Design Optimization for Robustness Considering the Gear Transmission Error

Shinn-Liang Chang, Jia-Hung Liu, Kai-Wei Jin, Ching-Hua Hung, and Shang-Hsin Chen

***Abstract*—Transmission error in gearing system is a critical index for product quality. Gears with appropriate tooth modification can always behave well in transmission error, i.e. the double crowned gears. Nevertheless, to manufacture a gear with double crowning by CNC gear shaving machine indeed costly. This paper investigated the manufacture of double crowned gears by traditional gear shaving machine. Taguchi Method was applied to find critical parameters in the shaving process and the transmission error had also been improved through the process of robust design.**

***Index Terms*—Gear shaving, Robust Design, Taguchi method, Transmission error.**

I. Introduction

With the increasing requirements of transmission systems for high rotation speed and compact size, the precision of gears, which are the most important components in transmission systems, is also highly required. Gear shaving is one of the most efficient and economical process for gear finishing after the rough cuttings of hobbing or shaping. The shaving process has the ability to correct errors in index, helix angle, tooth profile, and eccentricity by removing fine hair-like chips from gear tooth surfaces [1]. The basic meshing condition of 3D crossed-axis helical gear pair was derived by Litvin [2], which has been widely adopted as the fundamental assumption for simulation of gear shaving.

Gear tooth crowning can produce significant improvements in transmission. The double crowned gear (gear teeth crowned both in lead and profile directions) is an excellent example [3], and it can be manufactured efficiently by gear shaving with CNC shaving machine [4]. However, it is indeed costly for gear manufacturer to replace a traditional shaving machine by a CNC one. The coordinated motions of traditional shaving machine is driven by mechanism instead of controller, and, nevertheless, only lead crowning can be conducted. Chang et al. [5] derived the mathematical models of the shaving cutter, the traditional shaving machine and the shaved gears. The effects of machine parameters on gear tooth lead crowning and tooth contact analysis were then investigated.

Transmission error is one of the most important indication of gearing quality. Through optimum or robust design, the transmission error can be significantly improved. Chang et al. [6] studied the transmission error in a modified helical gear train, and the system was optimized by adjusting the helical angles of the gears. Sundaresan et al. [7] explored the influences of gear profile parameters on transmission error, and robust design of the whole assembly was then achieved. Based on the past researches, the main subject of this paper is to investigate the transmission error of the gear pair with double crowning manufactured by traditional shaving machine, and to carry out robust design thereafter by Taguchi Method [8]. The results of this research can provide the manufacturers of gears, shaving cutters and shaving machines with useful guidelines.

II. Gear Tooth Surface Manufactured by the Traditional Shaving Machine

As mentioned above, the coordinated motions of the traditional shaving machine can only produce tooth lead crowning. Therefore, to manufacture a double crowned gear by this machine, the shaving cutter needs to be modified in the tooth profile direction, and it can be generated by the parabolic rack cutter shown in Fig.1. a_c is the parabolic coefficient used to control the profile of the rack cutter. Involute profile of shaving cutter can also be generated by simply setting $a_c = 0$, represented as (1).

$$\mathbf{r}_a = \begin{bmatrix} u_c & -a_c u_c^2 & 0 & 1 \end{bmatrix}^T \tag{1}$$

Through the coordinate transformations from rack cutter to shaving cutter, which is shown in Figs. 2 and 3, the locus can be obtained shown as (2) and (3).

$$\mathbf{r}_c = \mathbf{M}_{ca}(\theta_c)\mathbf{r}_a(u_c) = [x_c(u_c,\theta_c) \quad y_c(u_c,\theta_c) \quad z_c(u_c,\theta_c)]^T \tag{2}$$

$$\begin{aligned}[\mathbf{r}_s \quad 1]^T &= \mathbf{M}_{sc}(\psi_s)[x_c(u_c,\theta_c) \quad y_c(u_c,\theta_c) \quad z_c(u_c,\theta_c) \quad 1]^T \\ &= [x_s(u_c,\theta_c,\psi_s) \quad y_s(u_c,\theta_c,\psi_s) \quad z_s(u_c,\theta_c,\psi_s) \quad 1]^T\end{aligned} \tag{3}$$

By deriving the normal vector (4) and the meshing equation (5), the tooth surface of shaving cutter can be obtained.

