

# Intrinsic integration in rational number games – A systematic literature review

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**Abstract.** This literature review on digital game-based rational number learning investigated the integration of rational number content and established rational number learning principles with core game mechanics. A total of 34 papers eventually met the defined inclusion criteria. The review showed that the literature on digital game-based rational number learning is sparse demonstrating that this particular field is still in its early stages. We found evidence to indicate that most of the game-based learning solutions were designed to facilitate understanding of magnitudes or measurement units that is in line with the current rational number learning recommendations. Nevertheless, more importantly, the results revealed that in most cases the rational number content was not well integrated to core game mechanics that may undermine the benefits of game-based learning. The conclusions drawn from this review raise a set of issues that, if unaddressed, could disturb the maturation of the whole digital game-based learning field.

**Keywords:** Game-based learning, Rational numbers, Intrinsic integration, Educational game design, Game mechanics.

## 1 Introduction

Recent meta-analyses about game-based learning have shown that digital learning games can be effective learning solutions [1,2]. In line with this, Devlin [3] has argued that digital games can provide new interfaces to learn mathematics that are far easier and more natural to use than symbolic expressions that we have used to employ in conventional education. Nevertheless, a meta-analyses about digital game-based learning for K–12 mathematics education revealed that although game-based math studies have shown statistically positive learning effects, the overall effect size is rather small [4].

In addition to mainly positive learning outcomes, games are usually argued to motivate and engage students more than traditional instructional methods [e.g. 5]. Although many individual studies seem to suggest this, Wouters et al. [1] did not find

such an effect in their meta-analysis. One reason for this might be that the instructional design of the games disturbs the playing experience. It is argued [1] that the instructional pop-up screens often embedded in game-based learning solutions may disturb the flow of the game and consequently undermine the engagement created by the game. In fact, Habgood and Ainsworth [6] have argued and empirically proven that deep integration of game's core mechanics and its learning content is crucial for creating intrinsically motivating and effective game-based learning solutions. Consequently, one should aim at minimizing extrinsically motivating elements and rather focus on making those elements an intrinsically motivating part of the game-based learning tasks. This might be achieved by aligning game and learning mechanics. Unfortunately, previous research has shown that systematic investigation of learning integration in games is lacking [7].

Game's feedback system provides one fruitful context for aligning game and learning mechanics as the feedback is generally one of the most powerful educational mechanisms to foster learning [8] and games are founded on continuous feedback loops. In fact, several game-based learning models emphasize the meaning of the feedback [e.g. 9, 10]. For example, the flow framework for educational games [9] states that games should provide immediate and cognitive feedback that directs players' attention on relevant learning content and triggers reflective processes supporting the development of conceptual understanding of the learning domain.

Obviously, well-designed game-based learning solutions can be beneficial especially in content domains in which students tend to struggle. This paper focuses on the rational number content domain, which is an important research area because previous research has shown that many children face huge difficulties with it [11]. Furthermore, as the fidelity of theories addressing the development of rational number learning has increased this content domain provides an interesting context to explore integration of game mechanics and learning content.

## **2 Research objective and methods**

### **2.1 Research objective**

In recent years, the importance of rational number understanding for mathematical proficiency has been well proven. At the same time it has become evident that new instructional approaches should be developed for teaching rational numbers and games could provide appropriate means to implement the recent findings of numerical cognition research into practice. For these reasons, it is important to review published papers about digital game-based rational number learning and explore to what extent the established scientific knowledge of rational number learning is taken into account in the digital game-based rational number studies and to what extent rational number content is integrated to core game mechanics.

## 2.2 Data collection and search terms

A two-phase literature search strategy was used including 1) a database search and 2) a reference search. Papers were searched from ScienceDirect, Scopus, and Web of Science databases. In Scopus and in Science Direct we used title, abstract and keywords search depth, and in Web of Science “topic” search depth was used. Terms referring to the game-based learning were derived from the recent literature review [12]. The search terms used were: (“serious game” OR “learning game” OR “instructional game” OR “game based learning” OR “game-based learning” OR “video game” OR “educational game”) AND (“fraction” OR “percentage” OR “decimal” OR “rational number”)

## 2.3 Inclusion/exclusion criteria and paper selection

To be included in the review, papers had to A) be related to research aims of the literature review, B) include empirical evidence, C) use a term “game” to describe the studied learning solution, D) be written in English, and e) be published before May 2018. Conference proceedings papers and book chapters that our university did not have rights to access were excluded from the review.

The search returned altogether 521 potential papers for the review and 323 papers after removing duplicates. 291 papers were excluded from the review based on inclusion and exclusion criteria: 280 papers did not meet inclusion criteria A, one paper did not meet inclusion criteria B, and three papers did not meet inclusion criteria C. Seven publications were excluded because of exclusion criteria D. Finally, we bought two journal articles that we did not have full-text rights. To ensure that we included most of the relevant publications related to the aims of the study, we searched references of the papers selected in database search phase. We were able to find two new papers that met our inclusion criteria [13,14]. The final data consisted of 34 scientific papers (17 journal articles, a book chapter, and 16 conference papers).

