ is *valid* if it is true at every precisification of every model in $\mathfrak{M}_{\mathbb{S}_0}$. In this case we write $\models_{\mathbb{S}_0} \phi$.

The logic $\mathbb{S}_0$ enables one to formalise the content of statements such as. "Arg is definitely either a crag or a boulder. This map labels it a crag, but I would say it could be called either, although a boulder is different from a crag." With Ba and Ca meaning respectively 'Arg is a boulder' and 'Arg is a crag', one could write:

$$\Box_{*}(\mathrm{Ba} \vee \mathrm{Ca}) \;\wedge\; \Box_{map}\mathrm{Ca} \;\wedge\; \Diamond_{me}\mathrm{Ba} \wedge \Diamond_{me}\mathrm{Ca} \wedge \Box_{me}\neg(\mathrm{Ba} \wedge \mathrm{Ca})$$

3.2. Variable Reference Logic

We now generalise the standpoint semantics framework to define a first-order variable reference logic $\mathbb{V}_1$ that can represent predication and quantification.[4]

[4] In this presentation, we omit specifying the semantics for propositional connectives and standpoint operators and just give the semantics for predication and quantification in terms of truth conditions at a particular precisification in a given model. A more comprehensive semantics could be given by incorporating, with slight modification, the relevant specifications from the propositional semantics.

3.2.1. Syntax

The language of $\mathbb{V}_1$ is built from a vocabulary $\mathscr{V} = \langle \mathscr{K}, \mathscr{P}, \mathscr{Q}, \mathscr{N}, \mathscr{X} \rangle$, comprising the following symbols:

- $\mathscr{K} = \{\ldots, \mathsf{K}_i, \ldots\}$ is a set of count-noun symbols (sortals),
- $\mathscr{P} = \{\ldots, \mathsf{P}_i, \ldots\}$ is a set of individual property predicates,
- $\mathscr{Q} = \{\ldots, \mathsf{Q}_i, \ldots\}$ is a set of precise entity property predicates, (e.g. exact spatial properties)
- $\mathscr{N} = \{\ldots, \mathsf{n}_i, \ldots\}$ is a set of proper name symbols.
- $\mathscr{X} = \{\ldots, x_i, \ldots\}$ is a set of nominal variable symbols.

The symbols of $\mathscr{K}$, $\mathscr{P}$ and $\mathscr{Q}$ can all be applied as predicates, with the sortal symbols of $\mathscr{K}$ also being used to specify a range of quantification. Symbols of both $\mathscr{N}$ and $\mathscr{X}$ can both occur as arguments of predicates, although the variable symbols of $\mathscr{X}$ are only meaningful in the context of quantification.

The set S of formulae of $\mathbb{V}_1$ is the smallest set such that:

- $\{\alpha(\tau) \mid \alpha \in (\mathscr{K} \cup \mathscr{P} \cup \mathscr{Q}), \tau \in (\mathscr{N} \cup \mathscr{X})\} \subseteq S$ (contains all atomic predication formulae)
- $\{\neg\phi,\ (\phi \wedge \psi),\ \Box_{s_i}\phi,\ \Box_{*}\phi\} \subseteq S$ for every $\phi, \psi \in S$ (closed under connectives)
- $(\forall \mathsf{K} : x)[\phi] \in S$ for every $\mathsf{K} \in \mathscr{K}$ every $x \in \mathscr{X}$ and every $\phi \in S$ (includes quantified formulae)

3.2.2. Semantics

The semantics for variable reference logic $\mathbb{V}_1$ will be based on structures $\langle E, \Pi, \mathscr{V}, \delta \rangle$ where:

- E is the set of precise entities.
- Π is the set of precisifications.
- $\mathscr{V} = \langle \mathscr{K}, \mathscr{P}, \mathscr{Q}, \mathscr{N}, \mathscr{X} \rangle$ is the non-logical vocabulary, as specified above,
- $\delta = \langle \delta_{\mathscr{K}}, \delta_{\mathscr{P}}, \delta_{\mathscr{Q}}, \delta_{\mathscr{N}}, \delta_{\mathscr{X}} \rangle$ is a denotation function that can be considered as divided into components specifying the denotations for each type of non-logical symbol (see below).

