On relating heterogeneous elements from different ontologies

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1 Introduction

The majority of formalisms for distributed ontology integration based on the p2p architecture provide *mapping languages* able to express semantic relations between concepts of different ontologies. These formalisms can express that a concept C in Ontology 1 is equivalent (less/more general than) a concept D in Ontology 2 (see [13] for a survey). Few mapping languages allow also to express semantic relations between properties [8, 6], and thus state that a relation R in Ontology 1 is equivalent (less/more general than) a relation S in Ontology 2. These mappings, hereafter called *homogeneous mappings*, are able to cope with a large, but not the totality of heterogeneities between ontologies.

Assume, for instance, that a knowledge engineer builds an ontology of family unions containing the binary relations `marriedWith` and `partnerOf` between two persons. Suppose also that a second ontology engineer, asked to design a ontology for the same purpose, defines a concept `Marriage`, whose instances are the actual civil or religious marriages, and the concept `civilUnion`, whose instances are all the civil unions. We can easily see that while the first ontology prefers to model unions as relations, the second represents them as concepts. Despite this difference of style in modelling, the concept `Marriage` and the relation `marriedWith` represent the same (or a very similar) real world aspect, and similarly with `partnerOf` and `civilUnion`. To reconcile heterogeneous representations of this sort (which are instances of so-called schematic differences. See [2]) we need a mapping language that allows to map concepts of one ontology into relations of another ontology.

Motivated by these observations, we have illustrated in [9] the need of rich mapping languages that incorporate homogeneous and *heterogeneous mappings*, such as mappings between concepts and relations. [9] contains a preliminary investigation on how to define such rich mapping language in Distributed Description Logics (DDL) [12], and [8] presents a logic and an algorithm for the representation and reasoning with homogeneous mappings in DDL.

Here we address the task of representing and reasoning with both homogeneous and heterogeneous mappings. In particular, we extend the semantics of DDL to deal with heterogeneous mappings. The idea behind this is the ability to relate triples of the form $\langle object\_1, relation\_name, object\_2\rangle$ in one ontology with objects in the domain of another ontology. We provide a sound and complete axiomatization of the effects of all mappings from a source ontology to a target ontology. This is the crucial step towards the axiomatization for an arbitrary network of ontologies as shown in [12].

2 A rich language for mappings

Distributed Description Logic (DDL) [12] is a *natural* generalisation of the Description Logic (DL) framework designed to formalise multiple ontologies *pairwise* linked by semantic mappings. In DDL, ontologies correspond to description logic theories (T-boxes), while semantic mappings correspond to collections of *bridge rules* ($\mathfrak{B}$).

Given a non empty set I of indexes, used to identify ontologies, let $\{\mathcal{D}L_i\}_{i\in I}$ be a collection of description logics[3]. For each $i \in I$ let us denote a T-box of $\mathcal{D}L_i$ as $\mathcal{T}_i$. In this paper, we assume that each $\mathcal{D}L_i$ is description logic weaker or at most equivalent to $\mathcal{ALCQI}_b$, which corresponds to $\mathcal{ALCQI}$ with role union, conjunction and difference (see [15]). Because of lack of space, we omit the precise description of $\mathcal{ALCQI}_b$, and we assume that the reader is familiar with DDL as described in [12].

We indicate with $\{\mathcal{T}_i\}_{i\in I}$ a family of T-Boxes indexed by I. Intuitively, $\mathcal{T}_i$ is the DL formalisation of the i-th ontology. To make every description distinct, we will prefix it with the index of ontology it belongs to. For instance, the concept C that occurs in the i-th ontology is denoted as $i : C$. Similarly, $i : C \sqsubseteq D$ indicates that the axiom $C \sqsubseteq D$ is being considered in the i-th ontology.

Semantic mappings between different ontologies are expressed via collections of *bridge rules*. In the following we use A, B, C and D as place-holders for concepts and R, S, P and Q as place-holders for roles. We instead use X and Y to denote both concepts and roles.