$$\mathbf{n}_s = \frac{\partial \mathbf{r}_s}{\partial u_c} \times \frac{\partial \mathbf{r}_s}{\partial \theta_c} \tag{4}$$

$$f_1(u_c,\theta_c,\psi_s) = \mathbf{n}_s \cdot \frac{\partial \mathbf{r}_s}{\partial \psi_s} = 0 \tag{5}$$

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To derive the locus equations of the shaved gear, the coordinate systems again have to be constructed. The crowning mechanism of the traditional gear shaving machine shown in Fig.4 can induce lead crowning on shaved gear by rocking the work table. In the motion, the pivot can be fed horizontally only, and the pin will move along the guideway. Once the angle θ between the guideway and the horizontal is specified ($\neq 0$) in the shaving process, the rocking motion of the work table can be achieved. When $\theta = 0$, the work table will move horizontally without rocking and will therefore not produce any crowning effect.

The crowning mechanism can be further parameterized as shown in Fig.5, where d_v and d_h are the vertical and horizontal distances between the pin and pivot at the initial position. While the pivot (work table) moves z_t horizontally in shaving from position I to position II, the pin will move a distance d_p along the guideway. The rotating angle of the work table ψ_t can be derived as shown in (6) [5].

$$\psi_t = \sin^{-1}\left(\frac{d_v}{\sqrt{d_h^2 + d_v^2}}\right) + \sin^{-1}\left(\frac{d_h \sin\theta - d_v \cos\theta + z_t \sin\theta}{\sqrt{d_h^2 + d_v^2}}\right) - \theta \tag{6}$$

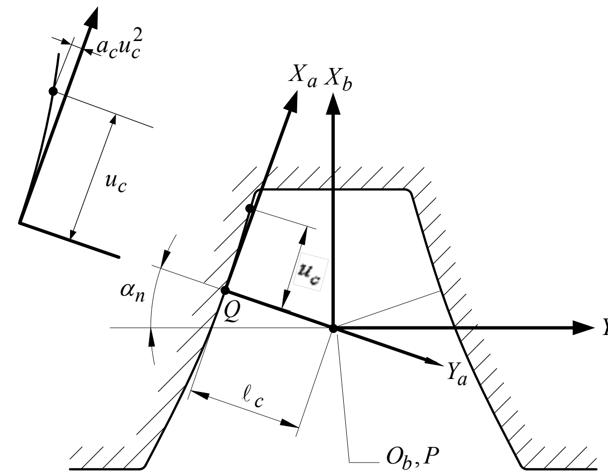

Fig.1. Normal section of the parabolic rack cutter

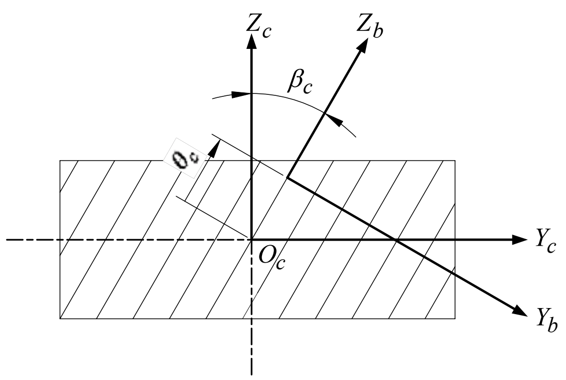

Fig. 2. Coordinate systems of the rack cutter from normal to axial section.

The coordinate systems of the shaving process can be simplified and illustrated by the coordinate systems shown in Fig.6, where the cutter assembly errors including horizontal, vertical, and center distance errors, are considered. The coordinate systems S_s and S_2' are connected to the shaving cutter and the work gear respectively while S_d is the fixed coordinate system; S_h' and S_v' are auxiliary coordinate systems for importing assembly errors into the horizontal and vertical directions; the angle Δh denotes the horizontal assembly error, the angle Δv denotes the vertical assembly error, and ΔE_0 indicates the error in the center distance. Other parameters in Fig.6 are also described as follows: z_t denotes the traveling distance of the shaving cutter along the axial direction of the work gear; C denotes the distance between the pivot and center of the work gear; γ denotes the angle between the two crossed axes; E_0 represents the center distance; ϕ_s and ϕ_2 represent the angles of rotation of the cutter and the gear respectively which are related to each other in the shaving operation.

