Skyrmions as Models for Nuclei

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Skyrme Field and Dynamics

- Skyrme model is nonlinear, effective field theory (EFT) of pions [T.H.R. Skyrme, 1961]. Its field equations have topological soliton solutions – Skyrmions – with surprising shapes.
- Skyrmions represent nucleons and larger nuclei. No explicit nucleon fields appear.
- Skyrme field

$$U(x) = \sigma(x)\,\mathbf{1}_2 + i\boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}$$

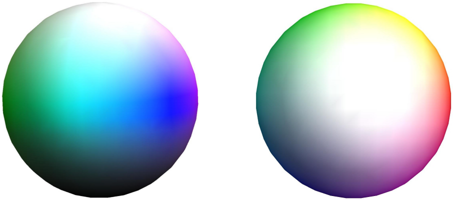
  requires $\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} = 1$, so $U \in SU(2)$. Field is smooth and needs no short-distance cutoff.
- $U \to \mathbf{1}_2$ asymptotically, and $U \simeq -\mathbf{1}_2$ in core of nucleons.

- Topological charge – the topological degree of U over space – is identified with baryon number B (atomic mass number).
- Skyrme field Lagrangian

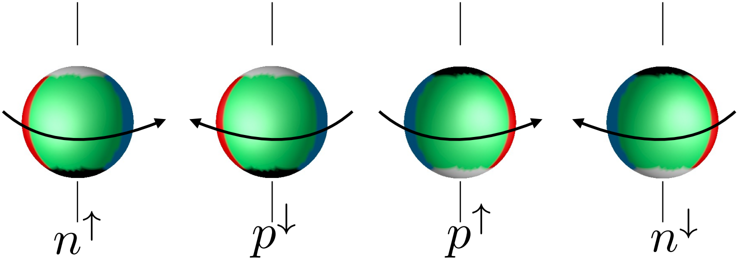
$$L = \int -\frac{1}{2}\mathrm{Tr}(\partial_\mu U\, \partial^\mu U^\dagger)\, d^3x + \text{higher order derivative terms} + \text{pion mass term}\,.$$

- Solve field equations to determine Skyrmion shapes and symmetries, energies, moments of inertia, vibrational frequencies. Analytic approximations are helpful.
- Can also couple ρ-meson and ω-meson fields – affects Skyrmion core structure and forces.

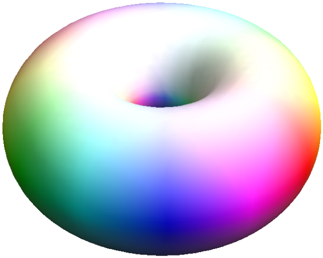
Skyrmions



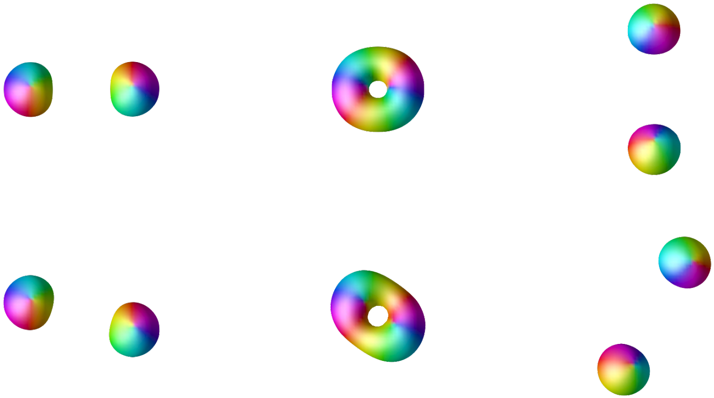
$B = 1$ Skyrmion in two orientations. These attract, clump together and slightly merge to form larger B Skyrmions. (Colours indicate pion field values on constant energy density surface.)



Classically spinning $B = 1$ Skyrmions, modelling quantised spin/isospin $\frac{1}{2}$ nucleons [D. Foster and NSM]. Spin/isospin $\frac{3}{2}$ delta resonances spin faster. Wavefunctions change sign under 2π rotation [Skyrme].

