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Ontologies for Dates and Duration

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Abstract

Reasoning with dates and duration has long been addressed by the community. Existing duration ontologies, however, lack complete axiomatizations of their intended models; many simply represent timedurations as real numbers and treat the duration function as a metric on the timeline. We show that such approaches are inadequate and provide a first-order ontology of duration that overcomes these limitations.

Introduction

Reasoning about durations in the context of a calendar composed of dates is a fundamental capability required for many applications, from supply-chain management and e-commerce to narrative analysis. For example, we would like a suite of ontologies that can be used to infer that an order is late if we know that today is Friday and the order was sent on Monday, but the order takes only two days to arrive.

Earlier approaches to ontologies for durations, dates, and calendars lack complete axiomatizations – intended semantics is specified in documentation but there are insufficient axioms to guarantee that all models are intended, or else arithmetic is used as the intended model (that is, reasoning about timedurations reduces to reasoning about real numbers). We want to find the weakest first-order theory with the minimal set of assumptions that axiomatizes the class of intended models, rather than use a specific intended model.

In this paper we introduce three first-order ontologies that axiomatize intuitions about durations, dates, and calendars. Existing ontologies for duration use a particular algebraic field (such as the rational or real numbers) to reason about duration. In the Duration Ontology presented in this paper, timedurations do not form a field, since we do not multiply timedurations, although we do want to multiply timedurations by a scalar (i.e. element of a field). Instead, timedurations form a vector space. The duration function is therefore a vector-valued function, and it cannot be a metric (which would be a function from the timeline to a field). Furthermore, the axioms for metric spaces alone are too weak to axiomatize the intended models that formalize the intuitions for the duration function.

The duration ontology presented in this paper also lays the foundations for an ontology of dates. Since dates have multiple repeatable occurrences at different timepoints, we define them as a class of activities in the Dates Ontology. Calendars are complex activities composed of dates, and the axioms that specify how such complex activities occur are introduced in the Calendar Ontology. Specific calendars, such as the Gregorian and lunar calendars, are domain theories that are extensions of the Calendar Ontology.

Methodology

The problem of ontology evaluation is becoming increasingly important. The obvious logical criterion for ontology evaluation is consistency. Strictly speaking, we only need to show that a model exists in order to demonstrate that a first-order theory is consistent. Constructing a single model, however, runs the risk of having demonstrated satisfiability for a restricted case; for example, one could construct a model of a process ontology in which no processes occur, without realizing that the axiomatization might mistakenly be inconsistent with any theory in which processes do occur. We therefore need a complete characterization of all models of the ontology up to isomorphism. One approach to this problem is to use representation theorems – we evaluate the adequacy of the ontology with respect to some well-understood class of mathematical structures (such as partial orderings, graph theory, and geometry) that capture the intended interpretations of the ontology's terms.

In order to specify and prove representation theorems, we first define various substructures of any $\mathcal{M} \in \mathfrak{M}$ [1]. For each class of substructures, we prove that the class exists and is nonempty, and then we provide a characterization of the structures in the class up to isomorphism. Next, we provide a definition of the class $\mathfrak{M}$, which will often include additional conditions on the ways in which the substructures

[1] In this paper, we will be using the following notation:
Structures are denoted by calligraphic font: $\mathcal{M}, \mathcal{N}, \mathcal{M}', \mathcal{M}_1, ...$
Classes of structures are denoted by $\mathfrak{M}, \mathfrak{N}$, ...
The domain of a structure $\mathcal{M}$ is denoted by M. Elements of a structure are denoted by **boldface** font.
The extension of an n-ary relation $\mathbf{R}$ in a structure $\mathcal{M}$ is denoted by the n-tuple $\langle \mathbf{a_1}, ..., \mathbf{a_n} \rangle \in \mathbf{R}$.

can be combined. For each relation, we provide a characterization of its extension with respect to the substructures defined earlier.

Given the definition of $\mathfrak{M}$, we then prove that the class exists and is nonempty; this is necessary because of the additional conditions in the definition of $\mathfrak{M}$ on the combination of substructures. This also provides a characterization of the structures in the class up to isomorphism. We prove the representation theorem in two parts – we prove that every structure in the class is a model of the ontology and then prove that every countable model of the ontology is elementary equivalent to some structure in the class.

The characterization up to isomorphism of the models of an ontology through representation theorems has several distinct advantages. First, unintended models are more easily identified, since the representation theorems characterize *all* models of the ontology. We also gain insight into any implicit assumptions within the axiomatization which may actually eliminate models that were intended. Second, any decidability and complexity results that have been established for the classes of mathematical structures in the representation theorems can be extended to the ontology itself. Finally, the characterization of models supports the specification of semantic mappings to other ontologies, since such mappings between ontologies preserve substructures of their models.

