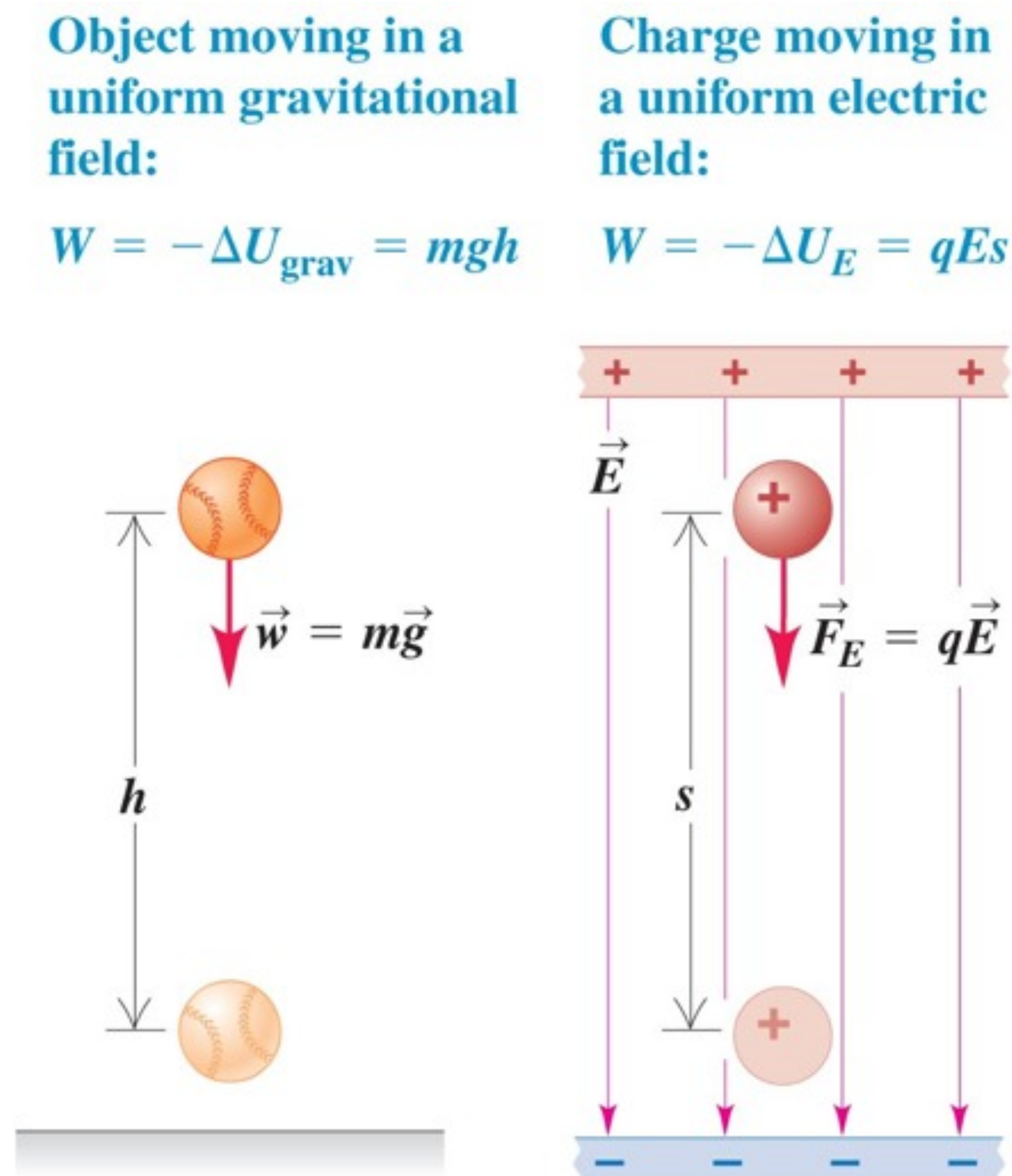
electric potential and capacitance

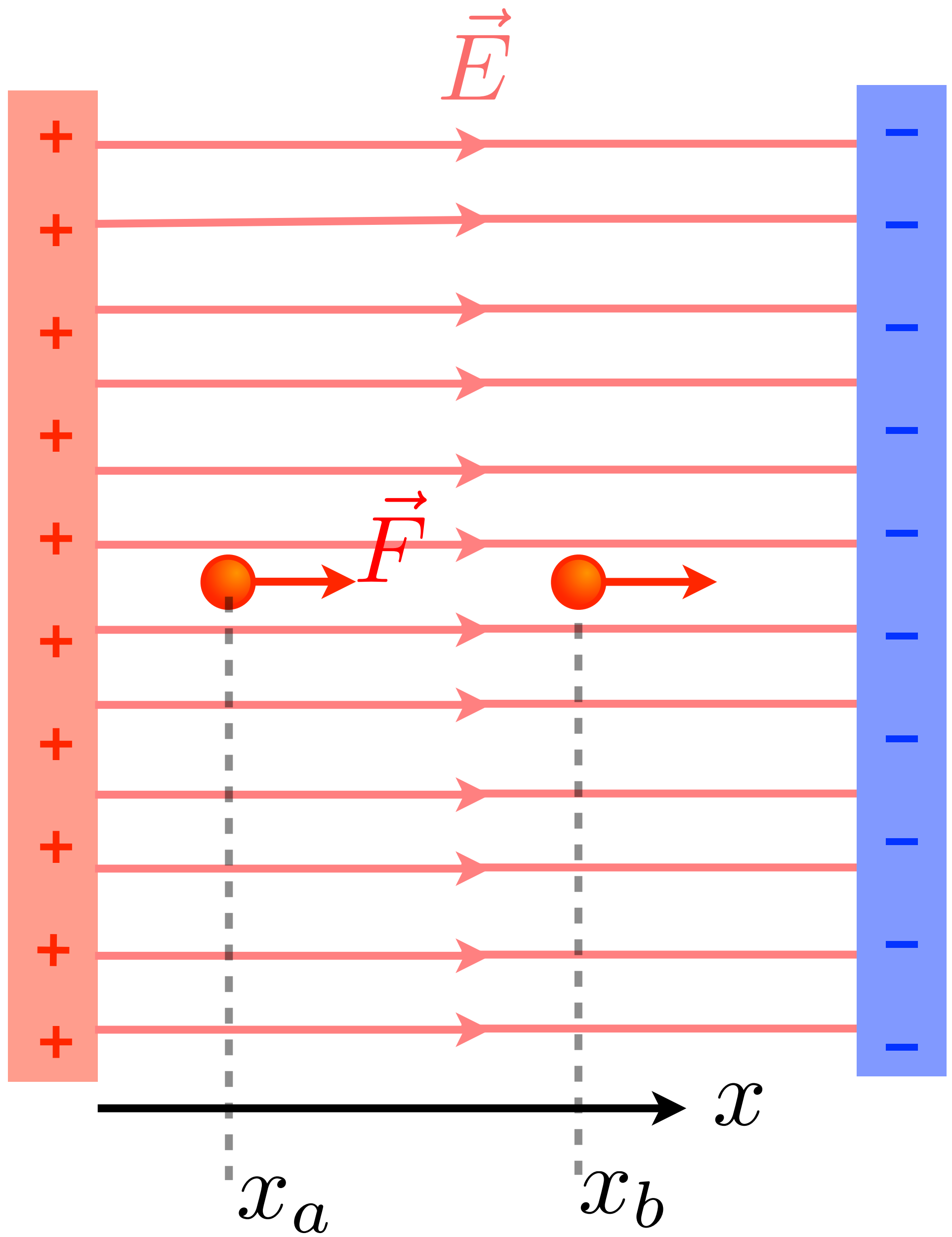
electric potential energy

➜ consider a uniform electric field (e.g. from parallel plates)

➜ note the analogy to gravitational force near the surface of the Earth



potential between parallel plates



$$W = F(x_b - x_a) = qE(x_b - x_a)$$

$$W = U_a - U_b$$

potential energy $\boxed{U = -qEx + c}$

define a quantity that depends only upon the field and not the value of the test charge

