Physics Informed Deep Learning for Well Test Analysis

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Abstract

Well Test Analysis is a section of reservoir engineering that best describes the reservoir characteristics with principles of fluid flow in porous media using pressure transient analysis. The transient pressure distribution for fluid flowing through the wellbore, across the porous reservoir model, at a constant terminal flow rate can be determined by solving the partial differential equation- diffusivity equation, along with the set of boundary conditions that define the reservoir model. Since the diffusivity equation has a non-linear quadratic term, it is either solved analytically by ignoring the quadratic term and thus compromising the model accuracy or solved using numerical approaches that is complex and time-consuming. This study provides an alternative and simpler approach to determine the pressure distribution using the Neural Networks method. This method could be applied to any type of reservoir that has a defined diffusivity equation and boundary conditions to predict the pressure distribution with good accuracy. To validate this approach and demonstrate the accuracy of the neural network with a greater level of confidence, for the purpose of this study, we have chosen to validate against analytical solution as it could be applied to all types of reservoir models in generic form.

Typical neural network-based approaches, however, were not yielding good results for Well Test Analysis as it needed bulk data since they typically ignored physical insights from the scientific system under consideration. In this paper, this problem is resolved by Physics informed neural networks that are trained to solve supervised learning tasks while honoring any given physics law.

Introduction

The measurement of transient pressure distribution in a single-phase homogeneous reservoir is significant to the researchers in the area of petroleum reservoir engineering, as this is the foundation for the Well Test Analysis, which helps in the determination of permeability distribution in the reservoir. This pressure distribution can be derived for a transient fluid flow through porous media by a non-linear diffusivity equation.

There have been several studies in solving this diffusivity equation analytically, by ignoring the non-linear term and linearizing it for various boundary conditions. In one study, the non-linear diffusivity equation was transformed into a linear diffusivity equation of dimensionless form, by substituting the variables like pressure ($p(r,t)$), radius of investigation(r), and time(t) with their dimensionless forms (p_d), (r_d) and (t_d) respectively (Ahmed and McKinney 2005). (Van Everdingen, Hurst et al. 1949) proposed an analytical solution for this linearized diffusivity equation for a specified list of assumptions. This work was expanded by (Chatas 1953) and (Lee 1982) for two cases, viz infinite-acting reservoir and finite-radial reservoir. In another study, (Matthews and Russell 1967) proposed an exponential integral (Ei) function solution to the linear diffusivity equation for the constant terminal rate scenario, which is further simplified in (Ahmed and McKinney 2005) for a specific range of Ei parameter value by log approximation.

Similarly, there have also been studies on analyzing the pressure distribution problem using the diffusivity equation without ignoring the quadratic gradient term. (Odeh, Babu et al. 1988) had arrived with the approximate solutions for the nonlinear PDE for three cases and compared the result with the solutions of the linear equation. Another notable work by (Chakrabarty, Ali, and Tortike 1993) solved the radial nonlinear PDE for a variety of boundary conditions to analyze the pressure distribution around a large diameter injection well, and presented the results for both constant pressure and constant discharge-rate (with wellbore storage) inner boundary conditions; the outer boundary conditions may be infinite, closed, or constant pressure. Similar work carried out by (Xu-long, Deng-ke, and Rui-he 2004) determined a solution to the nonlinear real space flow equation for both constant rate and constant pressure production using Weber Transform. They also solved the flow equation for a finite circular reservoir case using Henkel Transform and inferred that the difference between the nonlinear and the linear pressure solutions may reach about 8% in the long time. (Liu et al. 2016) further demonstrated Well Testing with Non-Linear PDE for a one-dimensional seepage flow problem with threshold pressure gradient, that represents the unconventional reservoir with low permeability and porosity, by constructing a moving boundary.

(Raissi, Perdikaris, and Karniadakis 2017b), (Raissi, Perdikaris, and Karniadakis 2017a) and (Raissi, Perdikaris, and Karniadakis 2019), introduced physics informed neural networks that are trained to solve supervised learning tasks while honoring any given physics law described by general PDEs. Driven by this work, we built a physics informed deep learning model for Well Test Analysis that enables the synergistic combination of physics law, which is diffusivity equation, and data, for regularizing the training in small data regimes to predict pressure.

