Developing GFO 2.0 Further – Initiating the Modules of Space and Material Objects

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Abstract.
Space and time are basic categories that account for fundamental assumptions of the mode of existence of those individuals that are said to be in space and time. For the General Formal Ontology (GFO), the basics of an ontology of space were axiomatized in 2016. Meanwhile, further development and transformation aim at providing a modularized system GFO 2.0. The present paper discusses continued research with several new results and sketches further extensions. The novelties include work on a module of material objects and its interrelation to the space module. Furthermore, a logical framework is outlined that supports axiomatic extensions of both modules. Such extensions serve as the formal basis for a systematic classification of material objects in reality.

Keywords.
Ontology of space, ontology of material objects, GFO

1. Introduction

The ontology GFO 2.0 [1] is under development in order to become the successor of the first version of the General Formal Ontology (GFO), released in 2006 [2]. The new version is organized into modules that are oriented at ontological regions and their corresponding levels of reality, as inspired by Poli [3] and Hartmann [4]. The modular architecture of GFO 2.0 fosters the development, maintenance and usage of the ontology. The overall theory of GFO is split into smaller fragments, each balancing high cohesion and low coupling. Modules are intended for specific, clear purposes and the interfaces between modules shall be defined explicitly. This will promote applications of the ontology in specific domains and/or tasks and it supports use cases of integration with other top-level ontologies. Once established, an organizing meta-ontology will integrate these modules by interrelating them by means of further ontological relations as well as by capturing logical connections between their axiomatizations.

The development of particular modules can already build on existing work. The present paper exemplifies such a case. It continues the work on GFO 2.0 based on the ontology of space [5] elaborated in the context of GFO earlier. In the course of our work, GFO-Space (for short) is turned into a module that additionally provides a logical framework for the systematic development of axiomatic extensions.

Another new module extends GFO 2.0 by the inclusion of material entities. Accordingly and content-wise, our primary contribution concerns the material ontological region of the world. Core sciences of the material region are the natural sciences, among others, physics, chemistry and biology. Various views are possible in accordance with the levels and sublevels associated with the domains of these

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sciences. In this paper, we assume the macro-physical level, as presented by Paul Needham [6]. This level is based on the phenomenology of those material objects which are assumed to have a middle size and which can be perceived by our senses. Hence, here we do not consider atoms, molecules, and other elementary particles. Spatiotemporal individuals on this level are classified into objects, processes, facts, and situations.

An ontology of space serves as a prerequisite for dealing with spatiotemporal individuals, thus a corresponding module is desired. GFO's ontology of space [5], (as well as that of time) is heavily inspired by Franz Brentano [7], whose ideas yield deep insights into the nature of space. Moreover, the theory of boundaries provides an appropriate understanding of dimension in space. A similar idea of dimension was developed by Karl Menger [8]. The systematic investigation and exploration of Brentano's ideas on space, time and material objects began about twenty years ago by work of Roderick M. Chisholm [9] as well as Barry Smith and Achille C. Varzi, cf. e.g. [10]. The present paper takes up this research and makes a further step towards an ontology of material objects in the spirit of Brentano.

The paper proceeds as follows. Additional information on the modular architecture of GFO 2.0, into which the modules under consideration will be embedded, is provided in section 2. Section 3 describes related work that is relevant for developing our theory. The main section 4 surveys the ontology of space briefly and develops a substantial outline of an ontology of material, middle-sized objects, including an axiomatization. Section 5 comprises additional analyses for extensions in connection with material objects, where the morphology of pure space entities or newly introduced distance relations play a major role. Final remarks conclude the paper in section 6.

2. Modular Architecture of GFO 2.0

Monolithic design of an ontology, similarly as in the case of other types of software components, imposes problems with its complexity that hinder the usability, maintenance and scalability [11]. In case of importing a monolithic ontology, which is one scenario of utilizing top-level ontologies, this can lead to unexpected inferences. As observed in [12], this can be one of the reasons of limited or no applications in areas such as Linked Data (LD). We believe that the non-monolithic, modular approach opens the doors for top-level applications in LD and the Semantic Web. In contrast, lack of modularization hinders top-level ontology application, their reuse, and mappings between them.

We have experienced issues with monolithic design when constructing and working with the first version of the GFO, since developing, managing and maintaining a theory comprising a few hundreds of FOL formulas as a monolithic artifact is hardly feasible. Therefore, GFO 2.0 is intended not as a monolith but as a modern modularized framework. Yet, not only modularization of a software component is a difficult art, as experiences over the decades of software engineering show, but additionally the discussion on the principles and metrics of ontology modularization [13] and category systems in general [14] is still open.

