

# Towards an Ontology of Categories and Relations

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**Abstract.** Categorization and association are fundamental for our understanding of the world, for which reason notions such as category, individual and relation are of major importance, for applied and formal ontology as well as for conceptual modeling. Several of the available foundational ontologies consider those notions at the meta level (only) and focus on individuals as their overall domain. This notwithstanding, we see a need for establishing well-founded ontological theories of categories and relations within the field of applied ontology, not at least as a foundation for conceptual modeling.

In this paper we report on our work on an ontology of categories and relations (and cognate notions), abbreviated  $\mathcal{CR}$ , which is intended to serve as a module for foundational ontologies. The approach has its background in the development of the General Formal Ontology (GFO) as well as in ontological semantics, a novel account of establishing semantics for representation or modeling languages on the basis of ontologies. The conceptual foundations of the ontology are introduced and we present initial axiomatization efforts.

**Keywords.** ontology, foundational ontology, top-level ontology, meta ontology, category, relation, relator

## 1. Introduction

Ontologies continue to be on the advance in many areas. Creating ontologies means to analyze a certain domain of interest in terms of the conceptual constituents by means of which the ontology is going to be formed. The major basic constituents of most ontologies today are *categories* and *relations*.<sup>3</sup> This is not surprising, as categorization / classification<sup>4</sup> and association are fundamental for our understanding of the world. Accord-

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<sup>2</sup>Parts of this submission are based on sections of [1] that have not been presented nor published elsewhere.

<sup>3</sup>Especially at this point, we assume only a rather weak semantic import of the words ‘category’ and ‘relation’. There are various alternative words / denominations in the literature that may be used for what we intend to refer to by those two, while one must be aware that all those denominations come with several more specialized readings in distinct contexts. Examples, ordered alphabetically, of alternatives for category are ‘class’, ‘concept’, ‘kind’, ‘notion’, ‘term’ (in a semantic reading), ‘type’ and ‘universal’. For relation, there are ‘association’, ‘property’ and ‘role’, among others. However, note already that ‘role’ is used to refer to another important notion herein, which is most closely related with relations.

<sup>4</sup>We use the word ‘categorization’ as a synonym of ‘classification’ herein. However, we employ only ‘category’ for a broadly conceived notion of instantiable entities. ‘Class’ is reserved for several notions that

dingly, they rank equally important in formal and applied ontology. We see it as a (if not *the*) central task of ontological analysis of any subject to address the questions of “Which (categories of) entities are there?” and “How are those entities interrelated?”.

Basically the same questions – we claim – are likewise present, at least implicitly, in almost all modeling enterprises. They are thus of similarly central nature for conceptual modeling, in particular, and for modeling in computer and information sciences in general. There the notions of category and relation frequently appear together among the very basic concepts of modeling approaches – of course, with varying terminology. For example, John Mylopoulos [3] reports that one novelty in M. Ross Quillian’s proposal of semantic networks [4] was to organize the information base in terms of ‘concepts’ and ‘associations’ [3, p. 131]. He further points out the distinction of ‘entities’ and ‘relationships’ in Peter P. Chen’s Entity-Relationship (ER) model [5]. From both, semantic networks and ER models, it seems to be only a short step in this regard to the Ontology Definition Metamodel (ODM) [6,7] and its meta model foundation, the Meta Object Facility (MOF) [8,9], which includes ‘classes’ and ‘associations’. Altogether, we see strong evidence for the fundamental importance of categories and relations, such that a well-founded theory of these notions appears desirable.

Categories and relations are further two highly generic/abstract notions, which leads us to top-level ontologies (TLOs) as well as to meta ontologies. TLOs are ontologies concerned with notions of very high generality, typically aiming at entities that appear in a large number of domain-specific areas. A meta ontology should provide notions that serve to characterize the constituents of an ontology and their interplay, which frequently also relates to language constructs that are employed in representing ontologies. The preceding considerations that categories and relations account for basic constituents of ontologies may thus suggest a meta-ontological nature of themselves, cf. also [1, sect. 2.3.3] and [10]. Indeed, for a number of the available TLOs the choice was made to focus solely on *individuals/particulars*<sup>5</sup> as entities that are subject to categorization and to being interrelated. But thereby categories and relations are used in many cases without a well-founded ontological understanding, which, we argue, should be developed. Especially if one aims at TLOs that are capable of classifying / covering all entities whatsoever,<sup>6</sup> analyzing category and relation as such remains an immediate (top-level) ontological problem, the solution of which may affect meta-ontological investigations nevertheless.