Mathematical Considerations

The constant terminal rate solution is a very important aspect of most of the transient test analysis methodologies, e.g., drawdown and pressure buildup analysis. The well is adjusted to produce at a constant flow rate and the pressure values i.e., $p(r,t)$ are measured as a function of time, in most of these tests (Ahmed and McKinney 2005). For the purpose of the experiment, the Ei function solution for constant terminal flow rate has been considered to generate the data set.

The system is assumed to follow the radial flow model where the slightly compressible fluid flows radially towards the fully penetrating vertical well at a constant terminal flow rate Q_o (STB/day) from a homogeneous reservoir of constant radius r_e (ft) and uniform thickness h (ft) and permeability k (md). The radius of investigation is r (ft). The reservoir is considered to be at constant reservoir pressure p_i (psi) at initial time ($t = 0$) and there is no flow across the reservoir boundary. By combining the continuity equation, transport equation, and compressibility equation with the boundary conditions, the non-linear diffusivity equation could be attained (Ahmed and McKinney 2005):

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \beta_f\left(\frac{\partial p}{\partial r}\right)^2 = \frac{1}{c}\frac{\partial p}{\partial t} \tag{1}$$

where β_f = Fluid compressibility (psi^{-1}) and c = Diffusivity constant (Equation (1) from (Chakrabarty, Ali, and Tortike 1993)).

The quadratic gradient term is ignored, and the linear diffusivity equation is expressed from equation (1.2.64) of (Ahmed and McKinney 2005) as:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{1}{c}\frac{\partial p}{\partial t} \tag{2}$$

Solving equation 2 through the form of Ei function arrived at the following line source solution (Matthews and Russell 1967) which is expressed from equation (1.2.66) of (Ahmed and McKinney 2005) as:

$$p(r,t) = p_i + \left[\frac{70.6 Q_o \mu B_o}{kh}\right] Ei\left[\frac{-948\phi\mu c_t r^2}{kt}\right] \tag{3}$$

where C_t = Total compressibility (psi^{-1}), μ = Viscosity (cp), B_o = Oil Formation Volume factor (bbl/STB) and ϕ = Porosity (fraction).

The exponential integral, Ei, can be approximated for the range of values with $x < 0.001$ and the final equation (Ahmed and McKinney 2005) of the form could be derived as mentioned in equation (1.2.70) of (Ahmed and McKinney 2005) as:

$$p(r,t) = p_i - \frac{162.6 Q_o \mu_o B_o}{kh}\left[log\left(\frac{kt}{\phi\mu c_t r_w^2}\right) - 3.23\right] \tag{4}$$

where $t > 9.48 * 10^4 \phi\mu c_t r^2/k$

The pressure distribution dataset obtained from equation 4 is considered as reference for computing the performance of the typical neural network and the physics informed neural network in the following sections. Precisely, starting from an initial condition $p(r, t = 1), r\epsilon[0.25, 15.25]$, and assuming periodic boundary conditions, we integrate equation up till final time $t = 120$ to generate the data set. The entire dataset is used as test data (29k) whereas the training dataset (0.1k) is a subset of initial and boundary condition data.

Alternately, when the non-linear term is considered, the diffusivity equation could not be simplified to an analytical form and could be solved only using numerical methods. For this condition, the parameters of the diffusivity equation, and its boundary conditions should be altered to best describe the reservoir conditions, to obtain the accurate pressure distribution confined to the considered reservoir model.

Model Architecture

Neural network architecture was adopted from (Raissi, Perdikaris, and Karniadakis 2017b) and (Raissi, Perdikaris, and Karniadakis 2017a). Network[1] is implemented using Tensorflow and is set up with 10 layers with 20 neurons per hidden layer. The hyperbolic tangent function is used as an activation function in all hidden layers. L-BFGS-B method is used to optimize the loss function.

Data Informed Model

The pressure of the reservoir at any given radius (r) and time (t) can be predicted by typical neural networks but needs to be trained on huge volume of high-quality historical data. With smaller training datasets, the model tends to overfit training data exhibiting poorer accuracy during extrapolation.

Given a set of $N_p = 100$ randomly distributed initial and boundary data, from the data generated from equation 4, this model learns the latent solution $p(r,t)$ using the mean squared error loss of equation 5. Root Mean Square Error (RMSE) of this model is 1.69.

$$MSE = \frac{1}{N_p}\sum_{i=1}^{N_p} |p(r^i, t^i) - p^i|^2 \tag{5}$$

Physics Informed Model

Physics informed surrogate models for Well Test Analysis using linear diffusivity equation is built to predict pressure. This approach is made possible by support from Tensorflow on automatic differentiation which differentiates neural networks with respect to their input variables and model parameters. The idea of utilizing prior domain knowledge

[1]https://github.com/maziarraissi/PINNs

in a neural network by exploiting automatic differentiation (Baydin et al. 2017) to differentiate neural networks is derived from (Raissi, Perdikaris, and Karniadakis 2017b) and (Raissi, Perdikaris, and Karniadakis 2017a).