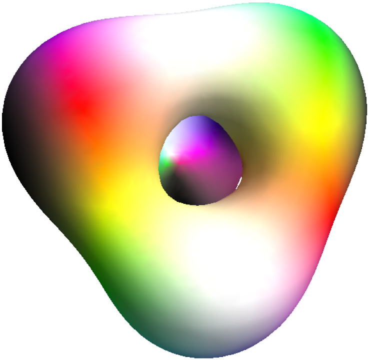


$B = 2$ Skyrmion

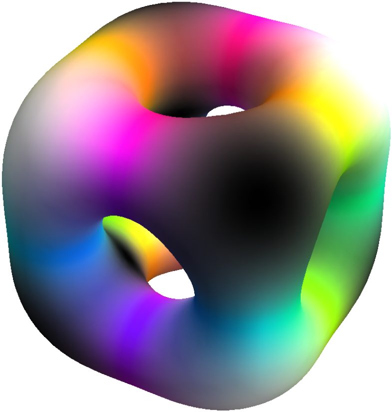


Scattering $B = 1$ Skyrmions [D. Foster and S. Krusch]

- Interaction between $B = 1$ Skyrmions is projected onto spin $\frac{1}{2}$ states to obtain nucleon-nucleon potentials.
- Need second-order perturbation theory to find central attraction and spin-orbit coupling [D. Harland and C.J. Halcrow].
- Larger B Skyrmions rotate and vibrate. Quantise to find spin/isospin of larger nuclei, and spectrum of excitations.
- S.B. Gudnason and C. Halcrow have website “Database of Skyrmion Vibrations”, showing vibrational modes up to $B = 8$.



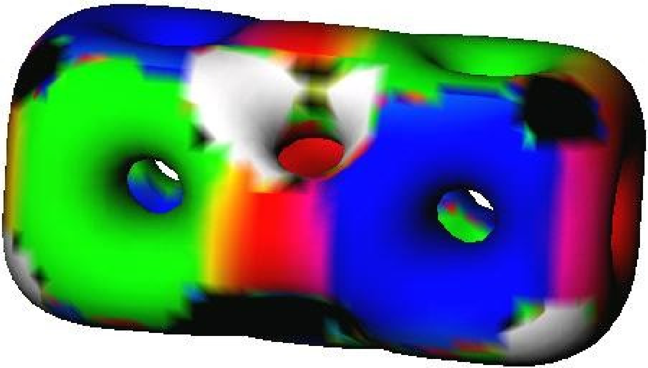
$B = 3$ Skyrmion [Braaten et al.]



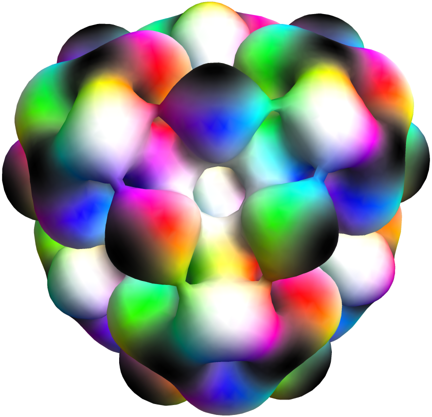
$B = 4$ Skyrmion

Skyrmions with Larger Baryon Numbers

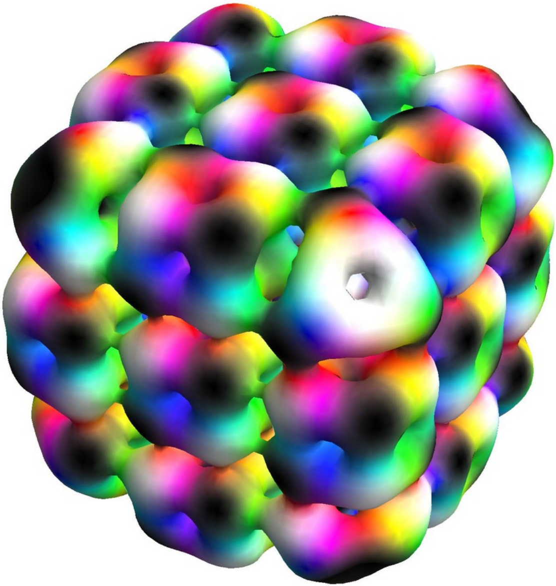
- (i) Glue $B = 4$ cubes with colours touching on faces. Dynamical $B = 4$ cubes give Skyrmion version of α-particle model.
- (ii) Tetrahedrally-symmetric Skyrmion clusters correspond to doubly-magic $N = Z$ nuclei (e.g. Oxygen-16 and Calcium-40). Have low-lying $0^+, 3^-$ states.
- (iii) Skyrmion crystal is cubic. Has exceptionally low energy per Skyrmion [Castillejo et al., Kugler and Shtrikman]. $B = 32, 100, 108$ Skyrmions are cubic crystal chunks.