For building GFO 2.0 we understand a module as a theory or conceptual model of some generic area. With the objective of reducing the overall design complexity, we follow a well-established rule of modular design adapted in the software engineering area of balancing low coupling and high cohesion. That is minimizing the internal and external complexity of the modules. Low coupling fosters the ease of using modules together, by reducing the complexity of connecting the modules to other modules, reducing the dependency between them and the degree of connections, as well as by making them explicit and easy for understanding without the insight into the internals of the modules. This increases flexibility, which supports interchangeability of modules, which is a key factor for a multi-ontology "democratic" environment. High cohesion, in turn, is going to be achieved by modules encapsulating a clear purpose and having clear modeling responsibility. In case of more complex modeling tasks, modules are split further into submodules.

GF0 2.0 modules are to be understood as theories (conceptually) and should not be identified with formalization/serialization artefacts in specific formal and semi-formal languages such as FOL, OWL or UML, which come with them. Therefore, for each single module more than one serialization and formalization is to be developed, depending on the current needs. Depending on the availability of those artifacts (and likewise others, such as guidelines and tutorials) the GFO 2.0 modules undergo versioning and maturity levels.

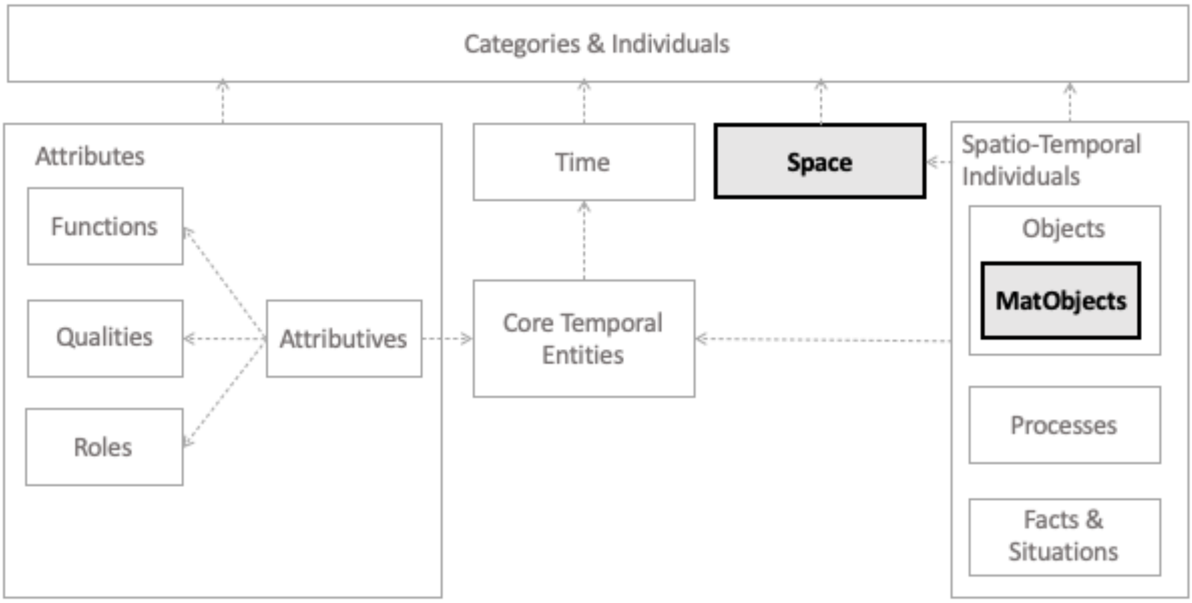

Figure 1. Overview of the main modules of GFO 2.0 and their dependencies. The modules of major relevance for this paper are highlighted.

Space and *Material Objects* are the two modules in the focus of this paper. We are working and report on their full provision currently. They are highlighted in Figure 1, which outlines the entire current modular architecture. (Some comments on what is not covered yet can be found in section 6.)

It is the responsibility of the Space module to represent all notions of space as such, independently of any entities that occupy it. The notions and axioms discussed in [5] (and briefly summarized in section 4.1 below) form the starting point for the ongoing provision of machine-processable formalizations and serializations. In particular, an OWL version of the Space module is under development.

The module Spatio-Temporal Individuals is responsible for covering all types of entities that occupy space and time, i.e., that are located in space and time. Clearly, the theory depicting spatio-temporal individuals relies on the notions of Space and therefore there is a dependency between the two modules. Additionally, the module depends on the notions of Individuals and Time as it also deals with individuals in time.

The Objects module's responsibility is to handle specific types of spatio-temporal individuals, in particular material objects. Accordingly, the module Material Objects is a major sub module of Objects and as such of Spatio-Temporal Individuals. In tandem with the Space module, an OWL serialization of the material presented in section 4.2 is work in progress.

3. Related Work

The introduction mentions that the ontology of GFO-Space is based on Brentano's theory of space, time, and continuum [7]. Here, various phases of later research can be distinguished. The first phase of research on Brentano's theories was established by R. M. Chisholm [9] and further developed by B. Smith in [10] and, together with A. C. Varzi, in [15]. Further investigations of relevance for the current paper can be found in R Casati's and A Varzi's "Parts and Places" [16], but also in their book on holes [17].