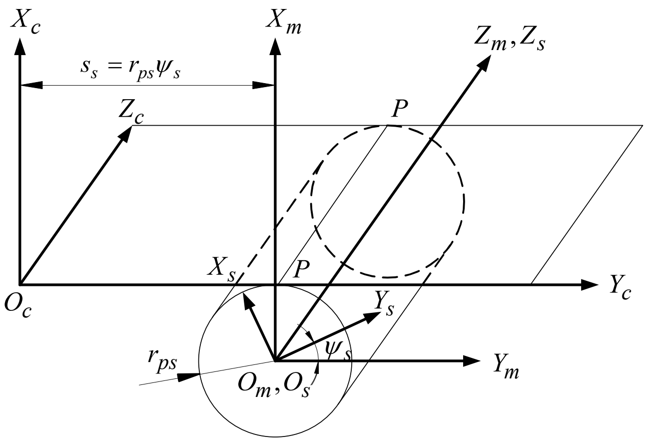

Fig. 3. Coordinate systems of the generating motion between the rack cutter and the shaving cutter.

The locus of shaving cutter on the shaved gear is derived by (7).

$$[\mathbf{r}_2(u_c, \theta_c, \psi_s, \varphi_s, z_t) \quad 1]^T = \mathbf{M}_{2s}(\varphi_s, z_t)[\mathbf{r}_s(u_c, \theta_c, \psi_s) \quad 1]^T \tag{7}$$

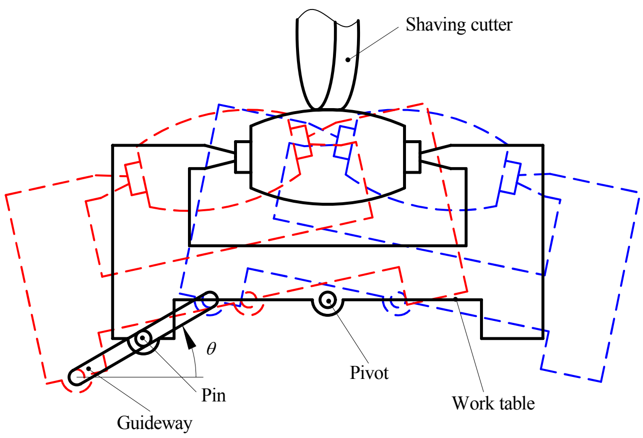

Fig. 4. Crowning mechanism of the traditional gear shaving machine.

The shaving process is controlled by two kinematic parameters φ_s and z_t so that two meshing equations (8) and (9) are required to solve the tooth surface of the shaved gear.

$$f_2(u_c, \theta_c, \psi_s, \varphi_s, z_t) = \mathbf{n}_2 \cdot \frac{\partial \mathbf{r}_2}{\partial \varphi_s} = 0 \quad (z_t = \text{constant}) \tag{8}$$

$$f_3(u_c, \theta_c, \psi_s, \varphi_s, z_t) = \mathbf{n}_2 \cdot \frac{\partial \mathbf{r}_2}{\partial z_t} = 0 \quad (\varphi_s = \text{constant}) \tag{9}$$

Note that the normal vector on S_2 is derived by (10), in which the 3×3 matrix $\mathbf{L}_{2s}$ is the sub-matrix of 4×4 matrix $\mathbf{M}_{2s}$.

$$\mathbf{n}_2 = \mathbf{L}_{2s}(\varphi_s, z_t)\mathbf{n}_s(u_c, \theta_c, \psi_s) \tag{10}$$

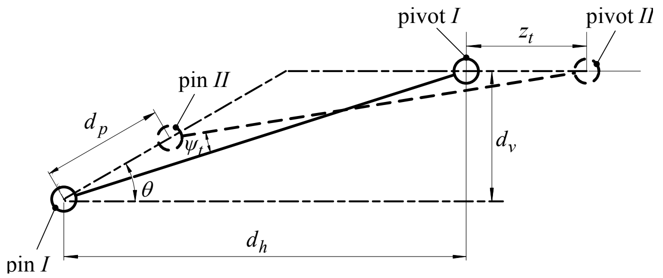

Fig. 5. Motion of pin on guideway of gear shaving machine.