$B = 8$ Skyrmion ($m_\pi = 1$)



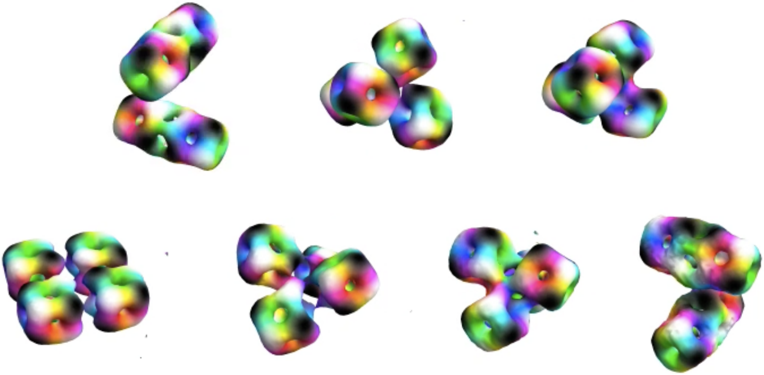
$B = 40$ Skyrmion



$B = 100$ Skyrmion [C. Lau and NSM]

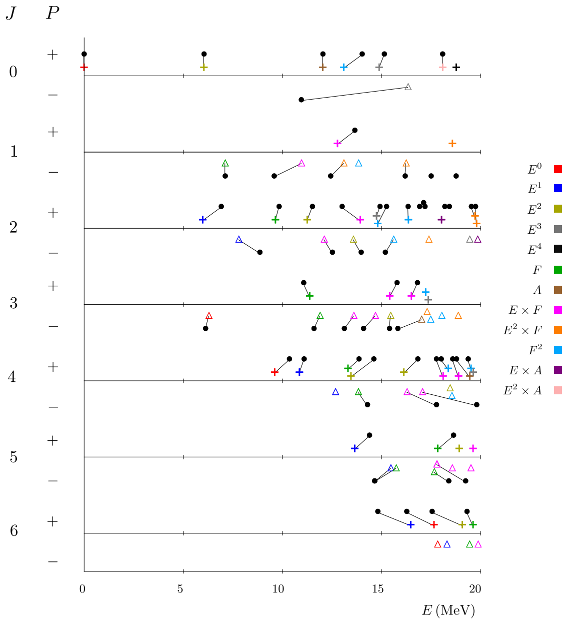
$B = 16$ Skyrmion and $^{16}\mathrm{O}$

$B = 16$ Skyrmion is a tetrahedral cluster of four $B = 4$ Skyrmions. The square cluster is a saddle point.



Cluster dynamics through tetrahedron, square and dual tetrahedron

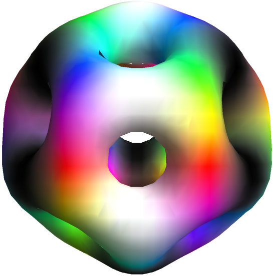
- Tetrahedral rotational states have spin/parity $0^+, 3^-, 4^+, 6^\pm, \ldots$ ($^{16}\mathrm{O}$ ground state band).
- Tetrahedral vibrations are A-mode (breather), E-mode (towards square) and F-mode (asymmetric break-up). E-mode has lowest frequency.
- Nonlinear, vibrational E-manifold of configurations permits tunnelling from tetrahedron to dual tetrahedron. Explains energy-splitting of 2^+ and 2^- E-phonon states [C.J. Halcrow, C. King and NSM].
- Inclusion of A- and F-phonons, and Coriolis effect of F-mode angular momentum gives $^{16}\mathrm{O}$ spectrum up to 20 MeV. Parameters fitted as in other rovibrational models [e.g. D. Robson, R. Bijker and F. Iachello].

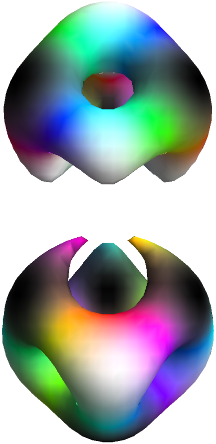


^{16}O energy spectrum – theory (colours), observed (black dots)

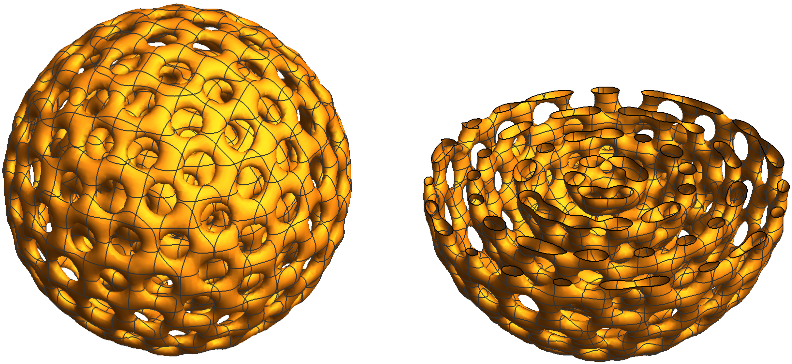
Icosahedral Skyrmions

- Icosahedrally-symmetric Skyrmions rather rare, but known for $B = 7, 17, 37, \ldots$. They can be combined in multi-layer structures.
- Icosahedral $B = 208$ Skyrmion has layers $7 + 37 + 67 + 97$. A. Heusler models selected high-spin states of $^{208}\mathrm{Pb}$ as icosahedral rotational band (spin/parity $0^+, 6^+, 10^+, 12^+, 15^+, \ldots$). Maybe 44 neutrons in two inner layers; 82 neutrons + 82 protons in outer layers.
- $^{44}\mathrm{Ti}$ may also have icosahedral band.