Some criticism of the work [15] is presented in [18]. Subsequent to this section, several new aspects of the theory of boundaries are addressed. These include the idea of an integration of different levels of granularity and abstraction and the need for a clear distinction between space boundaries and material boundaries. Only space boundaries may *coincide* (see section 4.1), whereas material boundaries may *touch* each other (see section 4.3), but cannot coincide. Furthermore, continuous and discontinuous material boundaries are introduced, generalizing the distinction between fiat and bona fide material boundaries.

Aameri, Grüninger and Ru have developed ontologies for the physical world [19, 20], following a very systematic modular approach. In our work, we aim at a model-theoretic framework that is yet a bit broader, such that the theories in [19], notably CardWorld, BoxWorld, and PolyWorld can be reconstructed. This is outlined in sections 5.1 and 5.2 below, the ontologies of which adopt balls and tetrahedrons and hence allow for the construction of a broader class of material objects. Classical results regarding balls originate from Alfred Tarski [21] and can likewise be utilized.

Finally, inquiring for a metric for Brentano space is a long-standing issue. Which metric is adequate for which purpose may be even more appropriate to ask, e.g., which metric is suitable to describe the visual space. There is a broad literature on these matters, including [22–24], with partly contradictory proposals. This influences our own views, touched upon in section 5.3.

4. Space and Material Objects in GFO

4.1. Overview on GFO-Space

Space is a very basic notion that plays a fundamental role in the material region of the world. On the one hand, space is generated and determined by material objects and the relations that hold between them. This *phenomenal space* of material objects appears to the mind, such that we claim its subject dependence. On the other hand, any material object has a subject-independent disposition, called *extension space*, which unfolds in the mind/subject as phenomenal space. Brentano's ideas on space, time and the continuum [7] have inspired the formalization of phenomenal space for GFO, expounded in [5]. This theory is well-established, but continues to be analyzed and refined.[2]

GFO-Space is based on four primitives: the category of *space regions*, the relations of being a *spatial part* and being a *spatial boundary*, as well as the relation of *spatial coincidence*. The (intended) universe of discourse for GFO-Space is the category of space entities, which divides into four pairwise disjoint categories, namely space regions, surface regions, line regions and point regions. They correspond to three-, two-, one-, and zero-dimensional space entities, respectively. Higher-dimensional entities cannot be reduced to lower-dimensional ones.[3]

A key notion for the next section is the relation of coincidence. If two distinct boundaries coincide, (1) they are of equal dimension, (2) intuitively speaking, they are "congruent", and (3) there is no distance between them. For instance, imagine a space cube and consider two halves of it, then each half has its own surface (where it "touches" the other half) and those two surfaces coincide, having no space between themselves. Spatial coincidence is an equivalence relation on equidimensional spatial boundaries.

Overall, GFO-Space comprises of 30 axioms in [5], covering mainly mereology, connectedness, coincidence and existence (e.g., of boundaries). The formulas involve the four primitives above and more than 30 defined categories and relations.

4.2. Material Entities

Material entities are those concrete entities that belong to the material ontological region of the world. What they have in common is that they consist of matter, that they have a mass, and that they occupy space. Natural sciences are associated to this ontological region, including, among others, physics, chemistry, and biology. The investigation of this region is conducted by various levels of *granularity and abstraction*. A heart, for example, exhibits various granularities and levels of abstraction[4]. At one level, the electrophysical properties of the heart can be described; such properties may then occur on an abstraction level of pathophysiology as ischemia of a certain heart artery. An electrophysical

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[2] For example, the decidability of the structure of its complete extensions is an open problem.
[3] But cf. the theorems on identity principles in [5, section 4.6].
[4] We use the notions of granularity and level of abstraction in an informal manner, whereas a detailed investigation is a research field of its own, cf. [3, 4].

measurement resulting in a "shape of the QRS-loop" and "ischemia of a heart artery" are under the current view expressions at different levels of abstraction.

Subsequently we expound various conditions and notions that serve as a basis for the explication of formal axioms.[5] An initial, work-in-progress version of the axioms is expounded in section 4.3. Having seen them might retrospectively enlighten this present section further.

4.2.1. General Conditions and Informal Notions

Firstly, material entities can be divided into solid bodies, fluids, and gaseous entities. We use the term *material object* for solid material entities.[6] We stipulate that in our considered domain any material entity is a fluid, a gas, or a material object (i.e., a solid). Further, we assume that any material entity has mass and density. One aspect of the size of a material object is reflected by its volume. As usual, the density of a material object Obj is defined by the formula d = m/V, where m is the mass and V the volume of Obj; consequently, its volume is determined by V = m/d. Two material objects are volume-equivalent if they possess the same volume.