Given a set of N_f = 500 collocation points, from the data generated from equation 4, this model learns the latent solution $p(r,t)$, obeying linear diffusivity equation which is represented as $f(r,t)$. MSE_f acts as a normalization mechanism that disciplines solutions to equation 6.

$f(r,t)$ is given by:

$$f := p_{rr} + \frac{1}{r} * p_r - \frac{1}{c} * p_t \tag{6}$$

where $c = 0.0002637k/\phi\mu c_t$

The trainable parameters shared between the neural networks $p(r,t)$ and $f(r,t)$ are learned by minimizing the mean squared error loss (Equation (4) from (Raissi, Perdikaris, and Karniadakis 2017b)):

$$MSE = MSE_p + MSE_f \tag{7}$$

where $MSE_p = \frac{1}{N_p}\sum_{i=1}^{N_p} |p(r^i,t^i) - p^i|^2$ and $MSE_f = \frac{1}{N_f}\sum_{i=1}^{N_f} |f(r_f^i,t_f^i)|^2$,

$p(r,t)$ denotes the data informed solution and $f(r,t)$ denotes physics informed solution,

r^i, t^i, p^i for $i = \{1, N_p\}$ denotes the initial and boundary training data on $p(r,t)$,

r_f^i, t_f^i for $i = \{1, N_f\}$ specifies the collocations points for $f(r,t)$.

Even with smaller training datasets, the model does not overfit training data exhibiting good accuracy during extrapolation. Root Mean Square Error (RMSE) of this model is 0.28.

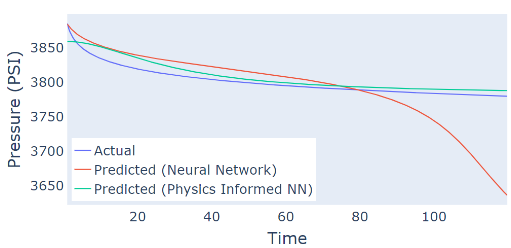

Figure 1: Model Performance at a randomly chosen radius (r = 8.5) for different time instants

At a randomly chosen radius ($r = 8.5$) for different time instants, Figure 1 displays model performance, Figure 2 displays Prediction error ($|p_{pred} - p_{actual}|$) and Figure 3 displays Nonconformity ($|f(r,t)|$) to linear diffusivity equation. Figure 1 shows that the pressure value predicted by the Physics informed model is closer to the actual value. Figure 2 shows the absolute error between the predicted and actual values from Figure 1. It is observed that the trends of Figure 2 and Figure 3 are similar indicating that the nonconformity to fundamental physics law impacts the models' performance.

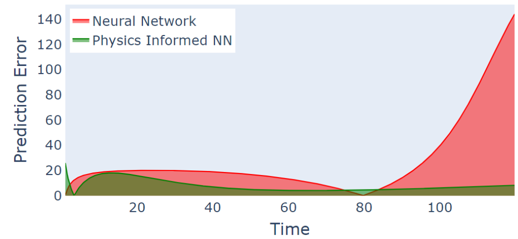

Figure 2: Prediction Error

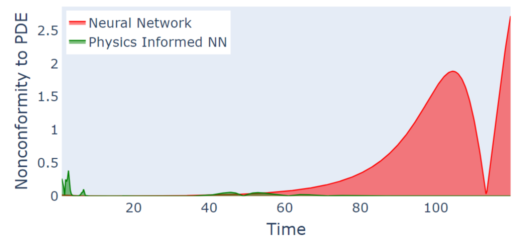

Figure 3: Nonconformity to PDE

Conclusion

This study confirms that the physics informed neural network can model Well Test Analysis for pressure drawdown using generic linear diffusivity equation with commendable accuracy. Apparently, PINN can model well testing using linear or non-linear PDE. Thus, the physics informed neural network model could be a simple and well-behaved alternative approach for pressure transient studies of any reservoir variant, given that the linear or non-linear diffusivity equation, its parameters and boundary conditions are defined.

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