SECTION – 1

1. A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as $T(t) = T_0(1 + \beta t^{1/4})$ where β is a constant with appropriate dimension of temperature. The heat capacity of metal is:

   (a) $\frac{4P\left(T(t)-T_0\right)^3}{\beta^4 T_0^4}$ (b) $\frac{4P\left(T(t)-T_0\right)^2}{\beta^4 T_0^3}$ (c) $\frac{4P\left(T(t)-T_0\right)^4}{\beta^4 T_0^5}$ (d) $\frac{4P\left(T(t)-T_0\right)}{\beta^4 T_0^2}$

Solution:

$$\text{Heat capacity } = \frac{dQ}{dT}$$

$$H = \frac{dQ}{dT} \Rightarrow \frac{dQ}{dt} = H.\frac{dT}{dt}$$

$$P = H\frac{d}{dt}T$$

$$= \frac{H\,T_0}{4}\beta \quad t^{-3/4}$$

$$\frac{4P}{T_0\beta} = H\ t^{-3/4}$$

$$t^{-3/4} = \left(\frac{T - T_0}{T_0\beta}\right)^3$$

$$H = \frac{4P\left(T - T_0\right)^3}{T_0^4\beta^4}$$

2. In a capillary tube of radius 0.2 mm the water rises up to height of 7.5 cm with angle of contact equal to zero. If another capillary with same radius but of different material dipped in the same liquid. The height of water raised in capillary will be, if angle of contact becomes 60°.

   (a) 7.5 cm (b) 15 cm (c) 3.75 cm (d) 30 cm

Solution:

$$T = \frac{Rh\rho g}{2\cos\theta}$$

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$$\frac{h}{\cos\theta} = const$$

$$\frac{7.5}{1} = \frac{h'}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow h' = 3.75cm$$

3. A sample of ${}_{19}K^{40}$ disintegrates into two nuclei Ca & Ar with decay constant $\lambda_{Ca} = 4.5\times10^{-10} S^{-1}$ and $\lambda_{Ar} = 0.5\times10^{-10} S^{-1}$ respectively. The time after which 99% of ${}_{19}K^{40}$ gets decayed is:

(a) 6.2×10^9 sec (b) 9.2×10^9 sec (c) 7.2×10^9 sec (d) 4.2×10^9 sec

Solution:

$$\lambda = \lambda$$

$$\frac{1}{100}\cancel{No} = \cancel{No}\ e^{-\lambda t}$$

$$\ln\left(\frac{1}{100}\right) = -(\lambda_1+\lambda_2)t + \ln 100 = +(\lambda_1+\lambda_2)t^2$$

$$\frac{2.303\times2}{5\times10^{-10}} = t$$

$$t = 9.2\times10^9 \text{ sec}$$

4. Consider a spherical gaseous cloud of mass density ρ(r) in a free space where r is the radial distance from its centre. The gaseous cloud is made of particles of equal mass m moving in circular orbits about their common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If ρ(r) is constant with time. The particle number density n(r) = ρ(r)/m is:
(g = universal gravitational constant)

(a) $\frac{3K}{\pi r^2 m^2 G}$ (b) $\frac{K}{2\pi r^2 m^2 G}$ (c) $\frac{K}{\pi r^2 m^2 G}$ (d) $\frac{K}{6\pi r^2 m^2 G}$

Solution:

$$\frac{GMm}{r} = \frac{m\upsilon^2}{r}$$

$$= \frac{2}{r}\frac{1}{2}m\upsilon^2$$

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$$=\frac{2}{r}k$$

$$M=\frac{2kr}{Gm}$$

$$4\pi r^2\,\cancel{dr}\,\rho=\frac{2k\,\cancel{dr}}{Gm}$$

$$\rho=\frac{k}{2\pi Gmr^2}$$

SECTION – 2

5. A thin spherical insulating shell of radius R caries a uniformly distributed charge such that the potential at its surface is V_0. A hole with small area $\alpha 4\pi R^2\left(\alpha<<1\right)$ is made in the shell without effecting the rest of the shell. Which one of the following is correct.

(a) The magnitude of $\vec{E}$ at a point located on a line passing through the hole and shell's centre on a distance 2R from the centre of spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

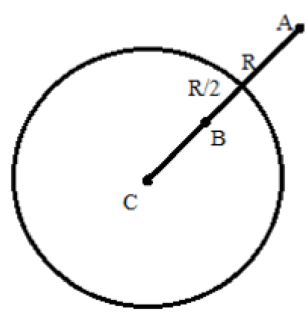
(b) Potential at the centre of shell is reduced by $2\alpha V_0$.