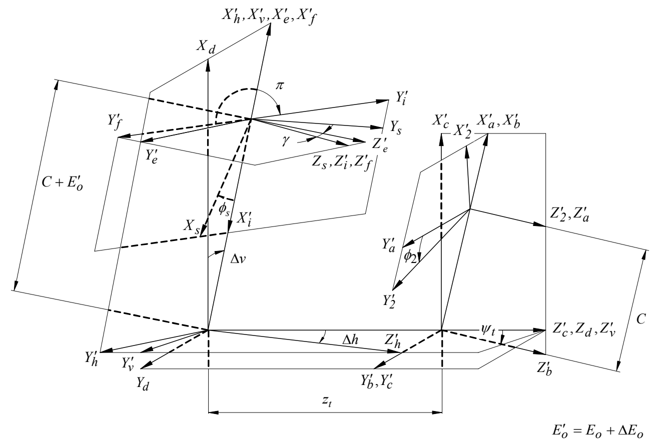

Fig. 6. Coordinate systems of shaving process.

III. Transmission Error Analysis of the Shaved Gear

Transmission error can be calculated by simulation of gear meshing. Considering a gear pair composed of a double crowned gear 2 and an involute gear 4, the coordinate systems can be illustrated as Fig.7. Coordinate system $S_2(X_2, Y_2, Z_2)$ is fixed on gear 2, and $S_4(X_4, Y_4, Z_4)$ is fixed on gear 4. The two gears rotate about axes Z_2 and Z_4 respectively. $S_h(X_h, Y_h, Z_h)$ and $S_v(X_v, Y_v, Z_v)$ are auxiliary coordinate systems for simulating assembly errors between gear 2 and gear 4 including horizontal error $\Delta\gamma_h$, vertical error $\Delta\gamma_v$ and the center distance error ΔE . ϕ'_2 and ϕ'_4 denote the real rotating angles of the two gears in operating. Transforming the vectors and unit normal vectors of the gear tooth surfaces to coordinate system $S_q(X_q, Y_q, Z_q)$, the two meshing surfaces Σ_2 and Σ_4 must satisfy

$$\mathbf{r}_q^{(2)} - \mathbf{r}_q^{(4)} = 0 \tag{11}$$

$$\mathbf{n}_q^{(2)} \times \mathbf{n}_q^{(4)} = 0 \tag{12}$$

, in which $\mathbf{r}_q^{(2)}$ and $\mathbf{r}_q^{(4)}$ are vectors while $\mathbf{n}_q^{(2)}$ and $\mathbf{n}_q^{(4)}$ are unit normal vectors of respective tooth surfaces, which is shown in Figs. 8

In the 3-D space, (11) and (12) include six scalar equations. By the given equation that for unit normal vectors $\left|\mathbf{n}_q^{(2)}\right| = \left|\mathbf{n}_q^{(4)}\right| = 1$, only five independent scalar equations need to be solved:

$$x_q^{(2)} - x_q^{(4)} = 0$$

$$y_q^{(2)} - y_q^{(4)} = 0$$

$$z_q^{(2)} - z_q^{(4)} = 0$$

$$n_{qy}^{(2)} n_{qz}^{(4)} - n_{qz}^{(2)} n_{qy}^{(4)} = 0$$

$$n_{qz}^{(2)} n_{qx}^{(4)} - n_{qx}^{(2)} n_{qz}^{(4)} = 0$$

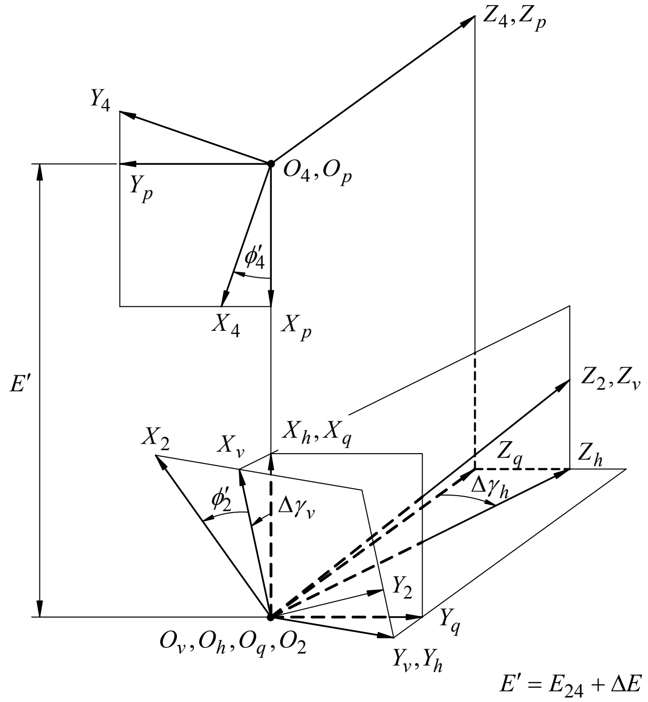

Fig. 7. Coordinate systems of the meshing gear pair.