To simplify the explanation of the semantics, I first define the semantic representation of a *indefinite individual.* (Here, and in the following, B^A denotes the set of all functions from domain set A into the range set B):

- $I = E^{\Pi}$ is the set of (indefinite) individuals, each of which is a mapping from the set of precisificaiton indices Π to the set of precise entities.

For each individual $i \in I$ and each precisification $\pi \in \Pi$, the value of $i(\pi)$ will be a precise version of individual i according to precisification π.

The denotation functions for all the non-logical vocabulary of the language as follows:

- $\delta_{\mathscr{K}} : \mathscr{K} \rightarrow \wp(I)^{\Pi}$ is a function mapping each sortal concept (count noun) in $\mathscr{K}$ to a function from precisifications to sets of indefinite individuals.

On the basis of $\delta_{\mathscr{K}}$ we define $I_\pi = \bigcup\{\delta_{\mathscr{K}}(\mathsf{K})(\pi) \mid \mathsf{K} \in \mathscr{K}\}$, the set of all individuals of any sort according to precisification π. We can now define:

- $\delta_{\mathscr{P}} : \mathscr{P} \to \wp(I)^\Pi$, such that, for all $\mathsf{P} \in \mathscr{P}$ we must have $\delta_{\mathscr{P}}(\mathsf{P})(\pi) \subseteq I_\pi$.
- $\delta_{\mathscr{Q}} : \mathscr{Q} \to \wp(E)$ (each precise predicate is associated with a set of precise entities)
- $\delta_{\mathscr{N}} : \mathscr{N} \to \wp(I)^\Pi$, subject to the condition that, for all $\mathsf{n} \in \mathscr{N}$ we must have $\delta_{\mathscr{N}}(\mathsf{n})(\pi) \in I_\pi$.
- $\delta_{\mathscr{X}} : \mathscr{X} \to \wp(I)$, (but the semantically relevant denotations of variables are determined by sortals occurring in quantifiers)

3.2.3. Interpretation of Reference and Predication

For a model $\mathscr{M} = \langle E, \Pi, \mathscr{V}, \delta \rangle$ and precisification $\pi \in \Pi$, the truth conditions for atomic predication formulae are as follows:

- $(\mathscr{M}, \pi) \models \mathsf{K}(\mathsf{n})$ if and only if $(\delta_{\mathscr{N}}(\mathsf{n}))(\pi) \in (\delta_{\mathscr{K}}(\mathsf{K}))(\pi)$
- $(\mathscr{M}, \pi) \models \mathsf{P}(\mathsf{n})$ if and only if $(\delta_{\mathscr{N}}(\mathsf{n}))(\pi) \in (\delta_{\mathscr{P}}(\mathsf{P}))(\pi)$
- $(\mathscr{M}, \pi) \models \mathsf{Q}(\mathsf{n})$ if and only if $((\delta_{\mathscr{N}}(\mathsf{n}))(\pi))(\pi) \in \delta_{\mathscr{Q}}(\mathsf{Q})$
- $(\mathscr{M}, \pi) \models \mathsf{K}(x)$ if and only if $\delta_{\mathscr{X}}(x) \in (\delta_{\mathscr{K}}(\mathsf{K}))(\pi)$
- $(\mathscr{M}, \pi) \models \mathsf{P}(x)$ if and only if $\delta_{\mathscr{X}}(x) \in (\delta_{\mathscr{P}}(\mathsf{P}))(\pi)$
- $(\mathscr{M}, \pi) \models \mathsf{Q}(x)$ if and only if $(\delta_{\mathscr{X}}(x))(\pi) \in \delta_{\mathscr{Q}}(\mathsf{Q})$

