From Free Will Debate to Embodiment of Fuzzy Logic into Washing Machines: On Fuzzy and Rough Sets Approaches to Vagueness Modeling*

Extended Abstract

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Deep scientific ideas have at least one distinctive property: they can be applied both by philosophers in abstract fundamental debates and by engineers in concrete practical applications. Mathematical approaches to modeling of vagueness also possess this property. Problems connected with vagueness have been discussed at the beginning of XXth century by philosophers, logicians and mathematicians in developing foundations of mathematics leading to clarification of logical semantics and establishing of mathematical logic and set theory. Those investigations led also to big step in the history of logic: introduction of three-valued logic. In the second half of XXth century some mathematical theories based on vagueness idea and suitable for modeling vague concepts were introduced, including fuzzy set theory proposed by Lotfi Zadeh in 1965 [16] and rough set theory proposed by Zdzisşaw Pawlak in 1982 [4] having many practical applications in various areas from engineering and computer science such as control theory, data mining, machine learning, knowledge discovery, artificial intelligence.

Concepts in classical philosophy and in mathematics are not vague. Classical theory of concepts requires that definition of concept C hast to provide exact rules of the following form:

if object x belongs to concept C, then x possess properties $P_1, P_2, \dots, P_n$;

if object x possess properties $P_1, P_2, \dots, P_n$, then x belongs to concept C.

In classical set theory concepts are sets and their non-vagueness (crispness) is expressed by characteristic functions: with every set $A \subseteq U$ there is function $\chi_A : U \longrightarrow \{0, 1\}$ such that $\chi_A(x) = 1$ iff $x \in A$, otherwise $\chi_A(x) = 0$.

Lotfi Zadeh introduced fuzzy sets as generalizations of characteristic functions together with operations based on Łukasiewicz's logical operations taken from three-valued logic: a fuzzy set X consisting of object from domain U is defined by membership functions $\mu_X : U \longrightarrow [0, 1]$, where $\mu_X(a)$ reflects a grade/degree in which object

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a belongs to fuzzy set X what corresponds to vagueness of concepts. In particular if $\mu_X(a) = 0$, then object a does not belong to fuzzy set X and if $\mu_X(a) = 1$, then object a belongs to fuzzy set X in full degree what corresponds to classical notion of belonging to a set.

In rough set theory concepts are represented by subsets of a given space U and objects are represented by granules, some collections of objects, in classical rough set theory these granules are equivalence classes of some equivalence relation R on U. In rough set theory sets are represented and analyzed by two operators: lower and upper approximations, denoted respectively by R_*, R^* and defined for set $X \subseteq U$ as follows:

$$R_*(X) = \bigcup\{Y \in U_{/R} : Y \subseteq X\} \qquad R^*(X) = \bigcup\{Y \in U_{/R} : Y \cap X \neq \emptyset\}.$$

Set $X \subseteq U$ is rough iff $R_*(X) \neq R^*(X)$. With every set $X \subseteq U$ there are associated three sets called regions:
positive region

$$POS(X) := R_*(X),$$

negative region

$$NEG(X) := U \setminus R^*(X) = R_*(X') = \bigcup\{E \in U_{/R} : E \cap X = \emptyset\}$$

where $X' = U \setminus X$ and boundary region

$$BND(X) := R^*(X) \setminus R_*(X) = U \setminus (POS(X) \cup NEG(X)).$$

Set $X \subseteq U$ is rough iff $BND(X) \neq \emptyset$. In the case of any set $X \subseteq U$ positive region of X can be interpreted as a set of objects from U which surely belongs to X, negative region can be interpreted as a set of objects from U which surely do not belong to X, whereas boundary region can be interpreted as a set of objects from U which possibly belong to X.

One can note that both approaches to modelling vagueness are some generalizations in which crisp sets are particular cases: in fuzzy set theory fuzzy set $X$of objects from U is crisp iff for all $a \in U$ either $\mu_X(a) = 1$ or $\mu_X(a) = 0$; in rough set theory set $X \subseteq U$ is crisp iff $BND(X) = \emptyset$. Both theories are also essentially connected with Łukasiewicz's ideas.