An *homogeneous bridge rule* from i to j is an expression defined as follows:

$$i : X \xrightarrow{\sqsubseteq} j : Y \qquad \textit{(into bridge rule)} \qquad (1)$$

$$i : X \xrightarrow{\sqsupseteq} j : Y \qquad \textit{(onto bridge rule)} \qquad (2)$$

where X and Y are either concepts of $\mathcal{D}L_i$ and $\mathcal{D}L_j$ respectively, or roles of $\mathcal{D}L_i$ and $\mathcal{D}L_j$ respectively. An *heterogeneous bridge rule* from i to j is as follows:

$$i : R \overset{\sqsubseteq}{\twoheadrightarrow} j : C \qquad \textit{(role-into-concept bridge rule)} \qquad (3)$$

$$i : R \overset{\sqsupseteq}{\twoheadrightarrow} j : C \qquad \textit{(role-onto-concept bridge rule)} \qquad (4)$$

$$i : C \overset{\sqsubseteq}{\twoheadrightarrow} j : R \qquad \textit{(concept-into-role bridge rule)} \qquad (5)$$

$$i : C \overset{\sqsupseteq}{\twoheadrightarrow} j : R \qquad \textit{(concept-onto-role bridge rule)} \qquad (6)$$

where R is a role and C is a concept. A *distributed T-box* (DTB) $\mathfrak{T} = \langle\{\mathcal{T}_i\}_{i\in I}, \mathfrak{B}\rangle$ consists of a collection $\{\mathcal{T}_i\}_{i\in I}$ of T-boxes, and a collection $\mathfrak{B} = \{\mathfrak{B}_{ij}\}_{i\neq j\in I}$ of bridge rules between them.

Bridge rules (3) and (4) state that, from the j-th point of view the role R in i is less general, resp. more general, than its local concept C. Similarly, bridge rules (5) and (6) state that, from the j-th point of view the concept C in i is less general, resp. more general, than its local role R. Thus, the bridge rule

$$i : \texttt{marriedInChurchWith} \overset{\sqsubseteq}{\twoheadrightarrow} j : \texttt{Marriage}$$

[3] We assume familiarity with Description Logic and related reasoning systems, described in [1].

expresses the fact that, according to ontology j, the relation `marriedInChurchWith` in ontology i is less general than its local concept `Marriage`, while

$$i : \texttt{civilUnion} \xrightarrow{\sqsubseteq}\!\!\!\!\rightarrow j : \texttt{partnerOf} \qquad\qquad i : \texttt{civilUnion} \xrightarrow{\sqsupseteq}\!\!\!\!\rightarrow j : \texttt{partnerOf}$$

say that, according to ontology j, the concept `civilUnion` in ontology j is equivalent to its local relation `partnerOf`.

In this paper we require that for every (into or onto) bridge rule between roles $i : P \longrightarrow j : R$ in $\mathfrak{B}_{ij}$, also $i : inv(P) \longrightarrow j : inv(R)$ is in $\mathfrak{B}_{ij}$ (where $inv(X)$ is the inverse of X). This to simplify the notation of the rules defined in Section 3.

The semantic of DDL assigns to each ontology $\mathcal{T}_i$ a *local interpretation domain*. The first component of an interpretation of a DTB is a family of interpretations $\{\mathcal{I}_i\}_{i\in I}$, one for each T-box $\mathcal{T}_i$. Each $\mathcal{I}_i$ is called a *local interpretation* and consists of a *possibly empty* domain $\Delta^{\mathcal{I}_i}$ and a valuation function $\cdot^{\mathcal{I}_i}$, which maps every concept to a subset of $\Delta^{\mathcal{I}_i}$, and every role to a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$. The interpretation on the empty domain is used to provide a semantics for distributed T-boxes in which some of the local T-boxes are inconsistent. The reader interested in this aspect of DDL can refer to [12].

The second component of the DDL semantics are families of domain relations. Domain relations define how the different T-box interact and are necessary to define the satisfiability of bridge rules.

Definition 1. *A* domain relation r_{ij} *from* i *to* j *is a subset of* $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$. *We use* $r_{ij}(d)$ *to denote* $\{d' \in \Delta^{\mathcal{I}_j} \mid \langle d, d'\rangle \in r_{ij}\}$*; for any subset* D *of* $\Delta^{\mathcal{I}_i}$*, we use* $r_{ij}(D)$ *to denote* $\bigcup_{d\in D} r_{ij}(d)$*; for any* $R \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$ *we use* $r_{ij}(R)$ *to denote* $\bigcup_{\langle d,d'\rangle\in R} r_{ij}(d) \times r_{ij}(d')$.