The overall goal of our work is to establish an ontology of categories and relations, which involves further cognate notions resulting from the analysis. There is a direct connection to the field of top-level ontologies, on the one hand, and to meta-ontological and semantic considerations, on the other hand. First of all, the project has emerged in the much wider context of constructing the General Formal Ontology (GFO) [11,12,13], see also [1, sect. 1.1.5]. GFO includes category and relation in its taxonomy, but that part still lacks an elaborate axiomatization yet. Hence we aim at proposing a corresponding ontology as a module for GFO. Due to the high degree of abstraction, it will be rather independent of other GFO specificities and may therefore be considered (1) in combi-

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arise in specific contexts in information and computer sciences, e.g. concerning the Semantic Web [2], object-orientation, and mathematical set or class theory.

<sup>5</sup>These two words are used interchangeably herein, albeit in-depth philosophical analyses may allow for distinguishing differing understandings. The term *logical individual*, however, leads to formal logic and Tarskian model theory, where it refers to a member of the universe of a formal model / interpretation structure.

<sup>6</sup>Surely, limitations based on the current state of science and knowledge apply.

nation with further TLOs and (2) as an ontological foundation for, primarily, meta modeling. The latter point has yet another intimate connection, namely to a novel semantic account that we propose in [1], called *ontological semantics*.<sup>7</sup> In a first step, such a semantics is intended to be defined for languages for ontology representation, before it can be extended to representations in general via translations. That first step requires itself an ontology to start from, which leads back to meta-ontological and meta-modeling issues.

In this paper we report on preliminary work in progress towards an ontology of categories and relations, named/abbreviated  $\mathcal{CR}$ . The conceptual foundations of the ontology are elaborated in sect. 2. Sect. 3 first summarizes a partial formalization which utilizes description logic / OWL2, which covers primarily a taxonomic fragment of the ontology. Secondly, a short preview on an axiomatization in first-order logic (FOL) is included. A discussion, also comprising some pointers to related work, is given in sect. 4 before the paper concludes with sect. 5.

## 2. Basic conceptions of categories and relations

### 2.1. Preliminary considerations and desiderata

Before expounding the view of categories and relations as it constitutes  $\mathcal{CR}$ , let us highlight some aspects that have an impact on developing the ontology. First of all, for both notions there is a large amount of material available in philosophy, cognitive science, computer and information sciences as well as in further areas that aims at defining or characterizing these notions. The amount becomes vast if we broaden the scope to literature which uses one or both notions or which is otherwise closely linked to them or just to aspects that one could consider in order to develop an ontology of categories and relations. Surveying even a fragment of this body of work deserves its own publication and cannot be accomplished herein. Therefore we present the subsequent theory of categories and relations without delivering a prior state of the art, while we shall mention connections to a very focused set of approaches in sect. 4.

That variety of literature comes along with an actual variety of differing views and understandings of categories and relations, e.g., cf. the analysis in [15] of seven of the many positions on the ontological status of categories within the philosophical literature. One intention that we should stress at this point is that especially for categories we are rather interested in a weak theory, such that various more specialized notions can fit under the  $\mathcal{CR}$  notion of category. For relations we adopt a more specific account which involves relation instances (called relators) that are composed of roles. This approach is already present in very early versions of GFO, where it was limited to so-called material relations. Besides finding the overall approach of role-based relations with relators very flexible and powerful, cf. e.g. [16, sect. 3.3], we drop that limitation and extend the approach to all relations.

Another guiding aspect is the desideratum of establishing a theory capable of self-reflection / self-analysis. More precisely, categorizing and interrelating the constituents of  $\mathcal{CR}$  should not require additional notions that are not already covered by  $\mathcal{CR}$ . That desideratum results from the more influential aim to eventually utilize the ontology in

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<sup>7</sup>It is not in any form closer related to the Ontological Semantics by Sergei Nirenburg, Victor Raskin and others [14], an approach for meaning extraction and representation from natural language texts.

the kind of “bootstrapping account” of semantics that is presented as ontological semantics in [1, ch. 4]<sup>8</sup>. We cannot sufficiently sketch ontological semantics here, given the spatial limitations. But such a self-reflective ontology can serve as a starting point for an ontological semantics, where certain notions of the ontology would serve as primitives in defining the semantics. This is not only another motivation for a clarified account of categories and relations, but it likewise yields a connection to meta-ontological matters, which is briefly reconsidered in sect. 4, with  $\mathcal{CR}$  already available. Yet, for all subsequent sections one should keep in mind that all constituents of  $\mathcal{CR}$  are also subject to the intended domain of discourse of that theory.