The main object of the paper is to present and compare fuzzy sets and rough sets approaches to vagueness and uncertainty modelling and analysis, in particular we will discuss representation of vague concepts in both theories. We will also present Łukasiewicz's arithmetization of propositional calculus semantics and Łukasiewicz's involvement in discussion on meaning and logical values of propositions about future which led to introduction of the third logical value and to proposing three-valued logic. Our comparison of fuzzy set theory and rough set theory approaches to vagueness modelling will be made with respect to a characterization of vagueness proposed in contemporary philosophy. This characterization includes a second order vagueness condition which, roughly speaking, requires that a boundary of a concept cannot be a crisp set. We will conclude the paper with presenting and discussion solutions to that problem in the rough set approach to vagueness modelling including that proposed by Andrzej Skowron in [11].

References

1. Keefe, R.: Theories of Vagueness. Cambridge Studies in Philosophy, Cambridge University Press (2000).
2. Keefe, R., Smith P.: (Eds.) Vagueness: A Reader. MIT Press, Massachusetts, MA.
3. Pawlak, Z.: Rough sets. International Journal of Computing and Information Sciences. **18** (1982) 341–356.
4. Pawlak, Z.: Rough sets and fuzzy sets. Polish Academy of Sciences, Institute of Computer Sciences Reports **540** (1984).
5. Pawlak, Z.: Rough sets. Theoretical Aspects of Reasoning About Data. Kluwer Academic Publishers (1991).
6. Pawlak, Z.: An Inquiry into Vagueness and Uncertainty. Warsaw University of Technology, Institute of Computer Science Reports **29** (1994).
7. Pawlak, Z.: Some Issues on Rough Sets. Transactions on Rough Sets, I, Journal Subline, Lectures Notes in Computer Science. **3100** (2004) 1–58.
8. Pawlak, Z., Skowron, A.: Rough Membership Functions. In R. R. Yager, M. Fedrizzi, J. Kacprzyk (Eds.), Advances in the Dempster-Schafer Theory of Evidence, John Wiley and Sons (1994), New York, 251Ű-271.
9. Pawlak, Z., Skowron, A.: Rudiments of rough sets. Information Science. **177** (2007) 3–27.
10. Read, R.: Thinking about Logic: An Introduction to the Philosophy of Logic. Oxford University Press, Oxford, New York (1995).
11. Skowron, A.: Rough sets and vague concepts. Fundamenta Informaticae **64**(1-4) (2005) 417–431.
12. Skowron, A., Stepaniuk, J.: Hierarchical modelling in searching for complex patterns: constrained sums of information systems. Journal of Experimentaland Theoretical Artificial Intelligence **17**(1-2) (2005) 83–102.
13. Skowron, A., Stepaniuk, J.: Approximation Spaces in RoughŰGranular Computing. Fundamenta Informaticae **100** (2010) 141-Ű157.
14. Skowron, A., Wasilewski, P.: Interactive information systems: Toward perception based computing, Theoretical Computer Science **454**, 240-260 (2012).
15. Wasilewski, P., Ślęzak, D.: Foundations of Rough Sets from Vagueness Perspective. In: A.E. Hassanien, Z. Suraj, D. Ślęzak, D., P. Lingras (Eds.) Rough Computing. Theories, Technologies and Applications. Information Science Refer. (2008) 1-Ű37.
16. Zadeh, L.A.: Fuzzy Sets. Information and Control 8(3), 338–353 (1965).
17. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning - Part I. Information Sciences 8(3), 199–249; Part II. Information Sciences 8(4), 301–357; Part III. Information Sciences 9(1), 43–80 (1975).
18. Zadeh, L.A.: Fuzzy sets as a basis for theory of possibility, Fuzzy Sets and Systems **1**, 3–28 (1978).
19. Zadeh, L.A.: Computing with Words: Principal Concepts and Ideas. Springer Verlag, Berlin, Heidelberg (2012).