To make sense these specifications you need to be aware that evaluation of a symbol may require zero, one, or two levels of de-referencing in relation to the precisification index π. You should first note that the K and P predications are semantically equivalent. When applied to a name constant, n, both the constant and the predicate get evaluated with respect to a precisification, so that the name denotes a particular individual and the predicate denotes a set of such individuals. When the argument is a variable rather than a name constant, the variable directly denotes an individual without any need for evaluation relative to a precisification. In the case of (exact) Q predications, individuals need to be further evaluated relative to the precisification in order to obtain a precise entity, which can be tested for membership of the precise set denoted by property Q. So although Q predicates are not themselves subject to variation in relation to π the impose an extra level of variability in the interpretation their argument symbol.

It may seem curious that the same precisification index π is used both for mapping names (and predicates) to individuals (and sets of individuals), and also for mapping from individuals to precise entities. Thus the individual denoted by a name n in precisification π is $(\delta_{\mathscr{N}}(\mathsf{n}))(\pi)$ and according to π it also refers to the precise entity $((\delta_{\mathscr{N}}(\mathsf{n}))(\pi))(\pi)$. This slightly simplifies the specification and does not appear to place any constraint on the semantics.

3.2.4. Interpretation of Quantification

To facilitate specification of the semantics for quantification, I define a meta-level operation $\mathcal{M}^{(x_i \Rightarrow \xi)}$ on interpretation structures that enables us to replace the value of a variable with a new value. For $\mathcal{M} = \langle E, \Pi, \mathcal{V}, \delta \rangle$, with $\mathcal{V} = \langle \mathcal{K}, \mathcal{P}, \mathcal{Q}, \mathcal{N}, \mathcal{X} \rangle$, $\delta = \langle \delta_{\mathcal{K}}, \delta_{\mathcal{P}}, \delta_{\mathcal{Q}}, \delta_{\mathcal{N}}, \delta_{\mathcal{X}} \rangle$, and $\xi \in E^{\Pi}$ let $\mathcal{M}^{(x_i \Rightarrow \xi)}$ be the structure $\mathcal{M}' = \langle E, \Pi, \mathcal{V}, \delta' \rangle$, where $\delta' = \langle \delta_{\mathcal{K}}, \delta_{\mathcal{P}}, \delta_{\mathcal{Q}}, \delta_{\mathcal{N}}, \delta'_{\mathcal{X}} \rangle$ and $\delta'_{\mathcal{X}}(x_j) = \delta_{\mathcal{X}}(x_j)$ for every $x_j \neq x_i$ and $\delta'_{\mathcal{X}}(x_i) = \xi$.

Finally we can specify the interpretation of a quantified formula:

- $(\mathcal{M}, \pi) \models (\forall\, \mathsf{K} : x)[\phi(x)]$ if and only if $(\mathcal{M}^{(x_i \Rightarrow \xi)}, \pi) \models \phi(x)$ for all $\xi \in \delta_{\mathcal{K}}(\mathsf{K})(\pi)$
- $(\mathcal{M}, \pi) \models (\exists\, \mathsf{K} : x)[\phi(x)]$ if and only if $(\mathcal{M}^{(x_i \Rightarrow \xi)}, \pi) \models \phi(x)$ for some $\xi \in \delta_{\mathcal{K}}(\mathsf{K})(\pi)$

This is much the same as how one would specify quantification in a classical sorted logic. The universally quantified formula is true just in case its immediate sub-formula is true for all possible values of the variable, taken from the range of the sortal predicate.