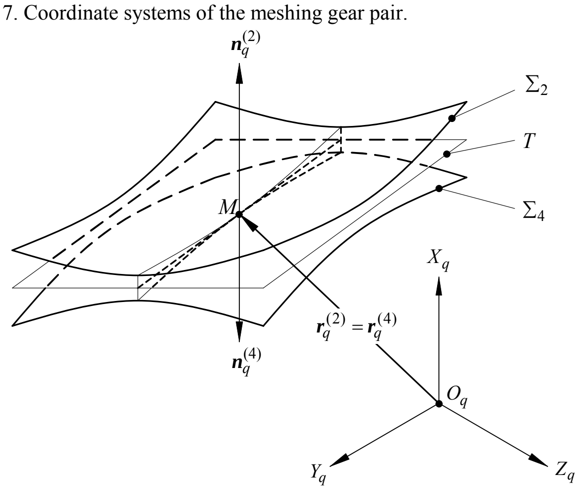

Fig. 8. Gearing contact between two tooth surfaces.

Considering the contact conditions listed in (11) and (12) the relation between real rotating angles $\phi'_4(\phi'_2)$ can be obtained calculated. Transmission error is defined as the difference between the real and the theoretical rotating angles, which can be represented as

$$\Delta\phi'_4(\phi'_2) = \phi'_4(\phi'_2) - T_2\phi'_2 / T_4 \tag{13}$$

, where T_2 and T_4 are tooth numbers of gear 2 and 4 respectively, and $T_2\varphi'_2/T_4$ is the theoretical rotating angle of gear 4.

IV. Robust Design for Transmission Error

In the constructed model for calculation of transmission error, many parameters can be tuned to observe their influences. They can be further divided into four categories: modification of shaving cutter, assembly errors of shaving cutter, assembly errors of gears and machine setting parameters, which are illustrated in the fishbone diagram illustrated as Fig. 8. Taguchi Method was adopted as the tool in this paper to investigate the system robustness of the gear shaving process.

Fundamental data of the gear pair for simulation are listed in Table 1. To plan the simulations, factors from different categories and their levels must be considered. As listed in Table 2, eleven factors and three levels for each factor were specified. The simulations of transmission error analysis were planned according to the L36 ($2^{11}\times3^{12}$) orthogonal array. The arrangements and results (S/N ratios) of the simulations are listed in Table 3.In the analyses, the value of transmission error was considered as the product quality, which was expected to be "smaller the better", and the formula for calculating the S/N ratio [8] is

$$S/N = -10Log\frac{\sum_{i=1}^{n} {y_i}^2}{n} \tag{14}$$

, in which n and y_i denote the number and result of simulation performed for each set of parameters in Table 3. Taking simulation 1 as an example, the S/N can be calculated as:

$$S/N = -10Log\frac{(1.2279)^2}{1} = -1.7838$$

by using (12).

Table 1 Parameters of the gear pair.

Item	Gear 2	Gear 4
Module (m_n)	2.65	2.65
Tooth number (T_i)	36	36
Helix angle (β_i)	10° L.H.	10° R.H.
Pressure angle (α_n)	20°	20°

Table 2 Factors and their levels in transmission error analysis.

Factor		Unit	Level		
			1	2	3
A	θ	degree	1°50′	2°50′	3°50′
B	d_v	*mm*	500	545	590
C	d_h	*mm*	350	385	420
D	C	*mm*	150	188	226
E	$\Delta\gamma_v$	degree	-0.02	0	0.02
F	$\Delta\gamma_h$	degree	-0.02	0	0.02
G	ΔE	*mm*	-1	0	1
H	Δv	degree	-0.02	0	0.02
I	Δh	degree	-0.02	0	0.02
J	ΔE_o	*mm*	-1	0	1
K	a_c	-	0.0001	0.0005	0.001