## 2.4 Coding of papers and interrater reliability

All the included papers were coded according to the created coding scheme. The coding dimensions and used codes are shown in Table 1. Game related dimensions were coded based on the pictures, provided web links, and game descriptions included in the papers.

**Table 1.** Coding scheme

Coding dimension	Possible codes
Publication type	Article, Book chapter, Conference paper
Game genre	Action games, Adventure games, Fighting games, Puzzle games, Role playing games, Simulations, Sports games, Strategy games, Platform games, Mini-games, Other games
Delivery platform	Video game console, Computer, Browser, Mobile device

Integration of game mechanics, Rational number (RN) knowledge & RN content	Scientific RN knowledge: Strong, Medium, Weak RN content with gameplay: Intrinsically integrated, Extrinsically integrated
Game's feedback system	Described (provides: cognitive feedback / general feedback) Not described

The integration of rational number content and gameplay was considered on two different levels. 1) We used integration of the rational number knowledge coding dimension to determine how well established scientific rational number learning knowledge (principles) was applied in the game design and justified in the paper [see 15 for more details of the principles]. In the strong integration category, rational number learning principles were clearly presented and the described gameplay was based on these principles. In the weak integration category rational number learning principles were not presented and the described gameplay was not based on rational number learning principles. In the medium integration category either the rational number learning principles or the gameplay were insufficiently presented, but the paper still included some relevant considerations of the integration. 2) We used intrinsic integration [6] coding dimension to determine whether the rational number content was intrinsically integrated to gameplay or not. This dimension determines whether the game provides such an representation of the learning content that the player explores the content through the interaction with core mechanics of the game (intrinsic integration). If the rational number content was separated from the gameplay, the game was classified to the extrinsic integration category. Moreover, we used an explorative approach to identify the most common rational number tasks and game mechanics that were used in the games.

First, one coder coded all papers independently according to the whole coding scheme. After that another coder coded all papers independently, but focused only on rational number knowledge integration dimensions of the coding scheme. This dimension were coded by two reviewers, because it was the most interpretative dimension. The Cohen's Kappa statistic was used to examine the interrater reliability with respect to the coding quality. According to Cohen [16] there was high agreement between reviewers' coding, ( $\kappa = .844$ ,  $p < .001$ ).

### 3 Results

#### 3.1 Games used in reviewed papers

Altogether 21 different games were used in papers. The Semideus game was the most popular and it was studied in 11 papers. According to these papers, the Semideus game was iteratively developed and thus different versions of the game were used in the papers. In fact, in [17] the authors noted that "we used our rational number game engine, Semideus, to develop a digital game for the training intervention". The basic mechanics of the different Semideus versions are quite similar and the major changes relate to content design and graphical implementation. In this paper we use the term 'Semideus games' when we generally refer to games created with the Semideus rational number

engine. The AnimalClass, Decimal Point, Math App, and UFraction games were each studied in 2 papers. The remaining eight games were each studied in a single paper.

### **3.2 Game delivery platforms and game genres**

The game delivery platform varied in studies. In 15 studies the games were played with computers, in 16 studies with mobile devices, and in one study with computer and mobile devices. Two papers failed to report what platform was used. The platform game genre was the most popular in the included papers ( $n = 11$ ). However, different versions of the Semideus game were used in these studies [e.g. 18-20]. The second most featured game genre was puzzle game which was used in seven studies. Four studies were based on mini games and four studies on quiz games. An adventure game with puzzles was used in two studies. Two studies that were based on a teachable agent approach and a study utilizing virtual reality based music game were categorized into other games genre. Two studies did not provide enough information for game genre categorization.

### **3.3 Rational number integration**

Table 2 summarizes the main results of rational number integration and identified rational number learning mechanics. First, we explored to what extent the established knowledge of rational number learning was taken into account in the papers. This classification reflects how well scientific rational number learning knowledge was applied in the game design and justified in the paper. The level of rational number knowledge integration was found to be strong in 16 papers, medium in 13 papers, and weak in five papers. Most of the strong integration category papers ( $n = 13$ ) were based on the different versions of the Semideus game. Paper [14] in which the effectiveness of the Motion Math game was studied is another example of strong integration. The paper provides clear theoretical justifications how the Motion Math game was designed to support players' understanding of how fractions are related to the number line. Another paper [21] presents a Save Patch game that turns the mathematical symbols of fraction arithmetic into objects that the student can manipulate on the number line. The theoretical grounding of the Save Patch game is sound, but the paper does not provide adequate theoretical justification of the game design and thus it was categorized to the medium integration category. The papers of the weak integration category did not manage to provide relevant theoretical basis for game designs or the descriptions of the games were poor. For example, in [22] students used fraction rods (physical manipulatives) along with a digital game, but authors of the paper do not give clear reason why the rods were used, how the gameplay was founded on rational number learning principles, and what rational number learning concepts were targeted to be learned. It is noteworthy that our categorization does not directly reflect the quality of the games with respect to rational number knowledge integration, because the categorization was based only on justifications and game descriptions included in the papers, while we did not had possibility to play all the games included in the reviewed papers.