Duration Ontology

One fundamental insight is that timedurations do not form a field, such as the reals or rationals, as many approaches ((Rescher & Urquhart 1971), (Navarrete *et al.* 2002), (Knight & Ma 1994b), (Knight & Ma 1994a), (Kautz & Ladkin 1991), (Hobbs & Pan 2004)) have assumed. Although we do want to be able to add durations together, the product of two timedurations is not a timeduration; thus, multiplication is not a function on the set of timedurations, and the underlying structure for timedurations cannot be a field. Nevertheless, we do want to be able to specify scalar multiples of timedurations (e.g. one task takes twice as long as another). If such scalars are elements of a field, then the intended models for timedurations must be vector spaces. If the scalars are elements of a ring (e.g. the integers), then the intended models for timedurations would be a module. If we consider problems in which the duration of an activity depends on distance and velocity, then we want to specify at least rational multiples of timedurations, and hence the scalars would form a field. Vector spaces have the additional advantage of being decidable (), whereas modules in general are not.

Timedurations alone are not sufficient for a duration ontology; we also need a function that assigns timedurations to time intervals or pairs of timepoints. Since earlier approaches have assumed that timedurations formed a field, they have treated this function as metric ((Hajnicz 1996), (Rescher & Urquhart 1971), (Barber 1993)). Given that timedurations actually form a vector space, the duration function is no longer a metric, and we must find a suitable class of vector-valued functions to adequately capture the intended models.

Structures for Representing Duration

Using these intuitions, we now formally specify the classes of structures that will be used to characterize the intended models of timedurations and the duration function, namely, timelines, ordered vector spaces, and vector maps.

Timelines Ontologies of time have been thoroughly studied in the literature, both from a model-theoretic (van Benthem 1991) and axiomatic (Hayes 1996) perspective. In this paper, we use the ontology whose models are a linear ordering over timepoints[2]; we do not impose any additional assumptions about density or discreteness.

Definition 1. *A structure $\mathcal{T} = \langle T, < \rangle$ is a timeline iff it is isomorphic to a countable linear ordering.*

Although the axioms that we introduce in $T_{duration}$ assume that there are no endpoints, the axioms can be easily be extended to time ontologies whose models do include endpoints at infinity.

Ordered Vector Spaces In addition to the above arguments in favor of timedurations forming a vector space, we also want to define an ordering over timedurations (e.g. a week is longer than a day but shorter than a week). This leads to the following class of structures:

Definition 2. $\mathcal{D} = \langle D, \mathbf{zero}, \mathbf{add}, \mathbf{mult}, \mathbf{one}, \mathbf{lesser} \rangle$ *is an ordered vector space iff $\mathcal{D}$ is a vector space whose elements are partially ordered.*

Vector Maps Within differential topology, a vector-valued function on the space $\mathbb{R}^n$ is known as a vector field. in which a unique vector is associated with each element of $\mathbb{R}^n$. The duration function is also a vector-valued function, since it maps pairs of timepoints to a unique timeduration. However, since we want a mapping δ from $\mathcal{T} \times \mathcal{T}$ to the vector space $\mathcal{D}$, we need to generalize the notion of vector field.

There are, of course, many possible functions from $\mathcal{T} \times \mathcal{T}$ to the vector space $\mathcal{D}$. We define the class of vector-valued functions for duration using the automorphisms of the structure, that is, mappings from the structure to itself that preserve values of the duration function δ:

$$\delta(\mathbf{t_1}, \mathbf{t_2}) = \delta(\mathbf{t_3}, \mathbf{t_4}) \Leftrightarrow \varphi(\delta(\mathbf{t_1}, \mathbf{t_2})) = \varphi(\delta(\mathbf{t_3}, \mathbf{t_4}))$$

Intuitively, automorphisms of the timeline (which form a group denoted by $Aut(\mathcal{T})$ should preserve the duration function – if we shift timepoints along the timeline while preserving their ordering, the values of the duration function should not change. This insight leads to the following class of structures:

Definition 3. *Let $\mathcal{T}$ be a timeline and let $\mathcal{D}$ be an ordered vector space.*

The structure $\mathbb{V}\langle \delta, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ is a vector map iff

$$\delta : \mathcal{T} \times \mathcal{T} \rightarrow \mathcal{D}$$

and

$$Aut(\mathbb{V}) \cong (Aut(\mathcal{T}) \times Aut(\mathcal{T})$$

[2]The axioms for T_{time} in CLIF (Common Logic Interchange Format) can be found at http://www.stl.mie.utoronto.ca/colore/linear-time.clif

Given all of these classes of structures, we are now in a position to define the class of structures $\mathfrak{M}^{duration}$ that will be used to characterize the models of the Duration Ontology.