The factors' responses can also be obtained from Table 3. For instance, the mean S/N of parameter A of level 1 can be calculated by:

$$\begin{aligned}A_1 &= \frac{1}{12}(R_1+R_4+R_7+R_{10}+R_{13}+R_{16}+R_{19}+R_{22}\\&\quad +R_{25}+R_{28}+R_{31}+R_{34})\\&= \frac{1}{12}[(-1.7838)+(-11.5664)+(-5.0211)+0+\\&\quad 7.1367+(-4.3953)+(-2.4226)+(-10.1193)\\&\quad +0+7.5297+(-5.1572)+(-16.4801)]\\&= -3.5233\end{aligned}$$

, in which R_i denotes the i-th simulation result. Similarly, those of level 2 and 3 can be obtained as -8.9130 (level 2) and -7.0376 (level 3). The effect of A is then calculated by subtracting the min from max.

$$Effect = (-3.5233)-(-8.9130) = 5.3897$$

After reorganizing, the factor response table and graph are shown in Table 4 and Fig. 9 respectively. It is obvious that the combination of factors A1, B1, C2, D2, E1, F3, G3, H3, I2, J2 and K1 can produce the best quality, namely, the shaving process with this set of parameters can manufacture the gear pair of the least transmission error. The ranking of the factor sensitivity (from the highest to the lowest) is K1, A1, J2, I2, H3, D2, B1, F3, E1, C2, and G3. The improvements on transmission error can be validated by the results shown in Fig.10. The dashed line represents the original set of parameter marked with shadow in Table 2 while the solid line represents the improved one. With the improved parameters, the curves of transmission error become smoother, which means that vibrations and noises can be reduced when the gear pair is operating.

TABLE 3 THE S/N RATIOS OF THE TRANSMISSION ERROR ANALYSES.

No.	1				2			3			4	S/N
	θ	d_v	d_h	C	$\Delta\gamma_v$	$\Delta\gamma_h$	ΔE	Δv	Δh	ΔE_0	a_c	
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	
1	1	1	1	1	1	1	1	1	1	1	1	-1.7838
2	2	2	2	2	2	2	2	2	2	2	2	-1.7838
3	3	3	3	3	3	3	3	3	3	3	3	-10.6055
4	1	1	1	1	2	2	2	2	3	3	3	-11.5664
5	2	2	2	2	3	3	3	3	1	1	1	-2.3489
6	3	3	3	3	1	1	1	1	2	2	2	-5.0211
7	1	1	2	3	1	2	3	3	1	2	2	-5.0211
8	2	2	3	1	2	3	1	1	2	3	3	-20.0087
9	3	3	1	2	3	1	2	2	3	1	1	-4.0242
10	1	1	3	2	1	3	2	3	2	1	3	0
11	2	2	1	3	2	1	3	1	3	2	1	0.6389
12	3	3	2	1	3	2	1	2	1	3	2	-12.3083
13	1	2	3	1	3	2	1	3	3	2	1	7.1367
14	2	3	1	2	1	3	2	1	1	3	2	-13.3246
15	3	1	2	3	2	1	3	2	2	1	3	-24.0182
16	1	2	3	2	1	1	3	2	3	3	2	-4.3953
17	2	3	1	3	2	2	1	3	1	1	3	-18.1906
18	3	1	2	1	3	3	2	1	2	2	1	8.3860
19	1	2	1	3	3	3	1	2	2	1	2	-2.4226
20	2	3	2	1	1	1	2	3	3	2	3	-15.3050
21	3	1	3	2	2	2	3	1	1	3	1	7.5373
22	1	2	2	3	3	1	2	1	1	3	3	-10.1193
23	2	3	3	1	1	2	3	2	2	1	1	-2.5605
24	3	1	1	2	2	3	1	3	3	2	2	-4.8046
25	1	3	2	1	2	3	3	1	3	1	2	0
26	2	1	3	2	3	1	1	2	1	2	3	-11.5493
27	3	2	1	3	1	2	2	3	2	3	1	5.8249
28	1	3	2	2	2	1	1	3	2	3	1	7.5297
29	2	1	3	3	3	2	2	1	3	1	2	-16.1962
30	3	2	1	1	1	3	3	2	1	2	3	-11.2518
31	1	3	3	3	2	3	2	2	1	2	1	-5.1572
32	2	1	1	1	3	1	3	3	2	3	2	-5.0117
33	3	2	2	2	1	2	1	1	3	1	3	-17.0885
34	1	3	1	2	3	2	3	1	2	2	3	-16.4801
35	2	1	2	3	1	3	1	2	3	3	1	-1.3159
36	3	2	3	1	2	1	2	3	1	1	2	-17.0768