As a final note, we admit that the phrase “ontology of categories and relations” may cause expectations to find an in-depth classification or taxonomy of kinds of categories, relations or further constituents of the theory. To counteract such expectation, let us state already that this is not the case, although occasionally we draw inspiration from having special cases in mind. But first of all, herein we are interested in the notions (which we deem relevant) in themselves and an axiomatic characterization of them. A limited taxonomy emerges on that basis nevertheless, yet substantial extensions, e.g. analyzing different kinds of categories, remain future tasks.

## 2.2. Entities, categories, individuals and instantiation

In the domain “surrounding” the term ‘category’, we first identify four principal notions, namely entity, category, individual, and instantiation, depicted together with major interdependencies in Fig. 1.<sup>9</sup> As a kind of minimal consensus for *categories* we merely require that they are subject to being instantiated<sup>10</sup>, hence to the *instantiation / categorization* relation in a specific way. This relation, in Fig. 1 shown as the UML association named by the double colon :: (as in the FOL formalization of  $\mathcal{CR}$ ), links entities to the categories that they instantiate. In this connection an entity is called an *instance*<sup>11</sup> of a category. Remembering the intent to constrain categories as weakly as possible, no assumptions are made regarding the degree of generality of categories. Insofar they can be very specific, e.g., (for another ontology than  $\mathcal{CR}$ ) as determined by the phrase “female lion that lived in South Africa between 1994 and 1999”, or they can be highly abstract categories as in top-level ontologies.

Indeed, *entity* is the category that is instantiated by anything as soon as that anything exists. Existence is thereby understood in the broadest conceivable sense, following GFO’s recourse to, a.o., Roman Ingarden and different modes of existence [19]. Consequently, literally everything is an instance of entity, i.e., entity is a *universal category*.

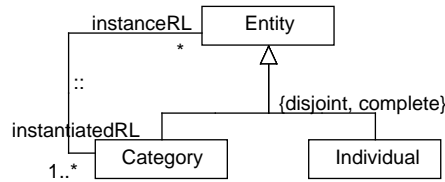
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<sup>8</sup>Alternatively, [17] outlines the general approach much more compactly, with core ideas already present in that earlier state of development.

<sup>9</sup>We employ class diagrams of the Unified Modeling Language version 2 (UML2) [18] solely for visualization purposes, assuming that the constraints so specified can be easily grasped by the reader. However, at the latest with hindsight of the overall conception of  $\mathcal{CR}$ , we interpret the diagrams within  $\mathcal{CR}$  itself.

<sup>10</sup>Note here already that “being subject to” does neither imply that every category *is instantiated*, nor even that it *can possibly be instantiated*, because neither non-instantiated categories are to be excluded, nor such that cannot be instantiated at all, like ‘round square’.

<sup>11</sup>The UML rolenames (following the terminology as in [18, p. 576 ff.], although ‘association end name’ is official for UML2) can better or only be explained after some parts of sect. 2.3. Besides the mnemonic parts of the role names, it may suffice here that the suffix ‘RL’ stands for ‘role’ itself and the difference between instance as just introduced and the instance role becomes clear in the section below.



**Figure 1.** Entities, Categories and Individuals in  $\mathcal{CR}$ .

Thus categories cannot only be instantiated, but they are instances themselves. Not at least, any category instantiates the category `Category` (indicating a technical  $\mathcal{CR}$  name by this font), which may be in conflict with one or another TLO. In line with many TLOs, however,  $\mathcal{CR}$  includes (also) *individuals* / particulars, i.e., entities that are instances of categories, but are not subject to being instantiated themselves. Entity is partitioned into categories and individuals, which entails that every entity instantiates either `Category` or `Individual`. In contrast, it is not enforced that categories must have any instances.

