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Some Thoughts On Well-Foundedness in Weighted Abstract Argumentation

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Abstract

We focus on the notion of well-foundedness originally provided by P. M. Dung in his pioneering work. We generalise such a property to different notions of defence in weighted frameworks, in order to finally obtain a single extension for different weighted semantics (uniqueness result).

Introduction and Motivations

Properties of extension-based semantics in *Abstract Argumentation Frameworks* (Dung 1995), or simply AAFs $\langle \mathscr{A}_{rgs}, R \rangle$ (a set of arguments and an attack relation between them), are important for the sake of the theoretical approach itself, but also due to practical reasons. For instance, they can be used to improve the performance of solvers (Bistarelli, Rossi, and Santini 2018a), as *ConArg*[1] (Bistarelli and Santini 2011): by studying a framework and derive from its structure that there is no stable-extension, or just only a preferred one, we can return an answer for decision problems as e.g. *enumeration* and *existence* of extensions.

Sufficient conditions for the well-foundedness of frameworks and unicity of extensions were presented in (Dung 1995). An AAF is well-founded if there exists no infinite sequence $a_1, a_2, \ldots, a_n, \ldots$ (with $a_i \in \mathscr{A}_{rgs}$) such that for each i, $R(a_{i+1}, a_i)$. The well-foundedness property is interesting because it points to a framework where all the notions of acceptability coincide: there exists only a set of arguments to be considered, under any semantics. According to Dung, every well-founded AAF has exactly one complete extension, which is also grounded, preferred and stable.

Extending well-foundedness with the purpose to consider a *synergy* in a combined attack has been already considered in (Nielsen and Parsons 2006). There, the authors generalise Dung' sequence of arguments to chain of sets: a set of arguments $\mathscr{B} \subseteq \mathscr{A}_{rgs}$ is a *minimal attack set* on an argument a if there is no sets $\mathscr{B}' \subsetneq \mathscr{B}$ such that $\mathscr{B}'$ attacks a. Note that Parson explicitly considers collective attacks "all together", and the idea is to capture the minimal set of arguments that is able (in synergy) to attack a given argument a: a set of arguments $\mathscr{B}$ attacks a, but no smaller set $\mathscr{B}'$ can attack a. For instance, "a_1: *Joe does not like Jack*", and "a_2: *There is a nail in Jack's antique coffee table*" do not separately attack "a_3: *Joe did not hammer a nail into Jack's antique coffee table*", but they do it only in conjunction.

[1] http://www.dmi.unipg.it/conarg/.

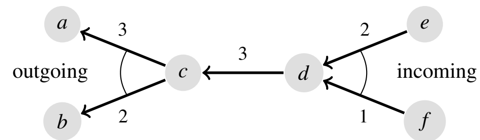

Figure 1: Incoming and outgoing synergies of attacks. The attack from d to c is outgoing for d and incoming for c.

In this paper we turn our attention to *Weighted AAFs* (*WAAF*), by considering AAFs where attacks are associated with a value. In particular, we mainly refer to *semiring-based WAAFs* (Bistarelli, Rossi, and Santini 2016; 2018b; Bistarelli and Santini 2017), which can be described by a tuple $\langle \mathscr{A}_{rgs}, R, W, \mathbb{S} \rangle$ encompassing a weight function W with domain in R, and a c-semiring $\mathbb{S}$ to be parametrically instantiated to specific metrics of weights. In (Bistarelli, Rossi, and Santini 2016; 2018b) the authors proved that (Martínez, García, and Simari 2008) and (Coste-Marquis et al. 2012) can be cast in the same semiring-based framework by just changing the notion of defence, while still using the same semiring-based operators.

The aim in (Nielsen and Parsons 2006) is to investigate synergies on classical Dung's frameworks. For this reason, weighted attacks are not considered at all, and, consequently, no compositionality of values can be adopted.

In this paper we address these additional features: weights and collective attacks obtained by composing classical single attacks. In addition, in (Nielsen and Parsons 2006) only the attacking synergy coming from different arguments towards a single one is considered (i.e., *incoming*, from e and f to d in Fig. 1): the synergy of multiple combined attacks from a single attacker towards a set of arguments (i.e., *outgoing*) is not modelled by minimal sets. Figure 1 shows this difference on a WAAF using the *Weighted semiring*, that is $\langle \mathbb{R}^+ \cup \{+\infty\}, \min, +, +\infty, 0 \rangle$. Argument d defends both a and b from c by adopting weighted defences as (Martínez, García, and Simari 2008) and (Coste-Marquis et al. 2012), since the defence has a weight at least as strong as each single attack: $3 \leq_{\mathbb{S}} 3$ and $3 \leq_{\mathbb{S}} 2$. However, if the attacks from c

are collectively considered instead, d does not defend a and b anymore: $3 \not\leq_{\mathbb{S}} 3+2$. This synergy can be represented only by the defence in (Bistarelli, Rossi, and Santini 2018b).