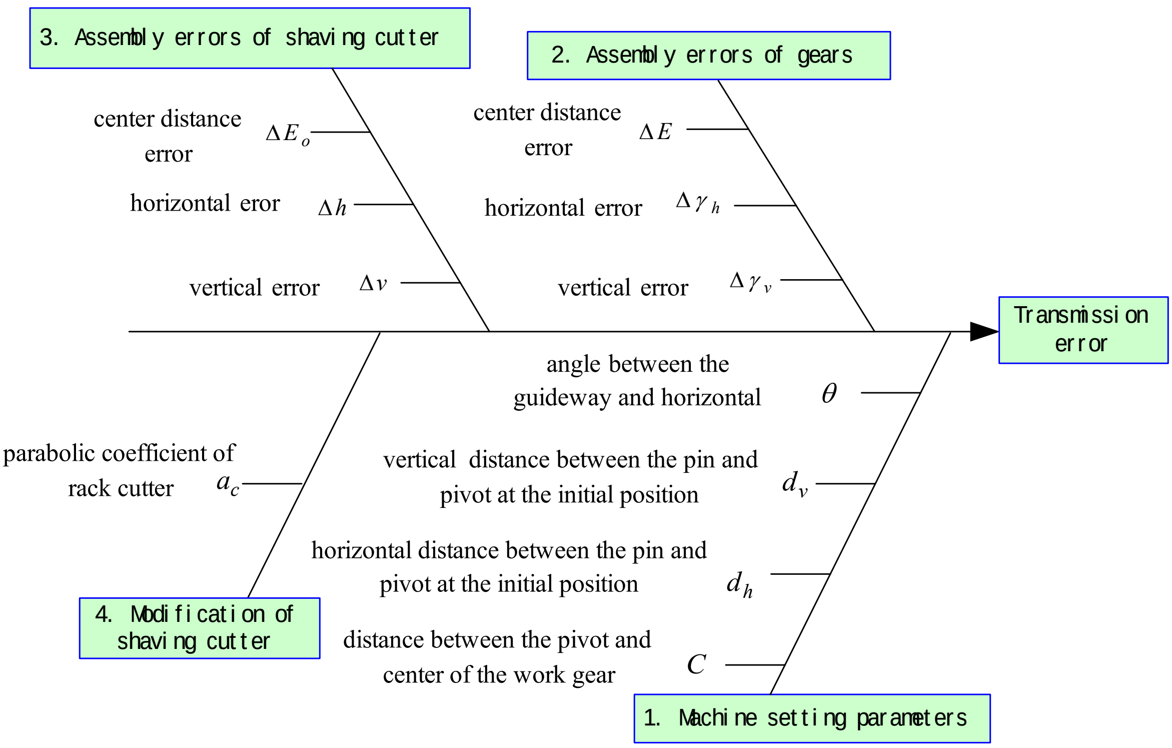

FIG. 8. FISHBONE DIAGRAM OF THE FACTORS.

TABLE 4 FACTOR RESPONSE TABLE FOR THE TRANSMISSION ERROR ANALYSES.

	A	B	C	D	E	F	G	H	I	J	K
Level 1	-3.5233	-5.4453	-6.8664	-6.7792	-5.9369	-7.5113	-6.6523	-6.9550	-8.3829	-8.8092	1.6553
Level 2	-8.9130	-6.0746	-6.1161	-5.0610	-7.2417	-6.7247	-6.6952	-7.6961	-4.6305	-5.0177	-7.2805
Level 3	-7.0376	-7.9540	-6.4914	-7.6337	-6.2953	-5.2378	-6.1264	-4.8227	-6.4605	-5.6470	-13.8486
Effect	5.3897	2.5086	0.7503	2.5726	1.3048	2.2735	0.5688	2.8734	3.7524	3.7915	15.5039
Ranking	2	7	10	6	9	8	11	5	4	3	1