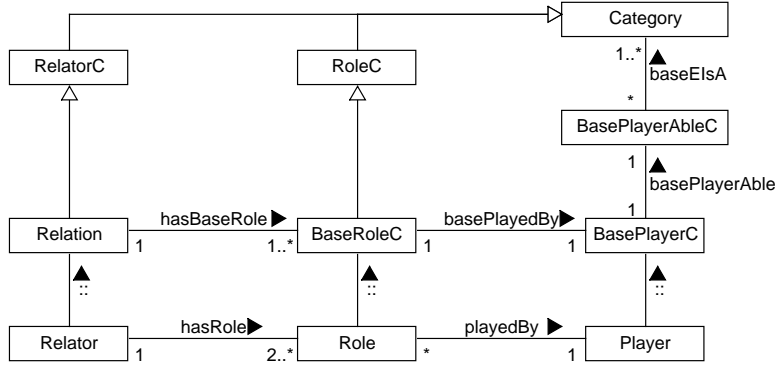
Considering instantiation anew, the above paragraphs imply that  $\mathcal{CR}$  yields no stratified nor well-founded account of instantiation. Without further restrictions, there may be instantiation chains of arbitrary length, as well as instantiation cycles. For example, we subscribe to the facts that Entity instantiates Entity (cycle of length 1), and that Entity instantiates Category and Category instantiates Entity (length 2). This is not too uncommon in applied ontology and possibly even more accepted in knowledge representation and the Semantic Web, cf. e.g. the notion of Resource in the Resource Description Framework (RDF) [2, ch. 2–3]. Without room for going into details (cf. [1, sect. 2.4.2.3] for a bit more), this affects relationships between  $\mathcal{CR}$  and set theory, for instance, if the notion of the *extension* of a category is considered, defined as the set of all and exactly the instances of this category. Then, relying on a wellfounded set theory, not every category may be equipped with a proper extension, or one adopts an alternative set-theoretic approach, such as non-wellfounded set theory [20].

A related and here final aspect is that of comprehension axioms for categories, which may easily lead to inconsistency of the theory, e.g., in analogy to Russell’s paradox [21]. At the present stage we expect to circumvent corresponding problems by not adopting any axioms for category comprehension for  $\mathcal{CR}$ .

### 2.3. Relations, relators, roles and non-relating entities

As already indicated in sect. 2.1, the part of  $\mathcal{CR}$  on relations is more specifically determined than the deliberately weak theory of categories. The basic view on relations combines a positionalist account<sup>12</sup> with one of individualized properties. More precisely, *relations* in  $\mathcal{CR}$  are categories which are instantiated by individuals, called *relators*, that are composed of roles and that have the power of relating / mediating between / gluing together arbitrary entities. Relators establish the relation that they instantiate between the entities / arguments / relata that they connect. At the categorial level, one may consider different kinds of relator categories, but relations are special insofar that they capture the connecting character of their instances among the entities related.

<sup>12</sup>See [22, esp. sect. 3.2] for a condensed exposition of that stance within conceptual modeling, with pointers to treatments in philosophy.



**Figure 2.** Relations with role base notions and corresponding categories of individuals. A C at the end of a name stands for category. Instantiation applies at the categorial level here, e.g., relators instantiate relations.

To discuss a concrete example and at the same time to demonstrate a case that is commonly seen as a formal relation (which holds immediately as soon as the relata are given), we consider in the domain of natural numbers that 1 is less than 2, commonly written  $1 < 2$ . Analyzing this fact in  $\mathcal{CR}$ , we can first distinguish four entities, namely the numbers 1 and 2, the less-than relation  $<$ , and a particular relator  $r^<$  instantiating  $<$  and mediating particularly between 1 and 2. With just that relator available, it is not clear in which – potentially distinct – ways 1 and 2 participate in the relation.

This is accounted for by *roles* (and so-called *base role categories* at the level of categories / relations), of which there is one per argument. Roles are individuals that can be seen as parts of relators, although within  $\mathcal{CR}$  we name that relation *role of*, symbolized by  $\dashv$  in the FOL formalization, or *has role*, denoted by  $\circ$ , in inverse reading.<sup>13</sup> In the example,  $r^<$  is composed of two roles, say,  $q_g$  and  $q_l$ . However, (grasping) the “semantics” / “nature” of the two regarding  $<$  relies on specific role categories that they instantiate. Indeed, the overall semantics of  $<$  depends on determining role categories such that relators composed of roles of those categories adequately reflect in which way each single argument participates in the relator (and thereby / loosely speaking, “in the relation”). We say in such cases that a role category is a *base role category* for a relation. For  $<$  let us name its base role categories *less* and *greater*.