3.2.5. What Can $\mathbb{V}_1$ Express?

So was the result worth all the work of defining that complicated semantics? Does it help us understand and represent semantic variability and its effect on reference, predication and quantification. Yes, I think so. Although the final definitions of the quantifiers seem simple, one needs to consider that significant work has been done by the denotation functions for the different kinds of symbol and the semantics given above for the different cases of predication. Their effect is to allow quantification to operate at an intermediate level in the interpretation of semantic variability of reference. The *individuation* of potential referents occurs prior to quantification by establishing individuals in relation to a given interpretation of sortal predicates. But these individuals can still be indeterminate in that they may correspond to many exact entities.

Consider the statement “There is definitely a mountain in Equador, that some say is on the equator and others say it is not”. This might be represented as:

$$\Box_{*}(\exists\, \mathsf{Mountain} : x)[\ \mathsf{InEquador}(x) \wedge \Diamond_{*}[\mathsf{OnEquator}(x) \wedge \Diamond_{*}[\neg \mathsf{OnEquator}(x)]\ ]$$

I believe that this kind of statement cannot be represented without using a semantics that can account for both *de dicto* and *de re* forms of vagueness. Its truth conditions require that there is something that is definitely a mountain and definitely in Equador but whose extent is ill defined such that it could reasonably be said either that it does or does not lie on the equator.

4. Co-Predication and Deep Polysemy of Sortal Concepts

I now turn to another issue involving polysemy and reference, that I admit not having properly considered during the whole time I was constructing the VRS logic described above: the issue of co-predication [28, 26, 29, 30, 31]. When I first became aware that people were studying this phenomenon as an issue in its own right, rather than just being a particular case of polysemy, I was somewhat surprised. And even after preliminary consideration, I assumed that it could be handled without special difficulty within the double level model of semantic indeterminacy provided by VRS. However, once I had got more deeply acquainted with the problem I came to realise that it does require special attention. This section will be a largely informal discussion of the topic.

4.1. The Problem of Co-Predication

A *sortal* concept is one that carries with it criteria for individuating and counting entities. Hence one might expect that sortals should at least be unambiguous when it comes to the fundamental criteria for being an instance or at least the general ontological category of the objects that can instantiate them. However this is not the case. Consider the sentence "There is a book by Olaf Stapledon on my bookshelf." With the simplification of treating being on my bookshelf as a simple predicate we would get the following 'naive' classical representation:

$$\exists x[\mathsf{Book}(x) \land \mathsf{By}(x, \mathsf{OS}) \land \mathsf{OnMyBookShelf}(x)]$$

A problem with this representation is that the predicate Book appears to be polysemous between the sense of being an informational artifact and being a physical object. And, moreover, whereas the authorship predicate, By, applies to the informational sense, the predicate OnMyBookShelf applies to the physical sense. It is this application to a single referent of two predicates that seem to describe very different kinds of object that is called co-predication. The issue is not just that Book has two senses but that the predications seem to imply the existence of an object that is an instance of both senses of Book.

One might expect that this problem could be avoided by adopting the view that books are a type of object that has both an information and a physical aspect and that different types of predicate that can be applied to books must be interpreted with respect to the aspect appropriate to the type of predicate (authorship applying to the informational aspect and physical properties to the physical aspect of book). But, unfortunately, the idea of multi-aspect objects can only account for very simple examples. Consider, the sentence: "There are two books by Olaf Stapledon on my bookshelf."

$$\begin{aligned}\exists x \exists y[x \neq y \land & \mathsf{Book}(x) \land \mathsf{Book}(y) \\ & \land \mathsf{By}(x, \mathsf{OS}) \land \mathsf{By}(y, \mathsf{OS}) \\ & \land \mathsf{OnMyBookShelf}(x) \land \mathsf{OnMyBookShelf}(y)]\end{aligned}$$

The problem is that determining how many books by Olaf Stapledon are on my self depends upon whether I count in terms of informational artifacts or physical volumes. I might have two copies of the same book, or two book titles contained within the same volume (or some more complex combination of volumes and contents). Thus, a claim regarding the number of books can only have definite meaning once I choose what kind of book object I wish to count. But accepting this leads to a recapitulation of the original problem. Once I choose between informational and physical books, I am no longer dealing with entities that can support both the informational property of the books content originating from a particular author and also the physical property of being a physical object located on a particular shelf.