A domain relation r_{ij} represents a possible way of mapping the elements of $\Delta^{\mathcal{I}_i}$ into its domain $\Delta^{\mathcal{I}_j}$, seen from j's perspective. The domain relation is used to interpret homogeneous bridge rules.

Definition 2. *The domain relation* r_{ij} *satisfies a homogeneous bridge rule w.r.t.,* $\mathcal{I}_i$ *and* $\mathcal{I}_j$*, written as* $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j\rangle \models br$*, when*

$$\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j\rangle \vDash i : X \xrightarrow{\sqsubseteq} j : Y \quad \textit{if} \quad r_{ij}(X^{\mathcal{I}_i}) \subseteq Y^{\mathcal{I}_j} \tag{7}$$

$$\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j\rangle \vDash i : X \xrightarrow{\sqsupseteq} j : Y \quad \textit{if} \quad r_{ij}(X^{\mathcal{I}_i}) \supseteq Y^{\mathcal{I}_j} \tag{8}$$

where X *and* Y *are either two concepts or two roles.*

Domain relations do not provide sufficient information to interpret heterogeneous mappings. Intuitively, an heterogeneous bridge rule between a relation R and a concept C connects a pair of objects related by R with an object which is in C. This suggests that, to evaluate heterogeneous bridge rules from roles in i to concepts in j we need a relation that maps triples of the form $\langle object\_1, relation\_name, object\_2\rangle$ from ontology i into objects of $\Delta^{\mathcal{I}_j}$. As an example we would like to map the triple $\langle \texttt{John}, \texttt{marriedWith}, \texttt{Mary}\rangle$ of the first ontology into the marriage `m123` of the second ontology, with the intuitive meaning that `m123` is the marriage which correspond to the married couple composed of `John` and `Mary`. We first formally introduce the triples $\langle object\_1, relation\_name, object\_2\rangle$ for a given ontology i.

Given a local interpretation $\mathcal{I}_i$ we consider the set of triples "induced" by the interpretation as the set of *admissible triples* $\Sigma^{\mathcal{I}_i}$. Let $\mathcal{I}_i$ be a local interpretation $\left\langle \Delta^{\mathcal{I}_i}, \cdot^{\mathcal{I}_i} \right\rangle$ for $\mathcal{D}L_i$, and $\mathcal{R}$ be the set of all atomic relations relations of $\mathcal{D}L_i$. We indicate with $\Sigma^{\mathcal{I}_i}$ the set of all triples $\langle x_1, X, x_2 \rangle$ such that $x_1, x_2 \in \Delta^{\mathcal{I}_i}$; $X \in \mathcal{R}$; and $(x_1, x_2) \in X^{\mathcal{I}_i}$.

Intuitively, $\langle \mathtt{John}, \mathtt{marriedWith}, \mathtt{Mary} \rangle$ is an admissible triple in $\Sigma^{\mathcal{I}_i}$ if `John` is married with `Mary`, or more formally if the pair $(\mathtt{John}, \mathtt{Mary})$ belongs to the interpretation of `marriedWith` in $\mathcal{I}_i$.

Definition 3. *A* concept-role domain relation cr_{ij} *from i to j is a subset of* $\Delta^{\mathcal{I}_i} \times \Sigma^{\mathcal{I}_j}$. *A* role-concept domain relation rc_{ij} *from i to j is a subset of* $\Sigma^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$.

The domain relation rc_{ij} represents a possible way of mapping pairs of $R^{\mathcal{I}_i}$ into elements of $C^{\mathcal{I}_j}$, seen from j's perspective. Concept-role and role-concept domain relations are used to interpret heterogeneous mappings.