Another connection is required to / from a role w.r.t. the corresponding argument of the relator, e.g., from  $q_l$  to 1. This connection is called *plays / plays role / fills role*, with the FOL predicate symbol  $\rightsquigarrow$  (or  $\leftarrow$ , if read reversely). To conclude our example coherently, we need  $1 \rightsquigarrow q_l$  and  $2 \rightsquigarrow q_g$ . Overall, the role analysis of  $1 < 2$  yields the subsequent formula, where all #. denote constant symbols with the same intended meaning of categories or relations as the predicate symbols without the prescript, thus reifying the latter (cf. also sect. 3).

$$r^< :: \#< \wedge q_l :: \#\text{less} \wedge q_g :: \#\text{greater} \wedge 1 \rightsquigarrow q_l \wedge q_l \dashv r^< \wedge r^< \circ q_g \wedge q_g \leftarrow 2$$

Meanwhile, not only are the three fundamental relations for  $\mathcal{CR}$  introduced, namely instantiation, role-playing and role-having, but the bottom layer of Fig. 2 is already covered.

<sup>13</sup>Importantly, this inversion is merely one of expressing the relation in a linear language, while ‘role of’ and ‘has role’ refer to the very same relation, from a positionalist stance. Cf. also [23, esp. sect. 4.1] on this aspect.

red by the preceding paragraphs. Even base role categories are mentioned above, yet they deserve further consideration. They capture the different ways / kinds in which entities may be involved in the specific relation, contributing to the comprehensibility of a relation. Their name originates from the notion of a *role base* for a relation, which comprises not only the base role categories, but possibly further constraints, cf. [1, sect. 2.4.3.3].

Here we restrict ourselves to considering the additional categories BasePlayerC and BasePlayerAbleC in Fig. 2, which should be less common (even under different names), but are likewise informative w.r.t. relations as well as useful in expressing certain connections. All three kinds of categories result from a trichotomy derivable from understanding role notions. Take ‘instance’ as an example, already introduced above, and observe that the role name near Entity in Fig. 1 is instanceRL, where RL abbreviates ‘role’. First, ‘instance’ may be said to be a role / base role category itself (and thus an appropriate UML rolename). If  $\mathcal{CR}$  is applied to instantiation itself, indeed, there is an instance role within any instantiation relator, which is played by the entity that instantiates something in virtue of that relator. Secondly, in  $r^< :: \#<$  we would say that  $r^<$  is the instance. From this point of view ‘instance’ is rather a category of (primarily) non-role entities that actually stand in the instantiation relation (to other entities). This categorization is one from the point of view of the relation. For that reason we use instanceRL for the role category, reserving ‘instance’ for the players of that role / base role category. Thirdly and associated with common domain and range specifications for relations, there is the question of which entities are at all subject to playing a role of a relation. That question can typically be given an answer from the point of view of the relation or the role. For example, all entities that can be instances are ‘categorizable’. Everything that can be instantiated is ‘instantiable’. These are cases of BasePlayerAbleC. Note that those are always role- or relation-based categories, in contrast to categories of role- and relation-*independent* entities. Claiming that Instantiable is equivalent to Category then yields that all and only categories are subject to being Instantiated, i.e., to playing the InstantiatedRL. Table 1 accounts for key components of the role bases of the three fundamental  $\mathcal{CR}$  relations.

Relation	BaseRoleC	BasePlayerC	BasePlayerAbleC	Category
::	InstanceRL	Instance	Categorizable	Entity
::	InstantiatedRL	Instantiated	Instantiable	Category
$\rightsquigarrow$	PlayerRL	Player	PlayerAble	Entity
$\rightsquigarrow$	PlayedRL	Played	Playable	Role
$\rightarrow$	RoleRL	RoleRole	RoleBeAble	Role
$\rightarrow$	ContextRL	Context	RoleHaveAble	Relator

**Table 1.** Role base notions of instantiation (::), plays role ( $\rightsquigarrow$ ) and role of ( $\rightarrow$ ), following Fig. 2. The suffix C in three column heads stands for ‘category’, RL in the base role column originates from ‘role’.

### 3. Towards formalizing the theory $\mathcal{CR}$

Our primary aim in formalizing  $\mathcal{CR}$  is to develop its axiomatization in first-order logic (FOL). Beforehand, we started with a partial axiomatization in description logic (DL). For the purposes of this paper, all formalizations are solely considered under standard set-theoretic semantics.