Any material entity consists of *stuff*, informally understood as matter, material, or as 'that which it is made up of'. Stuff is a category the instances of which are amounts or portions of stuff. A portion of stuff behaves similarly to an individual endurant/continuant in that it persists through time. However, it has an incomplete mode of existence. For example, in reality a portion of solid stuff always occurs combined with a form/shape. This relation between stuff and form can be analyzed differently. Let us consider an amount of clay. One approach is based on the idea that there are a category of forms and a category of stuff. Then the relation of inherence can connect instances of both categories. In this case we may say that the form of a vase (or of a statue) inheres in this portion of clay.

Another interpretation assumes that any portion of stuff always occurs in reality in unity with a form – even a lump of clay has a form. There cannot be a portion of stuff without form, more precisely, every portion of stuff is part of a portion of stuff (either another or itself) that has a form. Then the creation of a new form is understood as a transformation into another form. But where does the shape come from? We believe that the category of forms/shapes has two sources: one is based on a certain system of elementary forms that can be found in real objects and that are abstracted from them. Complex forms can then be constructed from elementary ones. This approach is similar to the theory of Geons, as developed by Irving Biederman [25]. The other source of forms or shapes is the platonic world of mathematics.[7]

4.2.2. Material Objects

Material objects persist through time and may change their properties from time point to time point. This behavior leads to certain conceptual difficulties. What is the space region, occupied by a material object, when this object moves? What are the material parts of a material object when this object loses parts through its lifetime? There is an approach to equip basic relations such as *part_of*(x,y) and *instance_of*(x,y) with a time-argument [28]. It turns out that these temporalized relations are unsatisfactory, and we believe that a revision of this approach is needed. GFO adopts the view that a material object, as it persists through time, at every time point of its lifetime exhibits a uniquely determined entity, which is wholly present at that time point. Such a snapshot is called a presential in GFO. A presential of a material object is called a material structure.

Albeit it is a major restriction for beginning the axiomatization, in the current paper we avoid the problems mentioned above by presupposing a fixed time point and considering all material objects at this time point. This notwithstanding, the axioms are formulated for material objects, as we speculate that a major part remains valid once that more temporal aspects and changes are taken into account, too.

[5] Time plays a minor role here. The phenomenology of material processes is studied in another paper.

[6] We are aware that solid bodies form the simplest case of material entities. We pursue the strategy to begin our ontological investigations with the simple cases, which already allow for the explication of many important ontological distinctions.

[7] Mathematics belongs to the platonic world of ideal entities. This platonic ontological region is considered to be independent of the material world and of the mind. How this world relates to the material world is a permanent debate in philosophy, cf. e.g. [26, 27].

Material objects – in context of the present paper – are not only understood as presentials of solid material bodies. We stipulate that any material object is embedded into an environment and that it exhibits a smooth boundary that demarcates it from the environment. Examples are biological organisms such as a cat or a human being, plants, or inorganic solid entities, e.g. a stone or a car. These material objects have a closed material boundary, called the outer closed boundary. Other material solid objects are not closed, for example a mountain. One may argue that a part of a mountain has a clear material boundary, though there is another part connected with the earth without a uniquely determined boundary. We call such material objects non-closed, but apart from that, we exclude them from our current investigations.

4.2.3. Boundaries of Material Objects

The ontology of (pure) space in GFO is established in [5], influenced by theories of Franz Brentano [7], cf. section 4.1 above. We hold that a material object is a three-dimensional entity, expressed by the condition that the space occupied by a material object is a space region. The boundary that demarcates a material object from its environment belongs to that object, specified by the relation mbd(x,y) := x is material boundary of the material object y. Moreover, material parts of a material object are three-dimensional entities; hence, the boundary of a material object is not a material part of this object.

Further, we assume that the boundary of a material object is a cognitive construction. The phenomenon of material boundaries occurs on the abstraction level of middle-sized objects. If we look at a finer granularity then a material body occurs as a structure of molecules and atoms with empty space between them. Our eyes do not see the atoms and molecules but perceive this object as a whole with a boundary. On the other hand, behind the impression of a whole entity with a boundary there is a physical entity, described by an arrangement of particles and forces holding them together. This physical system occurs to the mind as a middle-sized material object with a boundary. Hence, there is a law-like correspondence between the independent physical entity and the perception. The existence of such a correspondence is claimed by the principle of integrative realism, cf. e.g. [1, section 2]. The paradoxical statement in [15, p. 406] that material objects cannot be in contact results from mixing two different granularity and abstraction levels.

4.2.4. Parts of Material Objects

As stated, we assume that parts of material objects, called material parts, are three-dimensional entities. Most of the notions used for pure space are adopted for material objects. These notions include material parts, inner material parts, tangential material parts, and material hyper-parts. The outer material boundary of a material object is a material surface; an inner material part has no contact to the outer boundary. Any part of a material object defines new material boundaries that are assumed to be smooth, as well. We distinguish two-dimensional material boundaries (surfaces), 1-dimensional material boundaries (lines), and 0-dimensional boundaries (vertex/material points). According to Brentano's basic law, there are dependencies between the mentioned parts: a material surface depends on a three-dimensional material object, a material line is always a boundary of a material surface, and a material vertex is a material boundary of a material line.