(c) The magnitude of $\vec{E}$ at the centre of shell reduced by $\frac{\alpha V_0}{2R}$

(d) The ratio of potential at the centre of the shell to that of the point at $\frac{1}{2}$ R from centre towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

Solution:

$$dq=\frac{Q}{4\pi R^2}-dA=Q\alpha$$



$$V_C=\frac{KQ}{R}-\frac{K\alpha Q}{R}=V_0\left(1-\alpha\right)$$

$$V_B=\frac{KQ}{R}-\frac{K\alpha Q}{R/2}=V_0\left(1-2\alpha\right)$$

$$V_A = \frac{KQ}{(2R)^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$$

(reduced by $\frac{\alpha V_0}{R}$)

$$E @ C = \frac{K\alpha Q}{R^2} = \frac{\alpha V_0}{R}$$

Increased by $\frac{2V_0}{R}$

6. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r & with its center same as that of the shell. Here center of cylinder is a point on the axis of the cylinder which is equidistant from its top & bottom surfaces. Which of the following are correct.

(a) If h > 2R & r > R then $\phi = \frac{Q}{\varepsilon_0}$

(b) If $h < \frac{8R}{5}$ & $r = \frac{3R}{5}$ then ϕ = 0

(c) If h > 2R & r $= \frac{4R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

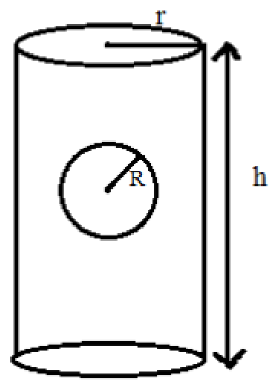
(d) If h > 2R & r $= \frac{3R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

Solution:

(1) If h > 2R

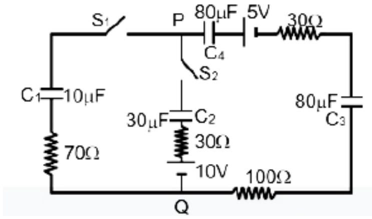
$$Q = \frac{Q}{E_0}$$

(2) $h = \frac{8R}{5}$ $\quad r = \frac{3R}{5}$



Using Gauss law concept ABD are correct

7. Which statements is/are correct:



(a) At time t = 0, the S_1 is closed instantaneous current in the closed circuit will be 25 mA

(b) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed at this time the instantaneous current across 30Ω resistor between P & Q will be 0.2A.

(c) If key S_1 is kept closed for long time such that capacitors are fully charged the voltage across C_1 will be 4V.

(d) If S_1 is kept closed for long time such that capacitors are fully charged the voltage difference between P & Q will be 10V.

8. A galvanometer of resistance 10 ohm and maximum current of 2μA is converted into voltmeter of range 100mV and when converted into ammeter then range is 1mA. When these voltmeter and ammeter are connected by a (ideal) battery in series with a resistance of R = 1000Ω, then

(a) Measured value of R is between 978Ω and 996Ω

(b) Resistance of voltmeter $10^5\Omega$

(c) Shunt resistance is 20mΩ

(d) If the ideal battery is replaced by non-ideal battery with internal resistance of 5Ω then R will be > 1000 Ω

Solution:

$$V = 100\times10^{-3}V$$

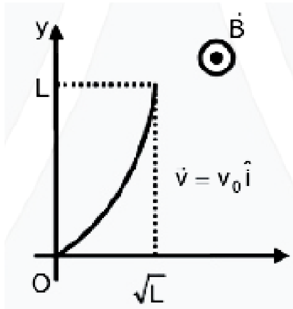
$$V = Ig\left(Rg+R\right)$$

$$= \frac{10^{-1}}{2\times10^{-6}} = \left(Rg+R\right)^{R_V}$$

$$R_V = 5\times10^{4}$$

$$S = \left(\frac{10}{\frac{10^{-3}}{2\times10^{-3}}-1}\right) = 20\,m\Omega$$

9. Conducting wire of parabolic shape, initially $y = x^2$ is moving with velocity $\vec{v} = v_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$ as shown in figure. If V_0, B_0, L & B are +ve constants & Δϕ is potential difference develop between the ends of wire, then correct statement(s) is/are