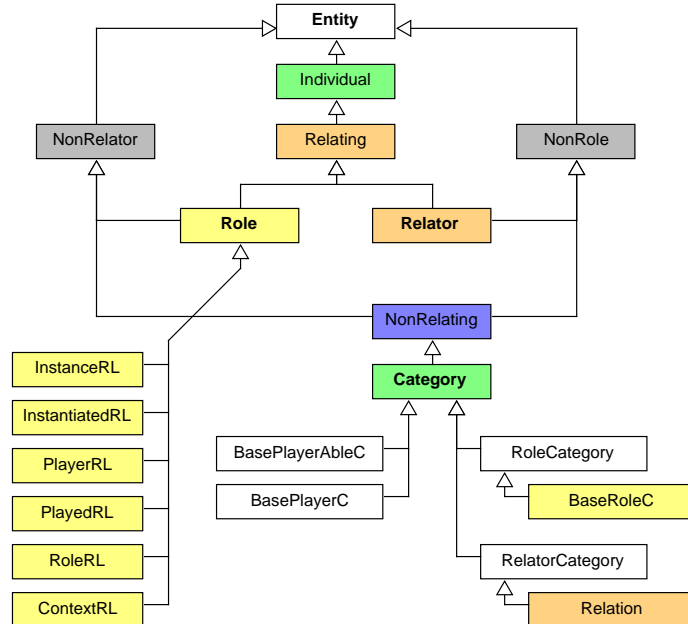
### 3.1. Taxonomic fragment in OWL

The informal exposition of  $\mathcal{CR}$  involves a number of<sup>14</sup> categories in intertwined ways. Therefore we focused on the taxonomy first, limited to DL and by utilizing Protégé [25] as OWL2 editor equipped with the Hermit reasoner [26,27] in version 1.3.8, for a first check for potentially unexpected effects.

The corresponding theory in DL is presented in [1, App. B, p. 281–283], such that we do not include specific DL axioms here. Moreover, this version of the ontology is available as an OWL2 file, see [24]. It is worth noting that this DL version does not include any self-reflection of  $\mathcal{CR}$ , for example, no DL individuals intended to reify  $\mathcal{CR}$  categories and/or relations. A corresponding extension is a future task for further testing, while the present version may be more amenable for reuse in other ontologies.

Still, some observations about this version of  $\mathcal{CR}$  appear to be of interest. First of all, the reasoner succeeded in classifying the ontology file without reporting inconsistency of the overall file, and without detecting incoherent, i.e., unsatisfiable DL concepts. A larger fragment of the resulting classified taxonomy of  $\mathcal{CR}$  is depicted in Fig. 3. It still includes some auxiliary notions that turned out useful in the course of the axiomatization. In particular, we expect that the categories Relating (orange; the union of Role and Relator) and NonRelating (blue; the complement of Relating relative to Entity / OWL Thing) are widely usable, e.g., as NonRelating is silently assumed in many cases of referring to Individual. Two major partitionings of Entity are (1) that into categories and individuals as in Fig. 1 and (2) that into non-relating entities, roles and relators.

<sup>14</sup>Indeed, not all categories discussed in [1, sect. 2.4 and 6.1] or contained in the OWL file `cr-dl.owl` [24] (therein 38 in total) could be discussed in the present paper.



**Figure 3.** Taxonomy fragment of selected  $\mathcal{CR}$  categories, as classified by Hermit [26]. Bold-face labels are representatives of groups of equivalent named categories. Colors provide only a vaguely defined grouping.



Overall, Fig. 3 corresponds well to our expectations. Yet there is one result that was not immediately obvious from the DL axioms, because it grows out of the domain and range constraints of  $\mathcal{CR}$  relations and some axioms about the notions introduced as role bases of those relations, cf. Table 1. In summary, the reasoner concludes the subsequent (extensional) equivalences among  $\mathcal{CR}$  categories. Each line corresponds to one of those nodes in Fig. 3 labeled in bold face. Although this high number of equivalent categories may seem awkward for an ontology in general, inspection shows that all of them are (in the light of  $\mathcal{CR}$ ) reasonable cases of intensionally distinct, but coextensional categories.

$$\text{Entity} \equiv \text{Categorizable} \equiv \text{Instance} \equiv \text{PlayerAble} \equiv \text{Player} \quad (1)$$

$$\text{Category} \equiv \text{Instantiable} \quad (2)$$

$$\text{Relator} \equiv \text{Context} \equiv \text{RoleHaveAble} \quad (3)$$

$$\text{Role} \equiv \text{Playable} \equiv \text{Played} \equiv \text{RoleRole} \equiv \text{RoleBeAble} \quad (4)$$