4.3. Towards an Axiomatization of Material Entities

Based on the analyses of the previous section 4.2 we introduce a selection of axioms that form the backbone of a formalized ontology for material objects.[8] The axiomatization makes use of the space module, as well. The most important relation connecting material entities to space is the occupation relation.

[8] We emphasize that these axioms are the beginning of the development of a full system of axioms for the whole domain of middle-sized material objects in the spirit of Paul Needham's work [6]. Further, we believe that this investigation is of use for qualitative process theory and artificial intelligence [29].

Signature selected from the module of space (B(3) for short, cf. [5]).

Conn(x)	(x is a connected space entity; renamed from C(x) in [5])
SReg(x)	(x is a space region)
sb(x,y)	(x is a spatial boundary of y)
scoinc(x,y)	(x and y are spatially coincident)
spart(x,y)	(x is a spatial part of y).

Signature for material entities.

Fluid(x)	(x is a fluid entity)
Gas(x)	(x is a gaseous entity)
MatE(x)	(x is a material entity)
ML(x)	(x is a material line)
MOb(x)	(x is a solid material entity with smooth boundary)
MS(x)	(x is a material surface)
MStr(x)	(x is a material structure)
MVert(x)	(x is a material vertex/point)
ObSit(x)	(x is an object-situation)
Stuff(x)	(x is an amount of material stuff)
consists_of(x,y)	(material object x consists of the stuff y)
contained_in(x,y)	(material object x is contained in material object y)
environ(x,y)	(x is an environment of y)
has_density(x,y)	(x has a density quality y)
has_mass(x,y)	(x has a mass quality y)
lifetime(x,y)	(x is the life time of the material object y)
maxbd(x,y)	(x is a maximal material boundary of y)
mbd(x,y)	(x is a material boundary of y).
mpart(x,y)	(x is a material part of y)
natmbd(x,y)	(x is a natural material boundary of y).
occ(x,y)	(material object x occupies space region y)
occbd(x,y)	(material boundary x occupies spatial boundary y)
touch(x,y)	(material boundaries x and y touch (or are in contact with) each other)

At the current stage of development, we treat all relations as primitives and capture them axiomatically. However, in future versions of the theory we expect that a few of them will be defined. For example, as far as we can see, touching of material boundaries should be definable by the condition that their occupied space boundaries coincide. Axiom M14 below captures only a part of that. Similarly, maximality of boundaries with respect to the material part-of relation could be defined. Yet again, at the moment all relations are characterized axiomatically.

The following axioms are grouped with respect to certain views.

(1) *Axioms on Brentano space.*

We assume the axioms A1–A30 in [5] for the domain of space. More precisely and for proper integration with the MatObj module, the Space axioms must be transformed into relativized editions, e.g., constraining their quantification to the domain of space.

(2) *General axioms on material entities.*

M1. $\forall x$ (MOb(x) $\lor$ Fluid(x) $\lor$ Gas(x) $\rightarrow$ MatE(x))

M2. $\forall x$ (MatE(x) $\rightarrow$ $\exists y$ (Stuff(y) $\land$ consists_of(x,y)))

M3. $\forall x$ (MatE(x) $\rightarrow$ $\exists yz$ (has_mass(x,y) $\land$ has_density(x,z)))

(3) *Basic axioms on material objects and their relation to space*

M4. $\forall x$ (MOb(x) $\rightarrow$ $\exists y$ (SReg(y) $\land$ occ(x, y) $\land$ Conn(y)))
Every material object occupies a connected space region. A stronger notion of connectedness, called material connectedness, requires forces that hold the material object together.

M5. $\forall x$ (MOb(x) $\rightarrow$ $\exists y$ (mbd(y,x)))
Note that the boundary that must exist is not assumed to be maximal.

M6. $\forall xyz$ (MOb(x) $\land$ mbd(y,x) $\land$ mpart(z,y) $\rightarrow$ mbd(z,x))
Every material part of the boundary of a material object is itself a material boundary.

M7. $\forall xyzu$ (MOb(x) $\land$ occ(x,y) $\land$ mbd(z,x) $\rightarrow$ $\exists! u$ (sb(u,y) $\land$ occ(z,u)))
If a material object occupies a space region, then any boundary of the material object occupies a uniquely determined boundary of the occupied space region.

M8. $\forall x$ (MOb(x) $\rightarrow$ $\exists y$ (environ(y,x)))
For every material object there exists an environment.