Definition 4. *The* role-concept domain relation rc_{ij} satisfies a role-(into/onto)-concept bridge rule w.r.t., $\mathcal{I}_i$ and $\mathcal{I}_j$, *written* $\langle \mathcal{I}_i, rc_{ij}, \mathcal{I}_j \rangle \models$ *br, when*

1. $(\mathcal{I}_i, rc_{ij}, \mathcal{I}_j) \models i : R \xrightarrow{\sqsubseteq} j : C$ *if for all* $(x_1, x_2) \in R^{\mathcal{I}_i}$ *and for all pairs* $((x_1, X, x_2), x) \in rc_{ij}$ *with* $X^{\mathcal{I}_i} \subseteq R^{\mathcal{I}_i}$, *we have that* $x \in C^{\mathcal{I}_j}$
2. $(\mathcal{I}_i, rc_{ij}, \mathcal{I}_j) \models i : R \xrightarrow{\sqsupseteq} j : C$ *if for all* $x \in C^{\mathcal{I}_j}$ *there is a pair* $((x_1, X, x_2), x) \in rc_{ij}$, *such that* $X^{\mathcal{I}_i} \subseteq R^{\mathcal{I}_i}$.

The concept-role domain relation cr_{ij} satisfies a concept-(into/onto)-role bridge rule w.r.t., $\mathcal{I}_i$ and $\mathcal{I}_j$, *written* $\langle \mathcal{I}_i, cr_{ij}, \mathcal{I}_j \rangle \models$ *br, when*

3. $(\mathcal{I}_i, cr_{ij}, \mathcal{I}_j) \models i : C \xrightarrow{\sqsubseteq} j : R$ *if for all* $x \in C^{\mathcal{I}_i}$, *and for all pairs* $(x, \langle x_1, X, x_2 \rangle) \in cr_{ij}$, *it is true that* $X^{\mathcal{I}_j} \subseteq R^{\mathcal{I}_j}$;
4. $(\mathcal{I}_i, cr_{ij}, \mathcal{I}_j) \models i : C \xrightarrow{\sqsupseteq} j : R$ *if for all* $(x_1, x_2) \in R^{\mathcal{I}_j}$ *there is a pair* $(x, \langle x_1, X, x_2 \rangle) \in cr_{ij}$, *such that* $X^{\mathcal{I}_j} \subseteq R^{\mathcal{I}_j}$ *and* $x \in C^{\mathcal{I}_i}$.

Satisfiability of a role-into-concept bridge rule forces the role-concept domain relation cr_{ij} to map pair of elements (x_1, x_2) which belong to $R^{\mathcal{I}_i}$ into elements x in $C^{\mathcal{I}_j}$. Note that, from the definition of role-concept domain relation two arbitrary objects y_1 and y_2 could occur in a pair $(\langle y_1, X, y_2 \rangle, y)$ with X different from R itself but such that $X^{\mathcal{I}_i} \subseteq R^{\mathcal{I}_i}$, Thus also this pair (y_1, y_2) belongs to $R^{\mathcal{I}_i}$ and we have to force also y to be in $C^{\mathcal{I}_j}$. In other words, we can say that satisfiability of a role-into-concept bridge rule forces the role-concept domain relation to map pairs of elements (x_1, x_2) which belong to R, or to any of its atomic subroles X, into elements x in $C^{\mathcal{I}_i}$.

A *distributed interpretation* $\mathfrak{I}$ of a DTB $\mathfrak{T}$ consists of the 4-tuple

$$\langle \{\mathcal{I}_i\}_{i \in I}, \{r_{ij}\}_{i \neq j \in I}, \{cr_{ij}\}_{i \neq j \in I}, \{rc_{ij}\}_{i \neq j \in I} \rangle .$$

$\mathfrak{I}$ *satisfies* the elements of a DTB $\mathfrak{T}$ if, for every $i, j \in I$:

1. $\mathfrak{I} \models i : A \sqsubseteq B$, if $\mathcal{I}_i \models A \sqsubseteq B$
2. $\mathfrak{I} \models \mathcal{T}_i$, if $\mathfrak{I} \models i : A \sqsubseteq B$ for all $A \sqsubseteq B$ in $\mathcal{T}_i$
3. $\mathfrak{I} \models \mathfrak{B}_{ij}$, if

- $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j \rangle$ satisfies all the homogeneous bridge rules in $\mathfrak{B}_{ij}$,
- $\langle \mathcal{I}_i, cr_{ij}, \mathcal{I}_j \rangle$ satisfies all the concept-to-role bridge rules in $\mathfrak{B}_{ij}$,
- $\langle \mathcal{I}_i, rc_{ij}, \mathcal{I}_j \rangle$ satisfies all the role-to-concept bridge rules in $\mathfrak{B}_{ij}$

4. $\mathfrak{I} \models \mathfrak{T}$, if for every $i, j \in I$, $\mathfrak{I} \models \mathcal{T}_i$ and $\mathfrak{I} \models \mathfrak{B}_{ij}$

Entailment and satisfiability of a single concept are defined in the usual way by means of the above satisfiability of a distributed T-Box (e.g. see [12]).