$B = 7$ Skyrmion and its deformation into clusters. Lowest spin/parity of icosahedral Skyrmion is $\frac{7}{2}^{-}$. Deformed Skyrmion models $\frac{3}{2}^{-}$ ground states of ${}^{7}\mathrm{Be}/{}^{7}\mathrm{Li}$.

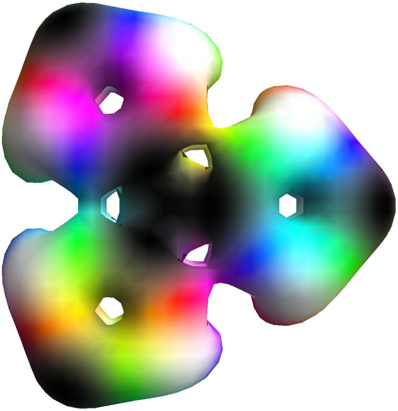


Icosahedral $B = 208$ Skyrmion [C.J. Halcrow]

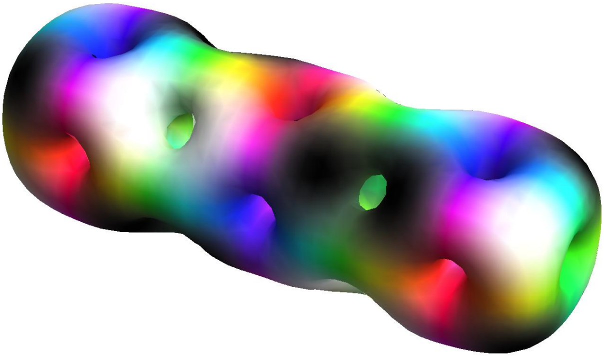
Summary

- Skyrmions are known for many baryon numbers B. They model how protons and neutrons change shape and partially merge into nuclei. Some Skyrmions have Platonic symmetry. Some resemble chunks of the cubic Skyrmion crystal.
- Rotational and vibrational quantisation of Skyrmions leads to nuclear spectra. Good spectra obtained for small B, for $B = 12$ (Carbon-12) [C. Lau and NSM, J. Rawlinson], $B = 16$ (Oxygen-16) and $B = 40$ (Calcium-40).
- Isobars less well modelled so far. Beryllium-8 is OK but spin of Lithium-8 is problematic (two closely-related Skyrmions). Coulomb energy needs further study.
- Recently, spin-orbit coupling better understood.

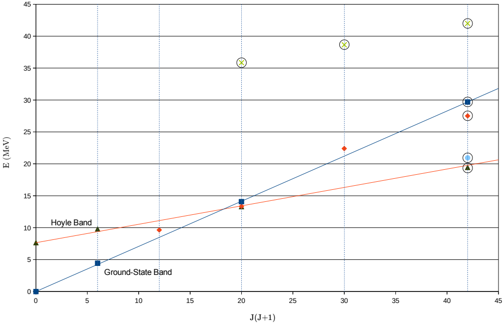
Extra Slides on Carbon-12



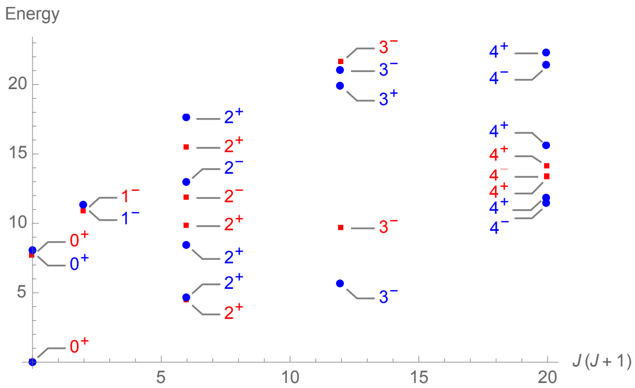
$B = 12$ Skyrmion with D_{3h} symmetry



$B = 12$ Skyrmion with D_{4h} symmetry



Carbon-12 states in the ground state band and Hoyle band



Carbon-12 Energy Levels, allowing for interpolation between Skyrmions: Experiment, Skyrme model