M9. $\forall xy$ (MOb(x) $\land$ environ(y,x) $\rightarrow$ ObSit(y) $\land$ contained_in(x,y))
The environment of a material object is an object-situation that contains this material object. This object-situation is not uniquely determined and can be arbitrarily extended. An object-situation can be understood as a complex material entity that may contain material entities of different states of aggregation (solid, gases, fluids). The environment of a fish, for example, may contain a part of a river with the water, water plants and stones at the river's ground.

(4) *Axioms on boundaries and material parts*

M10. $\forall x$ (MOb(x) $\rightarrow$ $\exists y$ (maxbd(y,x)))
Every material object has a maximal outer material boundary.

M11. $\forall xy$ (MOb(x) $\land$ maxbd(y,x) $\rightarrow$ Conn(y))
The maximal (outer) boundary of a material object is connected. Thus, there are no holes in the object.[9]

M12. $\forall xy$ (MOb(x) $\land$ mbd(y,x) $\rightarrow$ natmbd(y,x))
Any boundary of a material object is a natural material boundary.

M13. $\forall xy$ (mpart (y,x) $\land$ occ(x,z) $\rightarrow$ $\exists u$ (spart(u,z) $\land$ occ(y,u)))
Any material part of a material object occupies a spatial part of the space region occupied by the material object. We do not admit an axiom saying that for every spatial part of the occupied space region there exists a material part that occupies exactly this spatial part.

M14. $\forall xy$ (touch(x, y) $\land$ occbd(x, u) $\land$ occbd(y, v) $\rightarrow$ scoinc(u, v))
If two material boundaries touch, their occupied space boundaries coincide.

(5) *Axioms about the environment of a material object*

M15. $\forall xy$ (MOb(x) $\land$ environ(y,x) $\rightarrow$ $\neg\exists z$ (MatE(z) $\land$ mpart(z,x) $\land$ mpart(z,y)))
A material object has no common material part with an environment.

[9] This is a simplifying condition. The investigation of holes is a research field of its own. It is not addressed in this paper.

M16. $\forall xyz\ (MOb(x) \land environ(z,x) \land mbd(y,x) \rightarrow \exists uv\ (mpart(u, z) \land mbd(v,u) \land touch(y,v)))$
Every part of the outer material boundary of a material object is in contact with the material boundary of a part of an environment.

For the description of a material object, its inner boundaries are also relevant. These occur if we consider material parts of a material object. The boundaries of material parts of a material object are two-dimensional; they are material surfaces. A material line can be in contact with more than one material line; this is not possible for material surfaces. For any material surface S there exists at most one different surface T, being in contact with S. A material boundary x is said to be *discontinuous* if there exists a material boundary y, such that x and y are in contact, x is the boundary of the material entity u, y is the boundary of a material entity v, u and v have no common parts and u and v can be distinguished by different properties. If we take a material part of a material object, then this part may possess discontinuous boundaries. Let a ball B, for example, consist of two half balls, one made of gold and the other of iron. Then along the inner surfaces of these half balls, there occur two discontinuous two-dimensional inner material boundaries. Analogously, we introduce continuous boundaries that are not discontinuous; hence for a continuous (two-dimensional) boundary x this boundary cannot be distinguished by properties from any boundary y being in contact with x. Bona fide boundaries [15] are discontinuous, whereas fiat boundaries are continuous.[10]

(6) *Brentano's dependency axioms.*

M17. $\forall x\ (MVert(x) \rightarrow \exists y\ (ML(y) \land mbd(x,y)))$
For every material vertex exists a material line having this vertex as boundary.

M18. $\forall x\ (ML(x) \rightarrow \exists y\ (MS(y) \land mbd(x,y)))$
For every material line there exists a material surface having this line as a boundary.

M19. $\forall x\ (MS(x) \rightarrow \exists yz\ (MOb(y) \land mpart(z,y) \land mbd(x,z)))$
For very material surface there exists a material part of a material object having this surface as material boundary.

(7) *Mereology axioms.*

M20. $\forall x\ (MOb(x) \rightarrow mpart(x.x))$

M21. $\forall xy\ (MOb(x) \land MOb(y) \land mpart(x,y) \land mpart(y,x) \rightarrow x = y)$

M22. $\forall xyz\ (MOb(x) \land MOb(y) \land MOb(z) \land mpart(x,y) \land mpart(y,z) \rightarrow mpart(x,z))$

(M23-M26: Analogous axioms for material surfaces, and material lines.)

M27. $\forall x\ (MVert(x) \rightarrow \neg \exists y\ (mpart(y,x) \land x \neq y))$
A material vertex has no proper parts. Hence, vertices are the atoms of the mereology.

5. Analyses for Formal Ontology Extensions

In this section, we introduce and initially discuss some extensions that we plan to develop in detail in future research.