3 The effects of bridge rules

Bridge rules can be thought of as inter-theory axioms, which constrain the models of the theories representing the different ontologies. An important characteristic of mappings specified by DDL bridge rules is that they are directional, in the sense that they are defined from a source ontology O_s to a target ontology O_t, and they allow to transfer knowledge only from O_s to O_t, without any undesired back-flow effect. In this section we show that the semantic of mappings defined in the previous Section fulfills this requirement. Furthermore we characterize the effects of the bridge rules in terms of the knowledge they allow to propagate from O_s to O_t.

We start by characterizing the effects of mappings of a simple DTB $\langle \mathcal{T}_i, \mathcal{T}_j, \mathfrak{B}_{ij} \rangle$, composed of two T-boxes $\mathcal{T}_i$ and $\mathcal{T}_j$ and a set of bridge rules $\mathfrak{B}_{ij}$ from i to j. The first important property we prove is *directionality*:

Proposition 1. *$\langle \mathcal{T}_i, \mathcal{T}_j, \mathfrak{B}_{ij} \rangle \models i : X \sqsubseteq Y$ if and only if $\mathcal{T}_i \models X \sqsubseteq Y$*

The proof can be found in [7]. According to Proposition 1, bridge rules from i to j affect only the logical consequences in j, and leave the consequences in i unchanged. In the following we characterise the knowledge propagated from i (the source) to j (the target) using a set of *propagation rules* of the form:

$$\frac{\begin{array}{l}\text{axioms in } i\\ \text{bridge rules from } i \text{ to } j\end{array}}{\text{axiom in } j}$$

which must be read as: if $\mathcal{T}_i$ entails all the axioms in i, and $\mathfrak{B}_{ij}$ contains the bridge rules from i to j, then $\langle \mathcal{T}_i, \mathcal{T}_j, \mathfrak{B}_{ij} \rangle$ satisfies axioms in j.

Propagation rules for homogeneous mappings. Simple propagation rules which describe the effects of the homogeneous mappings are:

$$\frac{\begin{array}{l} i : A \sqsubseteq B \\ i : A \xrightarrow{\sqsupseteq} j : C \\ i : B \xrightarrow{\sqsubseteq} j : D \end{array}}{j : C \sqsubseteq D} \quad (9) \qquad \frac{\begin{array}{l} i : P \sqsubseteq Q, \\ i : P \xrightarrow{\sqsupseteq} j : R \\ i : Q \xrightarrow{\sqsubseteq} j : S \end{array}}{j : R \sqsubseteq S} \quad (10) \qquad \frac{\begin{array}{l} i : \exists P.\top \sqsubseteq B \\ i : P \xrightarrow{\sqsupseteq} j : R \\ i : B \xrightarrow{\sqsubseteq} j : D \end{array}}{j : \exists R.\top \sqsubseteq D} \quad (11)$$

Rule (9) describes a simple propagation of the concept hierarchy forced by bridge rules between concepts, and is widely described in [12]. This rule says that if $A \sqsubseteq B$ is a fact of the T-box $\mathcal{T}_i$, then the effect of the bridge rules $i : A \xrightarrow{\sqsupseteq} j : C$ and $i : B \xrightarrow{\sqsubseteq} j : D$ is that $C \sqsubseteq D$ is also a fact in $\mathcal{T}_j$. An analogous effect concerns the propagation of the role hierarchy due to bridge rules between roles, and is described by rule (10) where P, Q, R and S is either a role or an inverse role.[4] The effect of the combination of mappings between roles and mappings between concepts is the propagation of domain and range among relations linked by role-onto-role mappings. Propagation rule (11) describes a simple effect of these mappings, where P, R are roles and B, D are concepts. Rule (11) says that if the domain of P is contained in B and the appropriate bridge rules hold, then we can infer that the domain of R is contained in D. A similar rule allows to obtain $j : \exists R^{-}.\top \sqsubseteq D$ from $i : \exists P^{-}.\top \sqsubseteq B$ and the same bridge rules, thus expressing the propagation of the range restriction.