Second, we explored the intrinsic integration of rational number content and gameplay. This dimension determines whether the game provides such a representation of the learning content that the player can explore it through the interaction with the core game mechanics in an engaging game world. Rational number content of five different games studied in 15 papers were intrinsically integrated. For example, in the Refraction game the task of the player is to provide power to spaceships that are lost in space [23]. In order to successfully complete the tasks, the player had to spatially navigate the laser beam around obstacles and correctly split the whole laser into the fractions needed to power the spaceships. The remaining games ( $n = 17$ ) were classified to the extrinsic integration category. In these games the rational number content was either totally separated from the gameplay, only superficially attached to the core mechanics of the game, or the implementation of the game was poor (very low fidelity with respect to audio-visual implementation or game mechanics).

**Table 2.** Integration of rational number learning knowledge, rational number content and game mechanics (34 papers including 21 games)

Integration of rational number content and gameplay	
Intrinsically integrated	5 games
Extrinsically integrated	16 games
Level of scientific rational number knowledge integration	
Strong integration	16 papers
Medium integration	13 papers
Weak integration	5 papers
Identified main rational number learning mechanics	
Magnitude comparison or ordering	17 papers
Number line estimation	16 papers
Application of units or unit manipulation	9 papers
Multiple choice questions	3 papers

In order to have a deeper look on integration of rational number learning knowledge and game mechanics, we explored what kind of learning mechanics were used in the games. We managed to identify four main types of mechanics (see Table 2). The implementation of these mechanics in different games varied a lot.

Game mechanics in which players had to compare or order rational number magnitudes were included in 17 papers. In ten papers these mechanics were somehow linked to number lines and in 13 papers equivalent rational numbers were addressed. Only one game [24] relied on a gamified graphical multiple choice question approach in magnitude comparison (click a number that matches to a given statement). In two papers, the comparison mechanic was based on logical statements that players formalize in the game in order to teach relations between the given rational numbers to their virtual pet [25, 26] and in one game the comparison mechanic was based on the use of a scale [27]. Six papers reported that some of the comparison or ordering tasks

included in the game required comparison of magnitudes of the different rational number representations.

Although number line estimation appeared in 16 papers, only 5 different games that included number line estimation mechanics could be identified. Number lines were used in the Catch the Monsters [28], Motion Math [14], Save Patch game [21], and the Semideus games [e.g. 17, 29, 30] as well in one game of the Decimal Point mini-game collection [31, 32].

In four of the nine papers in which the game mechanics were based on unit manipulations, objects that are familiar to students were utilized. For example, in The Candy Factory, the task of the player is to manage specific types of candy orders that require slicing candies, copying candy slices, and measuring candies [33]. In two papers concrete physical manipulatives were used along the story driven UFraction mobile game [22, 34]. In the UFraction game players try to solve real-life fraction problems with the help of 12 different sized rods. Only one paper linked unit manipulations clearly to number line - the Save Patch game turns the mathematical symbols of fraction arithmetic into objects that the student can place and manipulate on the number line [21].

Only three papers were based on multiple choice question mechanics. However, the multiple choice questions were implemented in a visually rich game environment. For example, in the Monkey Tales game the player needs to shoot objects that correspond to a given mathematical statement as fast as possible to beat the monkey opponent [24].

Because it has been argued that the feedback that game-based learning solutions provide is crucial for educational effectiveness and the feedback system of the game should be used to integrate learning content to gameplay, we explored how well the implementation of games' feedback systems were reported. Surprisingly, the implementation of the feedback systems was considered only in ten papers. The games studied in nine papers provided cognitive feedback that was designed to facilitate learning. For example, games provided visual feedback about players solutions, explained why solutions were wrong, or provided possibilities to demand mathematical hints.