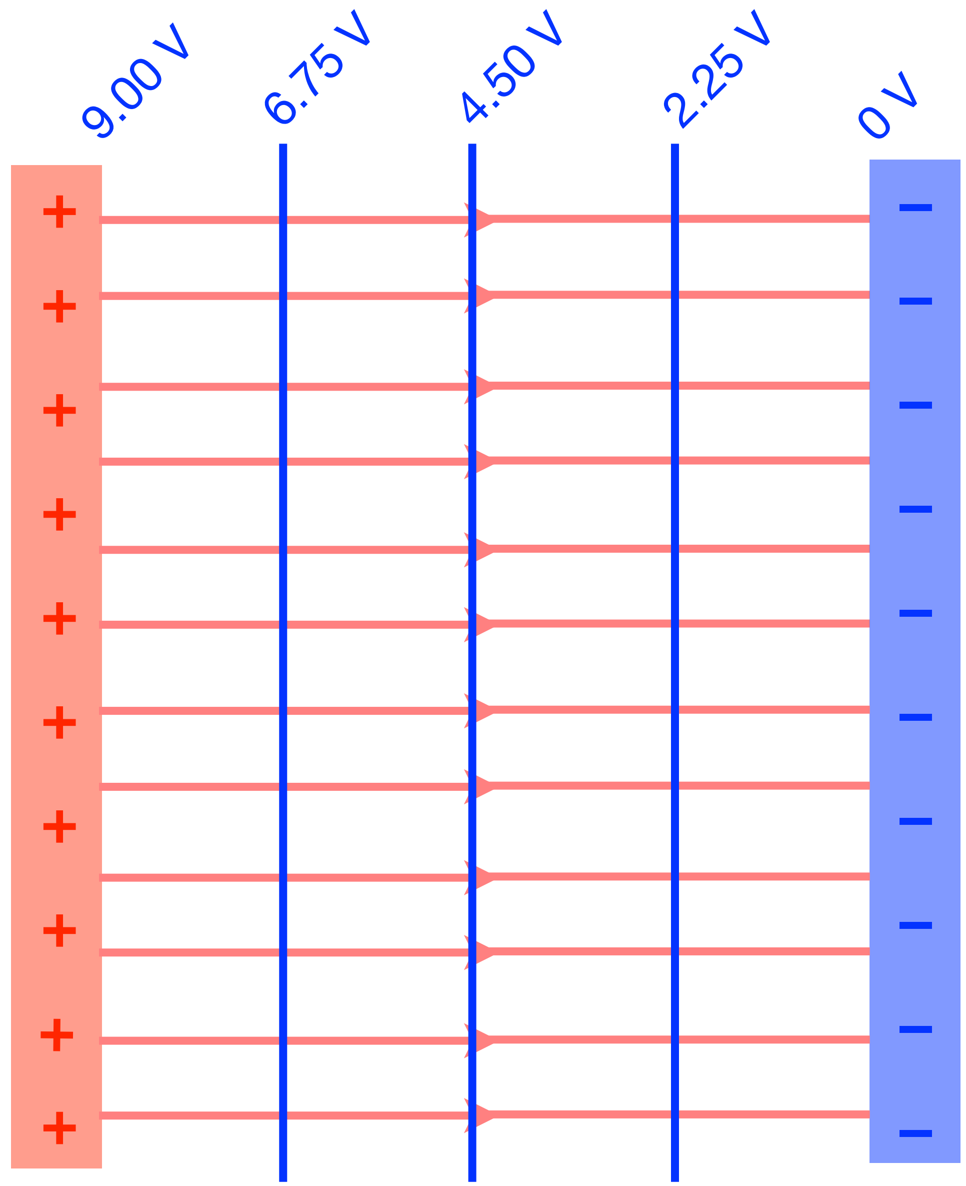
the 'potential' $V = \dfrac{U}{q}$

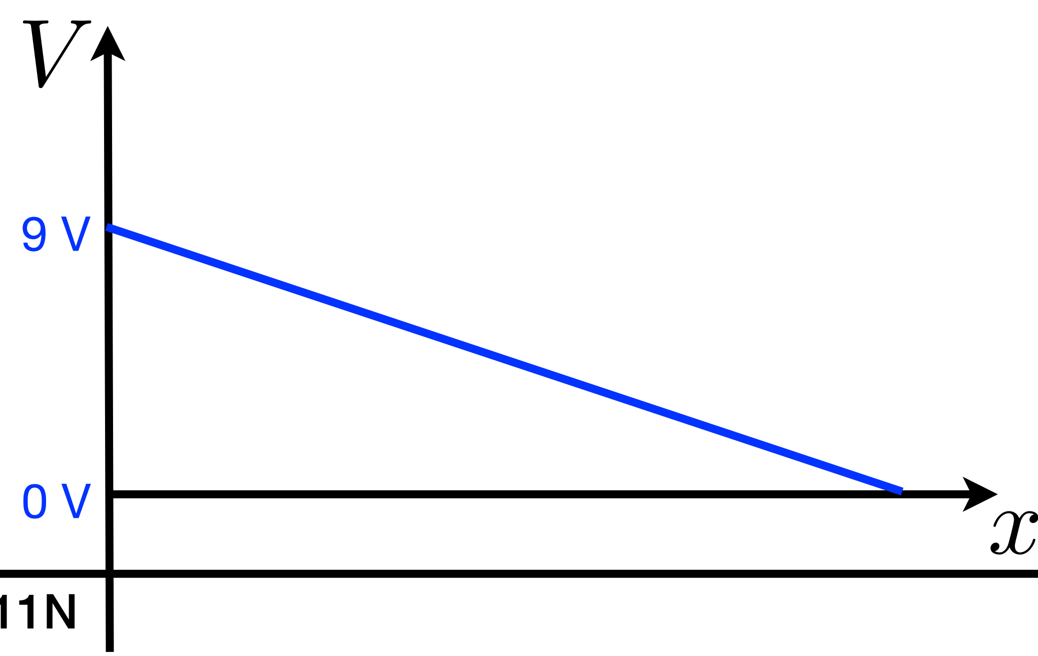
measured in Volts, V = J/C

$$\boxed{V = V_0 - Ex}$$

actually only differences of potential are meaningful, we can add a constant to V if we like