5.1. Classification of Space Entities by Axiomatic Extensions

An important extension of the Brentano space considers classes of space regions with a certain morphology. The ontology of space as presented in [5] does not contain a classification of pure space entities from this point of view. A space region has a form, which is associated with the boundary of

[10] The notion of a discontinuous and a continuous boundary depends on the admitted distinguishing properties, as shown in [18].

the region. Which properties of shapes can be defined using the basic signature Σ(0) of the space module B(3) (SReg(x), sb(x,y), scoinc(x,y), spart(x,y))? We must consider the shapes of regions, of surfaces and of lines. Surfaces (Surf(x)), lines (Lin(x)) and points (Pt(x)) are introduced by explicit definitions [5, p. 61]. We call such an extension Σ(1) a definitional extension of Σ(0).

To achieve a complete description of the shape of a space entity we must take into consideration the curvature of lines or of surfaces. This can be done only by using measurements and introducing metric notions, based on the concept of real numbers. We call the latter analytical properties, in contrast to the mereotopological properties, which are defined on the basic signature Σ(0) alone. A basic mereotopological classification of space entities in B(3) is presented in [5]: it includes space regions, connected space regions, with different types of connection, and analogously for surfaces, lines and points. In this framework a curved line cannot be distinguished from a straight line.

The mereotopological representation and formalization of space entities has the advantage that this kind of knowledge representation can be used in reasoning, which is more difficult for the analytical representations, as indicated in [19]. We want to introduce certain standard space-entities that capture mereotopological properties axiomatically in the spirit of Hilbert's method [30].

For this purpose the signature Σ(1) must be further extended to a signature Σ(2). This signature includes the following additional predicates:

PSurf(x) := x is a planar surface (of curvature zero), Egde(x) := x is a straight line with two endpoints, Vert(x) := is a vertex, Cube(x) := x is a cube, Ball(x) := x is a ball, Ellipsoid(x) := x is an Ellipsoid, TetraH(x):= x is a tetrahedron. Brentano's dependency axioms are preserved accordingly, hence an (elementary) planar surface is one of the surfaces of a cube and an (elementary) edge is a part of the boundary of an (elementary) surface. The construction of more complex space entities from the elementary ones is achieved by using the coincidence relation between spatial boundaries, i.e., space entities are glued together or joined by coincidence. There is a manifold of different kinds of joins between space entities, which are classified in [5] by different kinds of connectedness. From the axiomatically described basic space entities complex space entities can be constructed. Here the question arises about the expressive power of our framework. Will all relevant space entities be covered by the intended theory Th(Σ(2))[11]? We believe that this is – under certain additional conditions – possible and refer to the result of the triangulation of all three-dimensional manifolds. Moreover, a deeper exploration of the outlined framework suggests that the space ontologies related to CardWorld, BoxWorld and PolyWorld [19, 20] can be interpreted in our theory.

Finally, we collect some axioms on balls and ellipsoids.

B1 ∀x (Ellipsoid(x) → Conn(x))

B2. ∀x (Ball(x) → Ellipsoid(x))

B3. ∀x (Ball(x) → SReg(x))
Every ball is a spatial region.

B4: ∀x (SReg(x) → ∃y (Ball(y) ∧ spart(x,y))
Every region is a spatial part of a ball.

B5: ∀xy (Ball(x) ∧ maxbd(y,x) → ¬ ∃z (sb(z,y)))
The greatest boundary of a ball has no boundary (though any proper spatial part of the greatest boundary of a ball has a boundary.

B6 ∀xy (Ball(x) ∧ Ball(y) → Conn(x-y))
The mereological complement of spatial part of a ball being an intersection with another ball is connected.

B7. ∀xy (Ellipsoid(x) ∧ Ellipsoid(y) ∧ ¬ Conn(x-y) → ¬ Ball(x) ∨ ¬ Ball(y))
(This is essentially the contraposition of axiom B6.)

[11] The theory Th(Σ(2)) contains at present only few axioms. A more complete axiomatization is work in progress.

5.2. Mereotopology of Material Objects

Analogously as for space entities we consider basic properties for material objects. For this purpose we introduce a dual signature $\Delta(0)$ = (MatOb(x), mbd(x,y), touch(x,y), mpart(x,y)) for material objects, and from this we get MPt(x) := x is a material point, MLin(x) := x is a material line, MSurf(x) := x is a material surface. Similar as for $\Sigma(1)$ the signature $\Delta(1)$ presents a definitional extension of $\Delta(0)$. In analogy to $\Sigma(2)$ we introduce for $\Delta(2)$ certain basic types of material objects, namely; MPSurf(x) := material planar surface, MEdge(x) := material edge, MVert(x) := material vertex, and MCube(x) := material cubes, MBall(x) := material balls, and MTetrahy(x) := material tetrahedron.

By these basic predicates, representing certain standard material objects, more complex material objects can be constructed. In case of pure space regions, the only joining relation is the coincidence relation between pure space boundaries. This is not true for material boundaries. The connection between material objects is realized by an attachment relation that differs essentially from the touch-relation. If two material boundaries touch, then the corresponding occupied space boundaries coincide, though, touching does not imply attachment; both notions are different. The attachment of different material objects (and their material boundaries) can be realized by various means.

