Wavelet-type denoising for mechanical structures diagnosis

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Abstract: We propose an adaptive method for the analysis of the dynamical changes in mechanical structures. Using measurement techniques and the flexible Gabor-wavelet transform, we perform an optimal denoising of slowly variable band-limited signals for an improved mechanical structure fault diagnostics.

Key–Words: wavelets, mechanical structures, denoising, adaptive method

1 Introduction

Mechanical structures diagnosis is an esential processes for ensuring the safe running of machines. Signal analysis is the most common method used for condition monitoring and fault diagnostics. It allows the mechanical engineer to discover the important information contained in the signals and to isolate the vibration exposure. However, most of the mechanical signals recorded are contaminated with noise and also exhibit a non-linear behaviour and therefore for a proper indentification of the signal characteristics we need to apply a flexible transform followed by a suitable denoising procedure. Since the main classical tools of time-frequency (e.g. local Fourier transforms) and time-scale analysis (e.g. wavelet transforms) lack of flexible characteristics, there were many recently atempts to provide adaptive time-frequency representations with applications to vibrations like ([1]) that follows from previous work of Jones and Park ([2]) or for channel estimation in wireless communications like ([3]). Most of them are extensions of the STFT(Short-time Fourier transform) in the sense of adaptively modifying the bandwitdth or of the CWT(Continuos wavelet transform) by adaptively varying either the modulation or the variance parameters ([4]). We propose here an new approach based on a proper defined flexible transform with intrisic adaptability involving both translation-modulations and dilations, called the flexible Gabor-wavelet transform or the α-transform due to the parameter $\alpha \in [0, 1)$ that control its dependance of the frequency on the dilations. The theoretical foundations of this transform are related to the work of Peter Gröbner ([5]) under the guidance of Hans Feichtinger and also Nazaret and Holschneider ([10]), who were looking for a suitable decomposition of the frequency domain as an intermediate geometry between those of modulation and Besov spaces ([6]). Latter on, in an successful intent of theoretical discretization, Massimo Fornasier ([9]), obtained Banach frames for α–modulation spaces. Applied to the signal processing, the improved spectrum representations of this transform will most likely produce better results in concrete applications than the abovementioned methods. The structure of the paper is as follows. We start the first part below with the necessary theoretical background. The second section will introduce a numerical algorithm for denoising using the discrete Gabor-wavelet transform and in the last part we present some test results on mechanical simulated signals and conclusions.

Let $\omega, t \in \mathbf{R^d}$ with $\omega \cdot t$ the scalar product in $\mathbf{R^d}$, then the direct Fourier transform of $f \in L^1(\mathbf{R^d})$: $\hat{f}(\omega) = \int\limits_{-\infty}^{\infty} f(t)e^{-2\pi i\omega\cdot t}dt$, allows us a frequential representation of the signal, which can be then recovered by the inversion formula $f(t) = \int\limits_{-\infty}^{\infty} \hat{f}(\omega) \cdot e^{2\pi i\omega\cdot t}d\omega$. Let f, g, h etc. be signals, i.e. functions from one the Lebesgue spaces $L^p(\mathbf{R})(\mathbf{1} \leq \mathbf{p} \leq \infty)$.

Definition 1.1. *For $x, \omega \in \mathbf{R}^d$ and $s \in \mathbf{R} \setminus \{0\}$ we define the following operators: $T_x f(t) = f(t - x)$ (translation), $M_\omega f(t) = e^{2\pi i\omega\cdot t} f(t)$ (modulation), and $D_s f(t) = |s|^{-\frac{d}{2}} f\left(\frac{t}{s}\right)$ (dilation). The operators $T_x M_\omega$ or $M_\omega T_x$ are called time-frequency shifts.*

The next definition refers to a pair of functions f and g in the following two situations. If $f, g \in L^2(\mathbf{R}^d)$, then $f \cdot T_x\bar{g} \in L^1(\mathbf{R}^d)$, and the Fourier transform $(f \cdot T_x\bar{g})^\wedge(\omega)$ is punctually defined. If $g \in L^p(\mathbf{R}^d)$ and $f \in L^{p'}(\mathbf{R}^d)$, then by Hölder inequality $f \cdot T_x\bar{g} \in L^1(\mathbf{R}^d)$ and again the Fourier transform of the product is punctually defined.

Definition 1.2. *For f and g like above and $g \neq 0$, then the function*

$$V_g f(x,\omega) = \int\limits_{\mathbf{R}^d} f(t)\overline{g(t-x)}e^{-2\pi i t\cdot\omega}dt \text{ with } x,\omega \in \mathbf{R}^d$$

is called the short-time Fourier transform of the signal f with respect to the window g.

Remarks *1.Just for latter reference in the numerical implementation, we mention that the Fourier transform interacts with the dilation in the form: $\mathcal{F}D_s = D_{\frac{1}{s}}\mathcal{F}$.*
2. We will use windows $g \in L^1 \cap L^2(\mathbf{R^d})$ that satisfy the admisibility condition $\int\limits_{\mathbf{R}^d} g(x)dx = 0$.
3. We call a weight, a function defined on $\mathbf{R}^{2d}$ nonegative and locally integrable. We consider as a valid weight any continuous function, in order to drop further restrictions (e.g. moderate).