Outgoing synergy also stands in case of classical AAF (not weighted): for instance, "a_1: *I have enough money to book one single trip this summer*" does not attack either "a_2: *I will spend June in Norway*" or "a_2: *I will spend July in Greece*". However, a_1 attacks a_2 *and* a_3 when taken together.

Results

The framework we propose has as primitive representation of (binary) attacks as classical Dung's AAFs; however, it allows their composition and it adopts the $\otimes$ operator of a semiring (in its general form, $\mathbb{S} = \langle S, \oplus, \otimes, \bot, \top \rangle$) to compute the weight of the obtained collective attack, which represents the synergy obtained by the single attacks. Such a compositionality feature of the approach permits us to consider a simpler notion of cycles w.r.t. the minimal sets in (Nielsen and Parsons 2006).

We can state that, a positive check of the acyclicity in a given AAF (with just 1-to-1 attacks) is sufficient to ensure the acyclicity of any other synergic composition of such attacks (1-to-n and/or m-to-1) in the same (weighted) AAF.

Theorem 1. *The acyclicity of an AAF implies acyclicity in any hyper-graph obtained with the synergies.*

However, acyclicity is not enough to state well-foundedness in a given AAF when different notions of defence are adopted: for instance, weighted defences as in (Bistarelli, Rossi, and Santini 2016; 2018b) and (Martínez, García, and Simari 2008; Coste-Marquis et al. 2012). We now recall when a relation *Rel* is well-founded.

Definition 1. *A relation Rel is well-founded when its transitive closure (we call it Rel^+) is a directed acyclic graph.*

In case of AAFs, the transitive closure of the attack relation leads to the defence relation. In Dung, whenever we have an attack $R(a,b)$, and an attack $R(c,a)$, we can apply transitivity and say that c defends b from a. In case of weighted attacks, to be well-founded the transitive closure of the attack relation needs to consider also such weights. Hence, in Def. 2 we define these sufficient conditions.

Definition 2. *A WAAF is well-founded if it is acyclic and:*

wf_1 **(Martínez, García, and Simari 2008).** *For all arguments* a *that have both incoming and outgoing attacks, it holds that $\forall b.R(a,b)$ with $W(a,b) = w_{ab}$, then $\exists c.R(c,a)$ s.t. $W(c,a) = w_{ca}$ and $w_{ca} \leq_{\mathbb{S}} w_{ab}$.*

wf_2 **(Coste-Marquis et al. 2012).** *For all arguments* a *that have both incoming and outgoing attacks, it holds that, $\forall b.R(a,b)$ with $W(a,b) = w_{ab}$, then $\bigotimes_{c\,s.t.\,R(c,a)} W(c,a) = w_{ca}$ and $w_{ca} \leq_{\mathbb{S}} w_{ab}$.*

wf_w **(Bistarelli, Rossi, and Santini 2018b).** *For all arguments* a *that have both incoming and outgoing attacks, it holds that, $\bigotimes_{b\,s.t.\,R(a,b)} W(a,b) = w_{ab}$, then $\bigotimes_{c\,s.t.\,R(c,a)} W(c,a) = w_{ca}$ and $w_{ca} \leq_{\mathbb{S}} w_{ab}$.*

From Def. 2 we can derive some implications between the different notions of well-foundedness on a given AAF. To accomplish it, we exploit the implications between the notions of defence advanced in (Bistarelli, Rossi, and Santini 2016; 2018b). Theorem 2 collects these results.

Theorem 2 (Implications)**.** *On the same AAF, the following implications hold: $wf_w \Rightarrow wf_1$, $wf_2 \Rightarrow wf_1$, $wf_1 \Rightarrow wf_{Dung}$.*

In Th. 3, *w*-semantics rephrase Dung's semantics by using weighted defences: for this reason, the following theorem helps to find well-founded WAAF by using weighted semantics in (Martínez, García, and Simari 2008), (Coste-Marquis et al. 2012), and (Bistarelli, Rossi, and Santini 2018b),

Theorem 3 (Uniqueness of *w*-complete extension)**.** *Every well-founded WAAF has exactly one w-complete extension, which is also w-grounded, w-preferred, and w-stable.*

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