(a) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for β = 0

(b) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for β = 2

(c) $|\Delta\phi|$ is proportional to the length of wire projected on y-axis

(d) $|\Delta\phi|$ remains same if the parabolic wire is replaced by a straight wire, y = x, initially of length $\sqrt{2}\ell$

Solution:

$$d\varepsilon = BV_0dy$$

$$= B_0\left\{1+\left(\frac{y}{2}\right)^\beta\right\}V_0\,dy$$

$$\varepsilon = B_0\int_0^L 4+\left(\frac{y}{L}\right)^\beta\Bigg\}V_0\,dy$$

$$= B_0\ V_0L\left(1+\frac{1}{\beta+1}\right)$$

Let $\beta = 0 \quad \varepsilon = 2B_0V_0L$

$$\beta = 2 \quad \varepsilon = B_0V_0L\left(1+\frac{1}{3}\right)$$

$$= \frac{4}{3}B_0V_0L$$

B, C is correct

D is also correct because projection of wire on y axis is same

10. If in a hypothetical system if the angular momentum and mass are dimensionless. Then which of the following is true.

(a) The linear momentum varies as L^{-1} (b) The energy varies as L^{-2}

(c) The power varies as L^{-4} (d) The force varies as L^{-5}

Solution:

$$[M]=\left[M^0L^0T^0\right]$$

$$[J]=\left[ML^2T^{-1}\right]$$

$$\left[ML^2T^{-1}\right]=\left[M^0L^0T^0\right]$$

$$\Rightarrow \left[L^2\right]=[T]$$

Momentum

$$[P]=\left[MLT^{-2}.LT^{-1}\right]$$

$$=\left[ML^2T^{-3}\right]$$

$$=\left[L^2L^{-6}\right]$$

$$=\left[L^{-4}\right]$$

$$[E]=\left[MLT^{-2}.L\right]$$

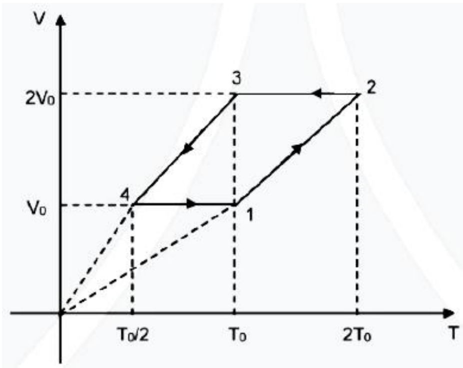
$$=L^2\ L^{-4}$$

$$=\left[L^{-2}\right]$$

$$[F]=\left[L\ \ L^{-4}\right]=\left[L^{-3}\right]$$

A, B, C

11. V – T diagram for n mol monoatomic gas is given below:



Choose the correct statement:

(a) $\left|\frac{\Delta Q_{1\to 2}}{\Delta Q_{3\to 4}}\right| = \frac{1}{2}$

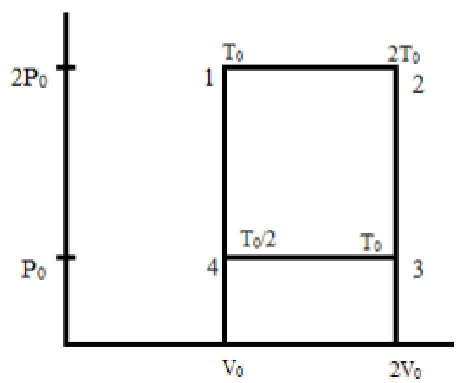
(b) $\left|\frac{\Delta Q_{1\to 2}}{\Delta Q_{2\to 3}}\right| = \frac{5}{3}$

(c) Work done in cyclic process is $\Delta W = \frac{nRT_0}{2}$

(d) There are only adiabatic and isochoric processes are involved.

Solution:

Corresponding PV entraps



(A) $\left|\frac{\Delta Q_{1-2}}{\Delta Q_{3\to 4}}\right| = \left[\frac{NC_P \Delta T_{1\to 2}}{NC_P \Delta T_{3-4}}\right] =$