### 3.2. Signature $\Sigma^{\mathcal{CR}}$ and preview on axioms in FOL

The axiomatization in first-order logic (FOL) is still in a preliminary stage, first of all. It follows the ideas of ontological semantics [1, ch. 4–5], which implies some methodological specialties. Nevertheless, all formulas specified in this section can properly be considered under classical set-theoretic semantics. We consider three binary relations of  $\mathcal{CR}$  as *fundamental relations*, which are the only ones that are solely axiomatically characterized. These relations are instantiation ( $::$ ), role playing ( $\rightsquigarrow$ ) and role having ( $\rightarrow$ ), where those symbols are binary predicate symbols that are used in infix notation.

Every other  $\mathcal{CR}$  notion is first reified by a logical constant symbol and thus subject to the domain of discourse.<sup>15</sup> Moreover, predicate symbols are typically introduced for convenience, but all of them must be “explained” by being equipped with a so-called *predication definition*, intended to fix the “semantics” of atomic formulas with that predicate. Notationally, like in sect. 2.3, we adopt the convention to let reifying constants be preceded by  $\#$ . For example, if  $\text{Ent}$  is a unary predicate symbol intended to denote the  $\mathcal{CR}$  category Entity, then  $\#\text{Ent}$  is a logical constant for which there is the same intent.  $\text{Pred}(\cdot)$  denotes all predicates of / in the argument,  $\text{Const}(\cdot)$  all constant symbols,  $\text{ar}(\cdot)$  the arity of its argument.

Subsequently we specify a very limited, selective preview on the FOL axiomatization over the signature  $\Sigma^{\mathcal{CR}}$ . Formulas that are / were intended as axioms, but are already shown to be entailed by others, are kept in the list of “axioms” / formulas, yet numbered with prefix C (with references to entailing formulas in their description), whereas the numbering of non-entailed and thus proper axioms has prefix A, that of definitions D.

#### Definitions

$$\text{D1. } \{P(x) \leftrightarrow_{df} x :: \#P \mid P \in \text{Pred}(\Sigma^{\mathcal{CR}}), \text{ar}(P) = 1\}$$

(uniform predication definitions for all unary predicates)

$$\text{D2. } \text{Ent}(x) \leftrightarrow_{df} \exists y . y = x \quad (x \text{ is an entity :iff } x \text{ exists})$$

<sup>15</sup>Introducing a constant symbol and thus reification applies likewise to the fundamental relations.

### Axioms and consequences

- A1.  $\{\#P \neq \#Q \mid \#P, \#Q \in \text{Const}(\Sigma^{\mathcal{CR}})\}$  (any two constants refer to distinct entities)  
A2.  $\{\exists x . P(x) \mid P \in \text{Pred}(\Sigma^{\mathcal{CR}}), \text{ar}(P) = 1\}$  (all  $\Sigma^{\mathcal{CR}}$  categories are instantiated)  
C1.  $\{\text{Cat}(\#P) \mid P \in \text{Pred}(\Sigma^{\mathcal{CR}}), \text{ar}(P) = 1\}$   
(categorization of constants as categories [A2, D1, A4])  
C2.  $\text{Ent}(x)$  (everything is an entity [D2])  
A3.  $\neg \exists x . \text{Role}(x) \wedge \text{Relator}(x)$  (role and relator are disjoint categories)  
A4.  $x :: y \rightarrow \text{Ent}(x) \wedge \text{Cat}(y)$  (instantiation relates entities with categories)  
A5.  $x :: y \wedge \text{Relation}(y) \rightarrow \text{Relator}(x)$  (instantiation of relations yields relators)  
A6.  $x :: y \wedge \text{RoleCat}(y) \rightarrow \text{Role}(x)$  (instantiation of role categories yields roles)  
A7.  $x \rightsquigarrow y \rightarrow \text{Ent}(x) \wedge \text{Role}(y)$  (plays relates entities with roles)  
A8.  $x \dashv y \rightarrow \text{Role}(x) \wedge \text{Relator}(y)$  (role-of relates roles with relators)  
A9.  $\text{Relator}(x) \rightarrow \exists y . \text{Relation}(y) \wedge x :: y$  (every relator instantiates  $\geq 1$  relation)  
A10.  $x \dashv y \wedge x \dashv z \rightarrow y = z$  (the relator of a role is uniquely determined)  
C3.  $x \dashv y \rightarrow (\neg \exists u u \dashv x) \wedge (\neg \exists v v \dashv y)$  (role-of is a single-step relation [A3])  
C4.  $(\exists y . y :: x) \rightarrow \text{Cat}(x)$  (if an entity is instantiated, it is a category [A4])  
C5.  $\text{Ent}(x) \leftrightarrow \exists y . \text{Cat}(y) \wedge x :: y$   
(every entity instantiates a category [C2, D1 for Ent, A4])