## **4 Discussion**

### **4.1 Summary of main findings**

A major challenge in educational game design is to incorporate instructional features in a way that trigger appropriate cognitive processing while players interact with the core game mechanics. In this review, we explored how the established rational number learning principles have been considered in the reviewed 34 papers and taken into account in the 21 games used in the studies. We found evidence to indicate that most of the game-based learning solutions that were used in the reviewed papers were designed to facilitate understanding of magnitudes or measurement units, which is in line with current rational number learning recommendations [e.g. 15]. Only five papers failed to provide a relevant theoretical basis for their game design, which indicates rather successful communication between game-based learning and rational number

learning research fields. However, the results revealed that intrinsic integration of rational number content and core game mechanics was rare. Although most of the games were based on appropriate rational number learning principles, the rational number content was not well integrated to core game mechanics or the implementation of the games were poor. This is alarming, as previous research has indicated that intrinsic integration tends to enhance learning gains and engagement [6]. We argue that stronger interdisciplinary cooperation between game developers, psychologists, and learning scientists might result in development of better learning solutions and interventions. For instance, game developers have deep knowledge on the diverse range of game mechanics to increase motivation and engagement and game designers can create engaging and aesthetically pleasing learning environments. On the other hand, psychologists and learning scientists can contribute in suggesting proven and relevant learning mechanics and pedagogical approaches for the game. Consequently, the combination of expert knowledge from all these different domains might facilitate the development of exhaustively intrinsically integrated game-based learning environments.

Another alarming result of the current review is the lack of discussion of feedback systems of the games. Specifically, the implementation of feedback was discussed only in six of the games even though the meaning of feedback that learning games provide has been emphasized in the literature [9, 10]. Game designers and researchers should consider the issue of feedback more carefully, as feedback solutions that do not trigger appropriate cognitive processing may undermine the benefits of game-based learning. One way of addressing the lack of feedback description and discussion is to aim at standardizing the descriptions of games in the game-based learning domain. This would ensure that most basic mechanics of every game can be described in similar way ensuring coherent and comprehensive descriptions. When thinking of such a standardized approach of describing educational games, feedback would be one of the most important mechanics that needs to be described sufficiently. From an educational perspective, feedback is generally one of the most powerful educational mechanisms to support and foster learning [for an overview see 8 and 35]. This emphasizes the importance of discussing feedback mechanisms in game-based learning studies. Discussing at least the most common types of feedback which include corrective feedback, directive feedback, and epistemic feedback would enhance the replicability of the effects found in game-based learning studies and decrease interpretation issues. Consequently, researchers could build upon this description to use feedback mechanisms that trigger appropriated cognitive processing in a diverse range of contexts.

#### **4.2 Research gaps and future research directions**

As only in 24% of the games rational number content and gameplay were intrinsically integrated and 65 % of the games were research prototypes, further research focusing on learning and affective outcomes with high-quality intrinsically integrated games are needed. Furthermore, more research that systematically examines the design of intrinsic integration between learning content and gameplay is needed.



The analysis of the papers revealed that conceptual change issues were considered only in three papers, even though previous research has pointed out that the acquisition of rational number knowledge seems to require radical changes in children's pre-existing concepts of the natural numbers [e.g. 15]. Thus, we argue that conceptual change theories should be taken better into account in game design.

The lack of coherent and standardized descriptions of certain game elements was another problem, which was particularly apparent for the discussion and description of feedback mechanisms. Digital game-based learning research would benefit from guidelines for describing the crucial elements of game-based learning solutions and integration between game mechanics and learning content allowing more comprehensive understanding of their (cognitive) effects.

### **4.3 Limitations and threats to validity**

Although the papers were reviewed systematically, there are some limitations and threats to validity. In order to overcome the possible negative influence of search terms, selected databases used, and publication time period, we expanded the basic search process by conducting a reference search. As the reference search produced only two publications, this may indicate that most publications relevant to the research questions are included in this review. If the number of publications found with reference search would have been greater, it could have indicated that database search could have had methodological faults. It is still possible that we may have missed certain studies that should have been included but are not widely referred or did not use search terms included in our search string, but this appears unlikely.

Initial coding table and coding schemes for the papers were created by two reviewers. The coding table and coding schemes were further developed and revisited as the coding of the papers progressed. In order to minimize coding mistakes any conflicts were discussed and resolved by the reviewers. With this approach, we tried to mitigate the threats due to personal bias. The poor description of the games used in the studies complicated the coding process and thus we had to resolve several conflicts.

## **5 Conclusions**

Taken together the field of digital game-based rational number learning seems to be in an early developmental stage and the literature base is sparse and fragmented. The review indicated a fruitful dialogue between game-based learning, numerical cognition, and mathematics education research fields. However, shortages in the integration of rational number content and game mechanics call for deeper integration of these research fields and more interdisciplinary studies and game development endeavors are needed. To be able to comprehensively map the instructional benefits of using games in rational number learning as well as other disciplines, it would be crucial that the scientific community creates a common game description template that defines the minimum and crucial information that should be reported in all empirical digital game-based learning papers. Our analyses revealed that the game descriptions included in the

reviewed papers tended to be too superficial, which made it challenging to judge the integration between learning content and gameplay. A common game description practice could support generalization of results and facilitate the evidence-based development of the game-based learning field.

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