Definition 1.3. *Let $\alpha \in [0,1)$ and $c > 0$. For all $g \in L^2(\mathbf{R})\backslash\{0\}$ and for $f \in L^2(\mathbf{R})$ we define the flexible Gabor-wavelet transform or the $\alpha - transform$ by the expression*

$$V_g^\alpha(f)(x,\omega) := \left\langle f, T_x M_\omega D_{c(1+|\omega|)^{-\alpha}}\right\rangle = \\ = \int\limits_{\mathbf{R}} f(t)\overline{T_x M_\omega D_{c(1+|\omega|)^{-\alpha}} g(t)}dt, \quad with \quad x,\omega \in \mathbf{R}.$$

For $\alpha = 0$ the transform V_g^α coincides with the short-time Fourier transform, while for $\alpha = 1$ is a slight modification of the wavelet transform. In particular, the intermediary case $\alpha = 1/2$ is exactly the Fourier-Bros-Iagolnitzer transform (FBI transform) ([7]).

2 The adaptive transform for denoising

In (10), the first author introduced an algorithm for computing the time-frequency representation of the flexible Gabor-wavelet transform. For the numerical algebraic algorithm description in the discrete setting will consider signals of length N.

Algorithm 2.1. *The flexible Gabor-wavelet transform can be computed in the following steps:*

1. Compute the Fourier transform of the signal f, with N samples on the interval T, using the Fast Fourier Transform. This step is executed only once.

2. For all ω corresponding to the all discrete frequencies.

2.1 Shift $\hat{f}$ with ω.

2.2 Compute the localized window $\hat{g}_\omega$.

2.3 Compute the punctual multiplication $C(T_{-\omega}\hat{f} \cdot \hat{g}_\omega)$.

3. Compute the inverse Fourier transform of the spectrum obtained by applying the convolution theorem in step 2.

4. Finish.

Making use, whenever it is possible, of the computational efficiency of the Fast Fourier Transform, we get a number of operation of the order $N(N + N\log N)$, when the flexible Gabor-wavelet transform has N^2 samples. All the steps can be performed in reverse order without any informations loss, so the inversion routing is also quite straightforward. The truncation error can be neglected since we are not using only local information. Therefore, the high precision of the estimates in the transform computation formula can be achieved without further assumptions on the window g or on the α parameter in the frame of the two theorems about the truncation error from ([9]).

In the follow, we will use this algorithm combined with a soft thresholding procedure for a more accurate denoising of vibration signals coming from mechanical vibrations measurements simulations. Typical vibration applications use accelerometers to measure vibration. An accelerometer consists of a piezoelectric element connected to a known mass. When the accelerometer is vibrated, the mass applies force to the piezoelectric element, generating an electrical charge that is proportional to the applied force. This charge can be measured to determine vibration characteristics. Most accelerometers require a current source of 4 mA and a compliance voltage of at least 18 V to drive their internal circuitry. Other accelerometers require a 2 mA current source, but have limitations in cable length and bandwidth. We will use for simulations vibrations signals taken using AC Coupling to measure low frequency signals accurately at the Nyquist sampling rate and loaded in Matlab. Our ideal anti-aliasing filter passes all signals in the band of interest and blocks all signals outside of that band. However, in practical use, the rolloff characteristics of the antialiasing filter allow some signals to pass above the filters cutoff frequency. By using AC coupling, we can eliminate any DC that may pass through the anti-aliasing filter. We will apply a denoising procedure using a wavelet denosing filter with threshold and the flexible Gabor-wavelet filter with the same threshold procedure and compare the results. The common denoising procedure for both the wavelet denoising and the flexible Gabor-wavelet denoising follows the next steps:

Algorithm 2.2. *Wavelet-based denoising with threshold:*

1. Decompose: Select a wavelet and select a level L. Compute the wavelet decomposition of the signal at level L.

2. Threshold detail coefficients: For each level from 1 to L, select a threshold and apply soft thresholding to the detail coefficients.

3. Reconstruct: Compute wavelet-based reconstruction using the original approximation coefficients of level L and the modified detail coefficients of levels from 1 to L.

4. Re-iterate or Finish.

3 Simulation of signal denoising

We have applied the denoising procedure based on the flexible Gabor-wavelet transform for highly perturbed and non-linear measurements in the sense of mixed stationary and transient components with an elevated signal-to-noise ratio, simulating the most complicated cases that can be recorded for mechanical structures. Our experiments were done in Matlab, after the acquisition of the sampled signal and with extra noise added. The improvements are expected to lead to better localization of defects in the clean signal.