5.3. Measuring Distance

We assume that the phenomenal space can be introspectively accessed without any metrics. The phenomenal space exhibits basic features, as continuity (i.e. there are no space atoms), the existence of boundaries as dependent entities, and the coincidence of space boundaries. The notion of dimension can be inductively defined by using the notion of boundary and the space entity being bound; this approach was proposed by Menger [8] and Poincare [31]. We stipulate that the phenomenal space includes space entities of the dimensions 0, 1, 2, 3. Furthermore, every space entity of dimension greater 0 can be extended along the same dimension. Metrics become relevant if we want to measure material objects, the size, the form, volume etc. Another important aspect, relevant for the metric is the dimension of the space. We adopt the condition that visual space, as an aspect of the phenomenal space, is three-dimensional. The three-dimensionality of visual space is defended by philosophers or mathematicians, such as Poincare [31] and Luneburg [22]. Another group assumes visual space to be two-dimensional, e.g. Helmholtz [32]. The metrics introduced for the phenomenal space should be compatible with the metrics of visual space, and other sense data spaces. Furthermore, we must admit that different points may have distance zero. This is the case if two boundaries are in contact. We conclude that an appropriate metric for the visual space, and hence for the phenomenal space, does not satisfy one of the conditions of a metrics. Hence, if we introduce a notion of distance between space entities then the boundary-based theories lead to pseudo-metric spaces: there are distinct point of distance zero. If we factorize such a pseudo-metric space with respect to classes of coinciding points (boundaries), then we get a metric, and we may ask which type of metric we should assume. One of the adopted metrics should be Euclidian. If the phenomenal space mirrors the features of the visual space, we may ask whether experimental investigations provide information about the metrics of visual space. Here exist competing approaches and claims. In [22], for example, it is claimed that visual space has a hyperbolic metric, whereas French [24] defends the idea that the visual space's metric is spherical. In contrast, Angell [23] states that the visual field is a non-Euclidean two-dimensional, elliptic geometry. Proponents of the claim that visual space possesses a Euclidean metric include Kant [33] and Strawson [34]. The investigations of the metrics of sense data spaces have not achieved a final stage, it is an active research area. The construction of various metrics is a topic of future research. We conjecture that the metrics related to our motor experience and tactile sense data is Euclidean whereas the metric of our visual sense data is non-Euclidean. The efficient transformation between these metrics is a basic assumption for a human being to react adequately to the real environment.

6. Conclusions and Future Work

In this paper we report on continued work on the ontology of space and of material entities in the spirit of Franz Brentano's theories, cf. [7]. The work is embedded in the context of establishing GFO 2.0 by following a modular approach. After a brief collection of related work and key aspects of the existing ontology of space [5], we focus on developing a module for material objects. Material objects are extended with much more detail in the ontology and the backbone of the module of material objects is developed. We discuss a novel view on bona fide and fiat boundaries and newly introduce the environment of a material object as an object-situation. The notions of level of granularity and level of abstraction are employed to put bona fide boundaries in contact, solving a problem in [15]. Furthermore, we discuss various conceptual extensions of GFO-Space that pertain to the problem of adding one or several metrics, and to a model-theoretic framework for studying classes of space regions, such as balls. The following items are on our future research agenda:

1. Development of a model theory for the space module and specification of the standard models of the theory.
2. Solution of the decision problem for theory B(3). We conjecture that the theory of space is undecidable (in contrast to GFO-Time [35]).
3. Investigation and introduction of various metrics:
   3.1. A Euclidean metric to bridge to classical mathematical spaces and theories.
   3.2. Various metrics with respect to the cognition of visual perception.
4. Inclusion of the theory of Biedermann's Geons [25] into the class of space regions and of material objects.
5. Investigation of the relation between shapes as ideal entities of mathematics and their realization in material objects. Development of a mereotopology of shapes of material objects.

For a final note, let us return briefly to the modular architecture of GFO 2.0 outlined in section 2 and more elaborately in [1]. Beyond the currently planned architecture (cf. Figure 1), we remark that the conceptual development of GFO has been continuously pursued during all years of its development. Accordingly, there are areas of GFO (and/or related to its application in various projects (see citations in [1, section 4])) that are partially conceptualized, but not captured by planned modules yet. For example, [36] presents a novel analysis and ontology of data. This is still closely related to attributives in GFO (and thus the corresponding module). Further examples can be gained from ontological regions and levels not yet considered. Future modules will thus need to cover aspects of the mental-psychological and the social regions, for instance, which cannot be reduced to the material region, also in accordance with integrative realism [1, section 2]. For those, the relations to the modules foreseen in Figure 1 are less clear and require significantly more effort.

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