Definition 4. *Let $\mathfrak{M}^{duration}$ be the following class of structures: $\mathcal{M} \in \mathfrak{M}^{duration}$ iff $\mathcal{M} = \mathcal{T} \cup \mathcal{D} \cup \mathbb{V}$, where*

1. $\mathcal{T} = \langle \mathbf{timepoint}, \mathbf{before} \rangle$ *is a timeline;*
2. $\mathcal{D} = \langle \mathbf{timeduration}, \mathbf{add}, \mathbf{mult}, \mathbf{zero}, \mathbf{one}, \mathbf{lesser} \rangle$ *is an ordered vector space;*
3. $\mathbb{V} = \langle \mathbf{duration}, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ *is a vector map.*

Characterization Theorems for $\mathfrak{M}^{duration}$

In this section, we show that the class of structures $\mathfrak{M}^{duration}$ is nonempty. The characterization of timelines up to isomorphism follows from (van Benthem 1991) and the characterization of ordered vector spaces is presented in (Peressini 1967). We therefore need to focus on the characterization of vector maps and show that vector-valued functions with the requisite automorphisms exist.

Definition 5. *Suppose $\mathbb{V} = \langle \delta, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ is a vector map.*

An integral curve $C(\mathbf{d})$ in $\mathbb{V}$ is a subset of $\mathcal{T} \times \mathcal{T}$ such that

$$C(\mathbf{d}) = \{(\mathbf{t_1}, \mathbf{t_2}) \,:\, \delta(\mathbf{t_1}, \mathbf{t_2}) = \mathbf{d}, \mathbf{d} \in D\}$$

In other words, an integral curve is an equivalence class of pairs of timepoints that map to the same timeduration. The following theorem shows that any vector map can be decomposed into a set of integral curves:

Theorem 1. *Let $\mathcal{T}$ be a timeline, let $\mathcal{D}$ be an ordered vector space, and suppose*

$$\delta \,:\, \mathcal{T} \times \mathcal{T} \rightarrow \mathcal{D}$$

The structure $\mathbb{V} = \langle \delta, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ is a vector map iff

1. *$\mathbb{V}$ is a set of disjoint integral curves which is isomorphic to a linearly ordered subgroup of $\mathcal{D}$ which is order-isomorphic to $\mathcal{T}$.*
2. *Each integral curve in $\mathbb{V}$ is order-isomorphic to $\mathcal{T}$.*

Axiomatization of $T_{duration}$

The axioms of $T_{duration}$ can be divided into two subtheories – the first captures the notion of timedurations as elements of ordered vector spaces, while the second capture the formalization of the duration function as a vector map. In particular, the second subtheory contains generalizations of the axioms for metric spaces, and also guarantees that automorphisms of the timeline also preserve the ordering over timedurations and that mappings that preserve the vector map are also automorphisms of the timeline.

$T_{duration}$ does not mention specific constants that denote particular timedurations such as $second$, $hour$, day, or $year$. The axioms for such constants are not contained in the Duration Ontology, but rather are specified in a domain theory (Gruninger 2009) that extends $T_{duration}$, just as linear equations are domain theories for general vector spaces. Thus, equations such as

$$hour = mult(60, second)$$

$$day = mult(24, hour)$$

define specific timedurations as the linear combinations of other timedurations. This allows for different time-keeping systems to share the same Duration Ontology.

Using a relative interpretation with the axiomatization in (Peressini 1967), together with Theorem 1, we can prove the representation theorem for models of $T_{duration}$:

Theorem 2. *Any a countable model of $T_{duration} \cup T_{time}$ is elementary equivalent to some $\mathcal{M} \in \mathfrak{M}^{duration}$, and any structure in $\mathfrak{M}^{duration}$ is isomorphic to a model of $T_{duration} \cup T_{time}$.*

Summary

Although reasoning with dates and duration has long been addressed by the community, earlier ontologies have either relied on an intended model such as the real numbers to represent the notion of duration, or they have used incomplete axiomatizations. In this paper, we have introduced first-order axiomatizations for durations. We have specified the classes of intended structures for these ontologies, proven characterization theorems for these classes of structures, and proven representation theorems for the models of the axioms. The resulting ontologies can serve as the basis for future standardization work in efforts such as the OMG Date-Time Vocabulary.

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