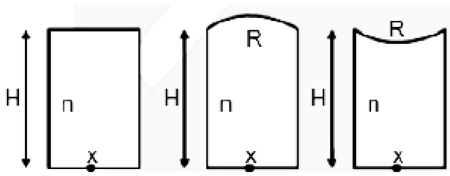
$$= \frac{T_0}{T_0 / 2} = 2$$

(B) $\left|\frac{\Delta Q_{1-2}}{\Delta Q_{2\to 3}}\right| = \left|\frac{NC_P \Delta T_{1\to 2}}{NC_V \Delta T_{2\to 3}}\right|$

$$= \frac{C_P}{C_V} = \frac{5}{3}$$

(C) $\omega = amu = P_0 V_0$

12. Apparent depth for point object x in all three cases are H_1, H_2 & H_3 respectively when seen from below given H = 30 cm, n = 1.5 & R = 3m, then



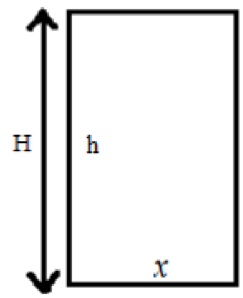
(a) $0.8 < H_2 - H_1 < 0.9$ (b) $H_2 > H_1$ (c) $H_2 > H_3$ (d) $H_3 > H_1$

Solution:

Case I

$$\frac{n_2}{n_1} = \frac{d^1}{d} \Rightarrow \frac{1}{n} = \frac{d^1}{(3Q)}$$

$$d^1 = \frac{30}{3} \times 2 = 20cm$$

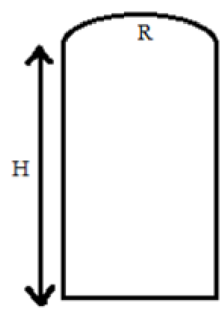


Case II

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{1^2}$$

$$\frac{1}{-v_2} - \frac{3}{-2 \times 30} = \frac{1 - 3/2}{-300}$$

$$H_2 = 20.684$$



Case II

$$V_3 = 19.354$$

SECTION – 3

13. Consider the following nuclear fission reaction

$${}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_2He^4 + Q.$$

In this fission reaction. Kinetic energy of α-particle emitted is 4.44 MeV. Find the energy emitted as γ – radiation in keV in this reaction.

$$m\left({}_{88}Ra^{226}\right) = 226.005\,amu$$

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$$m\left({}_{86}Rn^{222}\right) = 222.000\,amu$$

$$m\left({}_{2}He^{4}\right) = 4.000\,amu$$

Solution:

Δm = 0.005 amu

$$\frac{K_\alpha}{K_{Rn}} = \frac{m_{Rn}}{m_\alpha}$$

$$K_{Rn} = \frac{m_\alpha}{m_{Rn}} K_\alpha =$$

$$= \frac{4}{222} \times 4.44 = 0.08\,MeV$$

Energy of γ – photon

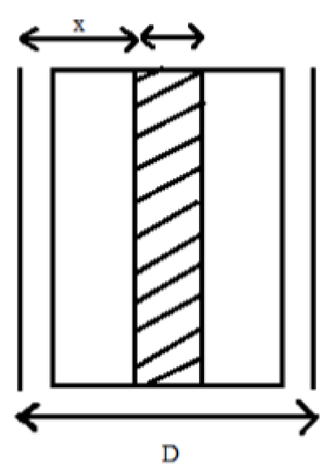
= 4.655 – (4.44 + 0.08)

= 0.135 MeV

14. N dielectrics are introduced in series in a capacitor of thickness D. Each dielectric have width d = D/N & dielectric constant of m^th dielectric is given by $K_m = K\left(1 + m/N\right)$: [N >> 10^3, Area of plates = A]

Net capacitance is given by $\dfrac{K\varepsilon_0 A}{\alpha D \ell n 2}$. Find value of α.

Solution:



$$\frac{x}{m} = \frac{\Delta}{N}$$

$$d\frac{1}{C} = \frac{dx}{Km\varepsilon_0 A} = \frac{dx}{K\varepsilon_0 A\left(H\dfrac{m}{N}\right)}$$

$$\int d\frac{1}{C} \quad = \int_0^D \frac{dx}{K\varepsilon_0 A\left(H\dfrac{x}{D}\right)}$$

Integrating we get

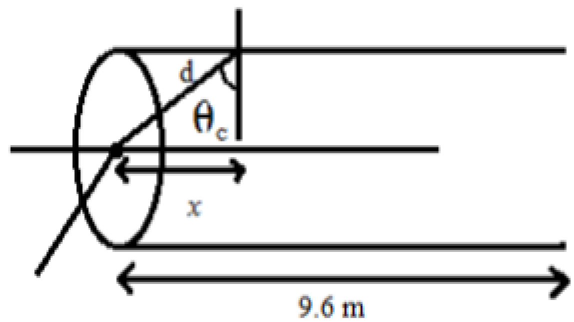
$$C_{eq} = \frac{K\varepsilon_0 A}{D\ell n 2}$$

$$\therefore\ \alpha = 1$$

15. If at angle θ the light takes maximum time to travel in optical fiber. Then the maximum time is $x \times 10^{-8}$, calculate x.