The general form of propagation rules (9)–(11) is given in Figure 1. Note that rule (10) can be obtained from rule (b) in Figure 1 by setting $l = 1, p = 0, m = 0$, while rule (11) can be obtained by setting $l = 0, p = 0, m = 1$. Analogously the rule for range restriction can be obtained by setting $l = 0, p = 1, m = 0$.

Propagation rules for heterogeneous mappings. The effects of the heterogeneous bridge rules is the propagation of the role hierarchy into the concept hierarchy and vice-versa. The simplest forms of these rules are:

$$\frac{\begin{array}{l} i : P \sqsubseteq Q \\ i : P \xrightarrow{\sqsupseteq} j : C \\ i : Q \xrightarrow{\sqsubseteq} j : D \end{array}}{j : C \sqsubseteq D} \quad (12) \qquad \frac{\begin{array}{l} i : A \sqsubseteq B \\ i : A \xrightarrow{\sqsupseteq} j : R \\ i : B \xrightarrow{\sqsubseteq} j : S \end{array}}{j : R \sqsubseteq S} \quad (13)$$

The general form of these rules is given in Figure 1. The expression $\bigsqcup_{k=1}^{n} S_k$ with $n = 0$ in rule (d) represents the empty role $R_{\bot}$, which is obtained with the axiom $\top \sqsubseteq \forall R_{\bot}.\bot$.

Given a set of bridge rules $\mathfrak{B}_{ij}$ from $\mathcal{DL}_i$ to $\mathcal{DL}_j$, we have defined four different rules, shown in Figure 1, which take as input a T-box $\mathcal{T}_i$ in $\mathcal{DL}_i$ and produce a T-box $\mathcal{T}_j$ in $\mathcal{DL}_j$. Starting from these rules we define an operator $\mathfrak{B}_{ij}(\cdot)$, taking as input $\mathcal{T}_i$ and producing a T-box $\mathcal{T}_j$, enriched with the conclusions of rules (a)–(d) in Figure 1.

Theorem 1 (Soundness and Completeness of $\mathfrak{B}_{ij}(\cdot)$). *Let $\mathfrak{T}_{ij} = \langle \mathcal{T}_i, \mathcal{T}_j, \mathfrak{B}_{ij} \rangle$ be a distributed T-box, where $\mathcal{T}_i$ and $\mathcal{T}_j$ are expressed in the $\mathcal{ALCQI}_b$ descriptive language. Then $\mathfrak{T}_{ij} \models j : X \sqsubseteq Y \Longleftrightarrow \mathcal{T}_j \cup \mathfrak{B}_{ij}(\mathcal{T}_i) \models X \sqsubseteq Y$.*

The proof can be found in [7]. The generalisation of the axiomatization for an arbitrary network of ontologies can be obtained following the technique used in [12].

As a final remark we can notice that the combination of homogeneous and heterogeneous bridge rules does not generate any effect in the logic proposed in this paper. This because the domain relation and the concept-role and role-concept domain relations do not affect each other. The investigation of more complex heterogeneous bridge rules, which can lead to this sort of interaction is left for future work. An additional open point concerns the extension of our framework in order to account for transitive roles. It is well known that the unrestricted interaction between number restriction and transitivity is a source of indecidability; moreover, the bridge rules may infer additional subsumption relations among the roles. Therefore, guaranteeing appropriate restrictions to ensure decidability is no longer a matter of analysing the "static" role hierarchy (e.g., a in the case of $\mathcal{SHIQ}$).