For comparison purposes, we have compared our denoising procedure with the stardard wavelet denoising at different level of decomposition. We observed that the 'cleaning' properties of the denoising procedure based on the flexible Gabor-wavelet transform are powerful from the first level of decomposition. Actually, we managed to improve in only one step the signal-to-noise ration with more than half a point above the results that can be obtained using the wavelet transform (e.g. SNRoriginal=3, SNRwavelet=2.5, SNRgaborwavelet=2). Even at a simple visual inspection of the figures on the right, it is obvious that at the same level of decomposition and using the same some threshold criteria the flexible Gabor-transform performs a better denoising than the standard wavelet procedure. One straightforward explanation of this effect is the following: meanwhile the wavelets are dealing well with rapid-variable components of high frequencies, our flexible Gabor-wavelet procedures performs a time-frequency representation both at high and low frequency levels where it is extremely difficult to separate the noise in the raw measurements. In this way, we can approach the denoising not only for band-limited signals but also for slowly-variable band-limited signals by dilating the analyzing window accordingly to the frequency content.

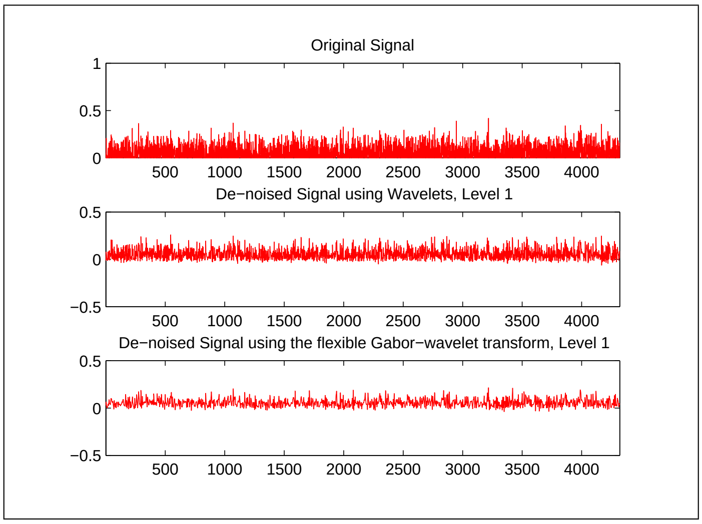

Figure 1: Denoising comparison at level 1

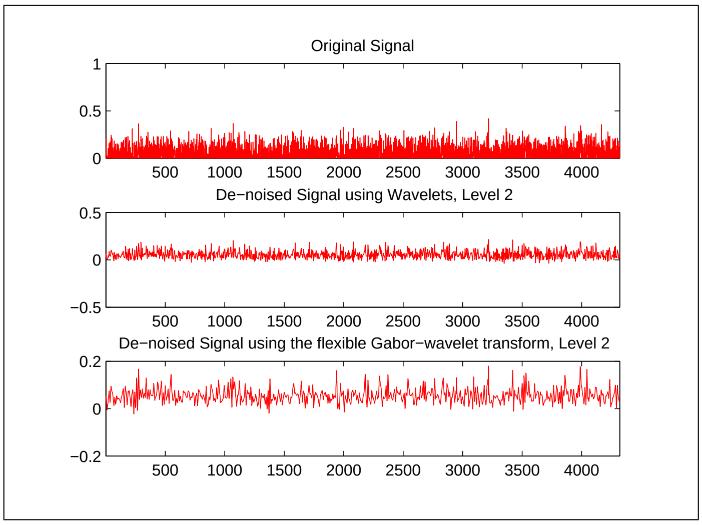

Figure 2: Denoising comparison at level 2

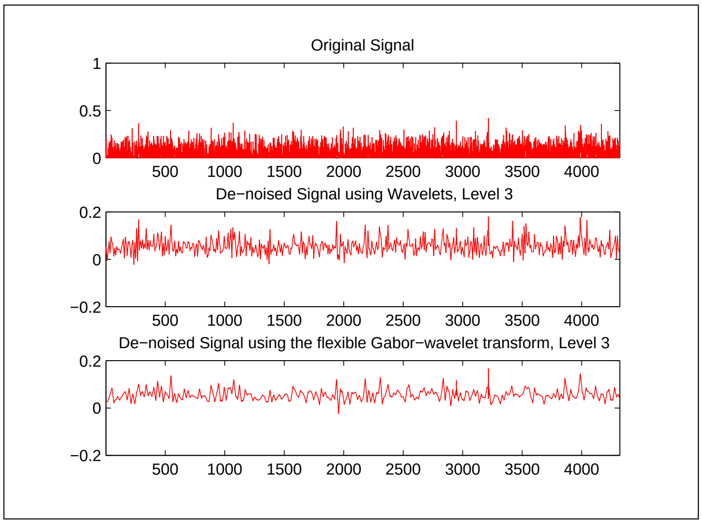

Figure 3: Denoising comparison at level 3

4 Conclusion

We introduced in this paper a new denoising procedure based on the flexible Gabor-wavelet transform and we compared the results with the typical wavelet denoising procedure. We observed that due to its adaptive characteristics, involving a well defined connection of frequencies and dilations the Gabor-wavelet denoising procedure is more accurate for complicated measurements of mechanical structures, simulated under heavy noise perturbations.

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