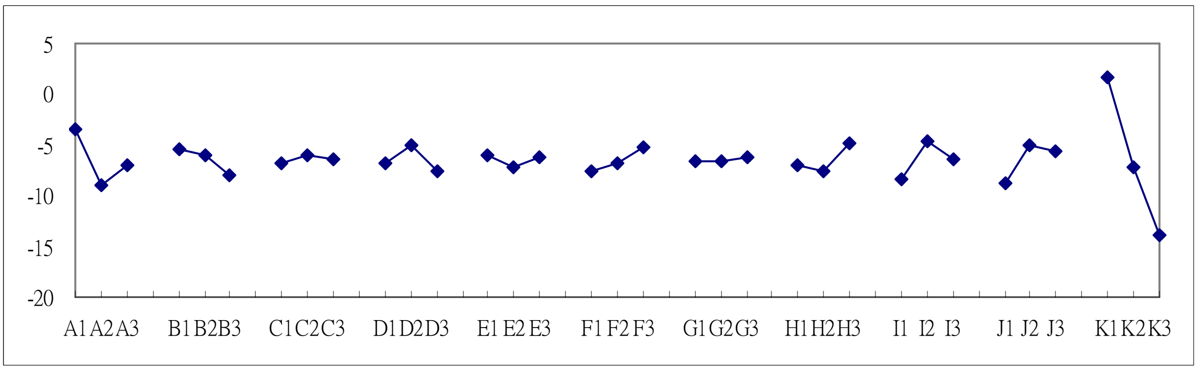

Fig. 9 Factor response graph for the transmission error analyses.

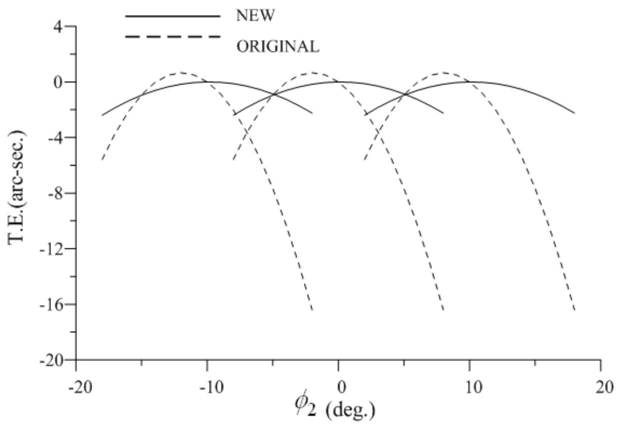

Fig. 10. Transmission errors caused by original and new parameters.

V. CONCLUSION

To have a double crowned gear shaved by traditional gear shaving machine for better performance in transmission error, four categories of parameter need to be considered: modification of shaving cutter, assembly errors of shaving cutter, assembly errors of gears and machine setting parameters. According to the studies in this paper, the conclusions can be made as the following:

1. The gear pair with new parameters indeed possesses better quality in transmission.
2. Among the eleven selected parameters, the coefficient a_c concerning the modification of shaving cutter and the angle θ between the guide way and horizontal on the shaving machine contribute the most to product quality (transmission error).

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REFERENCES

[1] Townsend, D.P., 1992. Dudley's Gear handbook: The Design, Manufacture, and Application of Gears, McGraw-Hill, New York.

[2] Litvin, F. L., 1994. Gear Geometry and Applied Theory, PTR Prentice Hall, New Jersey.

[3] Litvin, F.L., 2001. Helical and Spur Gear Drive with Double-Crowned Pinion Tooth Surfaces and Conjugated Gear Tooth Surfaces, United States Patent, Patent No: US 6205879.

[4] Litvin, F. L., Fan, Q., Vecchiato, D., Dememego, A., Handschuh, R. F., Sep, T. M., 2001. Computerized Generation and Simulation of Meshing of Modified Spur and Helical Gears Manufactured by Shaving, Computer Methods in Applied Mechanics and Engineering, vol. 190, pp.5037-5055.

[5] Chang, S. L., Lin, H. J., Chu, C. H., Liu, J. H., Hung, C. H., 2007. Simulation of Gear Shaving Machine and Tooth Contact Analysis of the Shaved Gears, Proceedings of the IAENG International Conference on Industrial Engineering, vol. 2, pp.2187-2192.

[6] Chang, S. L., Tsay, C. B., Tseng, C. H., 1997. Kinematic Optimization of a Modified Helical Simple Gear Train, ASME Journal of Mechanical Design, vol. 119, pp. 307-314.

[7] Sundaresan, S., Ishii, K., Houser, D.R., 1991. Design Optimization for Robustness Using Performance Simulation Programs, ASME Journal of Design Automation, vol. 1, no. 32, pp. 249-256.

[8] Taguchi, G., 1993. Taguchi methods: design of experiments, ASI Press, Dearborn, MI.