[4] The formula $R \sqsubseteq S$ is a shorthand for $\exists(R \sqcap \neg S).\top \sqsubseteq \bot$.

$$\frac{\begin{array}{l} i : A \sqsubseteq \bigsqcup_{k=1}^{n} B_k \\ i : A \xrightarrow{\sqsupseteq} j : C \\ i : B_k \xrightarrow{\sqsubseteq} j : D_k, \text{ for } 1 \leq k \leq n \end{array}}{j : C \sqsubseteq \bigsqcup_{k=1}^{n} D_k}$$

(a) Generalisation of rule (9).

$$\frac{\begin{array}{l} i : \exists (P \sqcap \neg (\bigsqcup_{h=0}^{l} Q_h)).\left(\neg \bigsqcup_{h=0}^{p} A_h\right) \sqsubseteq \left(\bigsqcup_{h=0}^{m} B_h\right) \\ i : P \xrightarrow{\sqsupseteq} j : R \\ i : Q_h \xrightarrow{\sqsubseteq} j : S_h, \text{ for } 1 \leq h \leq l \\ i : A_h \xrightarrow{\sqsubseteq} j : C_h, \text{ for } 1 \leq h \leq p \\ i : B_h \xrightarrow{\sqsubseteq} j : D_h, \text{ for } 1 \leq h \leq m \end{array}}{j : \exists (R \sqcap \neg (\bigsqcup_{h=1}^{l} S_h)).\left(\neg \bigsqcup_{h=1}^{p} C_h\right) \sqsubseteq \left(\bigsqcup_{k=1}^{m} D_k\right)}$$

(b) Generalisation of rules (10) and (11).

$$\frac{\begin{array}{l} i : P \sqsubseteq Q \\ i : P \overset{\sqsupseteq}{\twoheadrightarrow} j : C \\ i : Q \overset{\sqsubseteq}{\twoheadrightarrow} j : D \end{array}}{j : C \sqsubseteq D} \qquad \frac{\begin{array}{l} i : P \sqsubseteq \bot_R \\ i : P \overset{\sqsupseteq}{\twoheadrightarrow} j : C \end{array}}{j : C \sqsubseteq \bot}$$

(c) Generalisation of rule (12).

$$\frac{\begin{array}{l} i : A \sqsubseteq \bigsqcup_{k=1}^{n} B_k \\ i : A \overset{\sqsupseteq}{\twoheadrightarrow} j : R \\ i : B_k \overset{\sqsubseteq}{\twoheadrightarrow} j : S_k, \text{ for } 1 \leq k \leq n \end{array}}{j : R \sqsubseteq \bigsqcup_{k=1}^{n} S_k}$$

(d) Generalisation of rule (13).

Fig. 1. General version of propagation rules.

4 Related Work and Concluding Remarks

Recently, several proposals go in the direction of providing semantic mapping among different ontologies (e.g. [14, 12, 3]). However, to the best of our knowledge there is no specific work on heterogeneous mappings as described in this paper. This in spite of the fact that there are several attempts at providing some sort of mappings relating non-homogeneous elements. For example in [6], it is possible to express the mapping $\forall x.(\exists y.R(x, y) \rightarrow C(x))$; while, in the original version of DDL (see [12]), an analogous mappings can be established by means of the formula $1 : \exists R.\top \xrightarrow{\sqsubseteq} 2 : C$. Note that both cases cannot be considered heterogeneous mappings because they relates the domain of the relation R with the concept C; which are both concepts.

The work presented in this paper is clearly connected to the well known modelling process of *reification* (aka *objectification*) adopted in UML or ORM (see [10, 11]). As described in [10], this corresponds to think of certain relationship instances as objects. In UML this is supported by means of *association classes*, while in Entity-Relationship diagram this is often mediated by means of *weak entities*. Note that these modelling paradigms do not support rich inter-schema axioms in the spirit of ontology mappings as described in [14].

There are other modelling formalisms which enable the bridging between relations and classes in the context of Description Logics. In particular, the work on $\mathcal{DLR}$ (see [4]), specifically w.r.t. the technique for encoding n-ary relations within a standard Description Logic, and [5]. The advantage of our approach lies in the fact that the local semantics (i.e. the underlying semantics of the single ontology languages) does not need to be modified in order to consider the global semantics of the system. Specifically, there is no need to provide an explicit reification of relations since this is incorporated into the global semantics.

The language and the semantics presented in this paper constitute a genuine contribution in the direction of the integration of heterogeneous ontologies. The language proposed in this paper makes it possible to directly bind a concept with a relation in a different ontology, and vice-versa. At the semantic level we have introduced a domain relation that maps pairs of object appearing in a relation into objects and vice-versa. This also constitute a novelty in the semantics of knowledge integration. Finally we have proved soundness and completeness of the effects of the mappings and we leave the study of decidability and the definition of a reasoning algorithm for future work.

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