Admittedly, this is just a minimal selection of axioms, primarily demonstrating some axioms resulting from the reification aspect and then focusing on constraints of the fundamental relations. One can observe that some assumptions on the reifying constants have already effects on those relations. More of that can already be found in [1, sect. 6.1.2], e.g., where most of the common properties of orderings turn out not to be satisfied by instantiation, based on few axioms, a.o., having entity as a universal category.

The full theory is still under development. This goes in tandem with working on a consistency proof for the FOL theory.

## 4. Discussion and related work

With  $\mathcal{CR}$  we aim at a theory of categories and relations that is well-suited as a foundation for the semantics of languages as well as for other ontologies. The latter aspect concerns meta-ontological capabilities of  $\mathcal{CR}$ , i.e.,  $\mathcal{CR}$  notions lend themselves to an ontological analysis of the constituents of ontologies. That aim of serving as a foundation for other ontologies is as broad as to cover arbitrary ontologies, including  $\mathcal{CR}$  itself. It is therefore pursued by proposing a self-contained / self-analytic theory. As may be clear from the sections above, all notions of  $\mathcal{CR}$  are themselves considered to be in the intended domain of discourse of the theory, such that, for example, all  $\mathcal{CR}$  constituents can be categorized by  $\mathcal{CR}$  categories. Moreover, literally *all* relations of  $\mathcal{CR}$  are subject to the same analytic account based on relators composed of roles. This applies even to the three fundamental binary relations, cf. the discussion of that aspect in [1, sect. 2.4.4.3 and near the end of sect. 6.1.2]. A price to be paid for this is that  $\mathcal{CR}$  is subject to infinite regresses, which lead not only to infinite (logical) models of the full theory, but which are often considered as harmful, if not necessary to be avoided. Unfortunately, we lack the space herein to argue in detail against necessarily seeing a problem with the particular regresses that occur, on philosophical, but also logical grounds. At least, we accept them in favor of

the – for us – very attractive feature that  $\mathcal{CR}$  does not rely on any additional assumptions nor on any kinds of entities that are considered solely external to the theory or that are declared to be not analyzable.

A few words remain to be devoted to related work. Focusing on applied ontology and existing ontologies first, we mention the Unified Foundational Ontology (UFO) [28] as the TLO with the most elaborate accounts of categories and relations that we are aware of. Therefore, UFO is the prime candidate for an in-depth comparison after  $\mathcal{CR}$  is fully established. A few other TLOs cover notions that  $\mathcal{CR}$  deals with, as well, e.g., the Suggested Upper Merged Ontology (SUMO) [29].

In the domain of conceptual modeling, we see two closely related lines of research. Admittedly, only recently having become much more clearly aware of the abundance of modeling and meta modeling approaches involving a positionalist account of relations (and thus roles), see e.g. [22, esp. sect. 3.2], one may consider to what extent  $\mathcal{CR}$  might yield novel contributions conceptually, for instance, remembering the discussion of base role, base player and base playerable categories, and / or whether it may serve as a foundation for ontology-driven conceptual (meta) modeling, e.g., in the sense of [22]. Similarly, we see a connection to fairly recent work on multi-level modeling such as [30], as  $\mathcal{CR}$  allows for complex categorization levels.

## 5. Conclusions

In this paper we argue for the striking importance of well-developed accounts of categories and relations and the remaining need to establish formal ontological theories for this subject matter. We are in the process of working on one candidate theory, abbreviated as  $\mathcal{CR}$ . Primarily its conceptual approach is elaborated herein to some extent, following [1], accompanied by reports on the status of our formalization efforts.

Not surprisingly, we consider  $\mathcal{CR}$  as a promising approach, while that largely remains a claim herein. Our next steps comprise of completing the first-order axiomatization and its meta-theoretic analysis. Afterwards or possibly starting in parallel, it will be instructive to apply and relate  $\mathcal{CR}$  not only to itself, but to other ontologies as well as in the foundation of conceptual modeling.

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