Before considering some proposals made in the literature regarding how one might account for co-predication, let us first note that the VRS theory of predicates and vague objects given above (Section 3.2) fails to provide a straightforward explanation of typical co-predication examples. That semantics both allows vagueness of sortal predicates that categorise and individuate objects over which we may quantify and also allows these objects to have properties that are not fully determinate, such as a mountain, whose boundary is unclear. So maybe the complicated VRS semantics could make sense of the following formulation:

$$(\exists\,\mathsf{Book} : x)[\mathsf{ByOS}(x) \wedge \mathsf{OnMyBookShelf}(x)]$$

But for this to work, we would need to assume that the polysemy of book is within the semantic object denoted by x so that the two predicates can be evaluated with respect to different aspects of some multi-faceted book object that has both physical and informational aspects. However, the problem of individuation and counting still remains. Although VRS allows that Book can have different senses both in terms of individuation and in terms of exact entities, it is based on the assumption that any ambiguity in individuation criteria is resolved prior to determining the set of individuals over which the quantifier $(\exists\,\mathsf{Book} : x)$ will range. Hence, although we can have both indeterminate counting criteria and indeterminate individuals, we cannot have indeterminate individuals whose precise correlates satisfy different counting criteria. In the case of vague of geographic features, for example, one might say that ‘individuation must precede demarcation’.

4.2. Towards a Solution

Many approaches, some very clever and most rather complex, have been proposed for accounting for co-predication. I will not consider these in the current paper except to mention that some have proposed that co-predication involves mechanism for *coercion* or *inheritance* between different categories of entity that can be referred to by polysemous nouns that exhibit co-predication phenomena [28, 30], some have proposed special kinds of compound ontological categories (e.g. [26]) and some have proposed complex semantics of individuation (e.g. [29]). None of the proposals I have encountered so far seems completely satisfactory to me.

My view is that different individuation conditions must be associated with different precisifications of the count noun. I don’t think we can individuate objects unless we have restricted the interpretation of the term to the extent that it enables individuation. The VRS approach does allow different modes of individuation to be associated with different senses of a sortal concept, which could account for the radically different ontological types associated with certain count nouns such as ‘book’. But if this is the case, we still need to account for co-predication examples where it seems that individual objects exhibit multiple ontologically diverse aspects. Such possibilities would need to be specified in terms of type conversions that can occur due to context, and are often unambiguous due to corresponding entities of a different type being uniquely determined in many cases (e.g. in many particular situations a physical book instance uniquely determines and informational book and a reference to an informational book determines a unique physical book).

5. Conclusions and Further Work

The paper has explored the issue of multiple possible references of linguistic terms, that may arise due to vagueness of terms or differences in opinions on how they should be interpreted, and how such variability effects the semantics of reference, predication and quantification, as conceived within a denotational approach to semantics. I have proposed the framework of *variable reference semantics* to interpret a logical language in a way that can account for both *de dicto* vagueness in predicates and also *de re*, or, as I would prefer to call it *de sensu* variability in the 'objects' that are referred to.

The system presented is intended more as a proof of concept than a workable formal language. The semantics is rather complex but when formulating representations within the object language of the logic $\mathbb{V}_1$ much of this complexity is hidden under the bonnet. However, the utility of the system remains to be demonstrated beyond a few relatively simple examples. The main direction for further work would be to consider a much wider range of examples and evaluate the strengths and limitations of the proposed formalism. As for formal logic in general, I do not really envisage the system being used in its full blown form, but rather some aspects of the language and semantics might be used in a restricted form to support various kinds of application, such as, for example, querying of information systems.

The other direction in which I would like to extend the work is with further consideration of issues such as co-predication, which would test the limitations of the system in dealing with semantic phenomena related to semantic variability.

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