Solution:

$$1.5 \sin\theta_C = 1.44 \sin 90°$$



$$\sin\theta_C = \frac{1.44}{1.5} = \frac{24}{25}$$

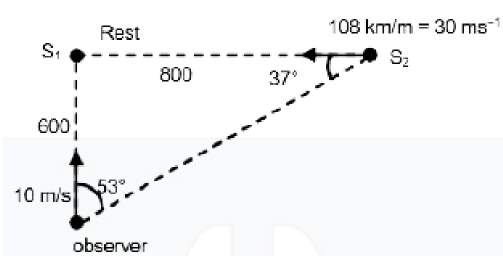
$$\sin\theta_C = \frac{x}{d}$$

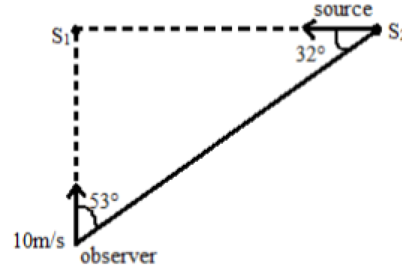
$$\Rightarrow d = \frac{25x}{24}$$

$$t = \frac{L}{\left(\frac{C}{n_2}\right)} = \frac{10}{\frac{3\times10^8}{1.5}}$$

$$t = 5\times10^8 s$$

16. The source S_1 is at rest. The observer and the source S_2 are moving towards S_1 as shown in figure. The roof beats observed by the observer if both sources have frequency 120 Hz and speed of sound 330 m/s in is



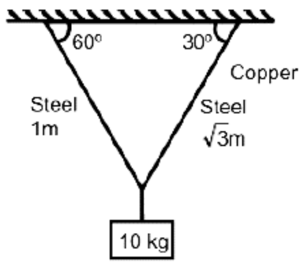


Solution:

$$f_b = 120\left(\frac{330+10\cos 53}{330-30\cos 37}\right)-\left(\frac{330+10}{330}\right)$$

$= 8.128$ Hz

17. A weight of 100 N is suspended by two wires made by steel and copper as shown in figure length of steel wire is 1 m and copper wire is $\sqrt{3}m.$ Find ratio of change in length of copper wire $\left(\Delta\ell_o\right)$ to change in length of steel wire $\left(\Delta\ell_s\right)$. Given Young's modulus: $Y_{steel} = 2 \times 10^{11}$ N/m^2, $Y_{copper} = 1 \times 10^{11}$ N/m^2.



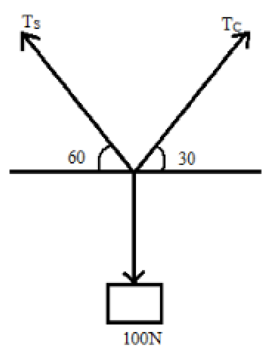
Solution:

$$\frac{T_S}{Z} = T_C\frac{\sqrt{3}}{2}$$

$$T_S = \sqrt{3}\ T_C$$

$$\frac{\Delta\lambda_C}{\Delta\lambda_S} = \left(\frac{T_C}{T_S}\right)\left(\frac{L_C}{L_S}\right)\frac{Y_S}{Y_C}$$

$$= \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{1}\right)\frac{2\times10^{11}}{1\times10^{11}} = 2$$



18. An optical bench, to measure the focal length of lens, is 1.5 m long and on the bench marks are with spacing $\frac{1}{4}$ cm. Now a lens is placed at 75 cm and pin type object is placed at 45 cm marks on the bench. If its image is formed at 135 cm find maximum possible error in calculation of focal length.

Solution:

V = 30 cm dv = 0.5 cm

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V = 60 cm dv = 0.5 cm

$$\frac{1}{y}-\frac{1}{u}=\frac{1}{f}\Rightarrow f=20cm$$

$$\frac{-dv}{v^2}+\frac{-dv}{u^2}=\frac{-df}{f^2}$$

$$\frac{df}{f}\times 100=f\left[\frac{dv}{v^2}+\frac{du}{u^2}\right]$$

= 1.38 and 1.39

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