potential between parallel plates





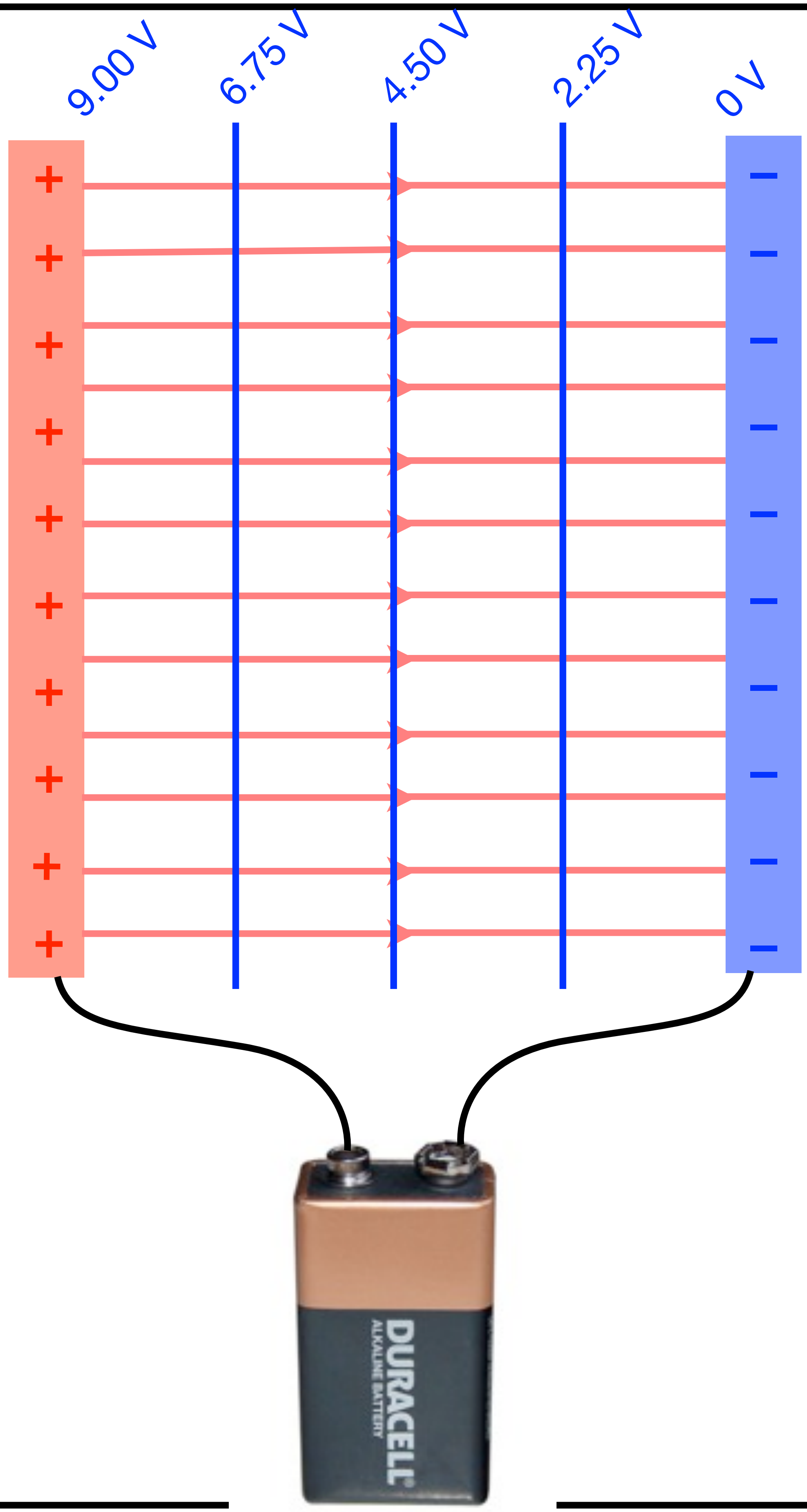
equipotential lines

lines of equal value of potential

$$V = V_0 - Ex$$

arbitrarily choose V=0 at the right-hand plate

a capacitor

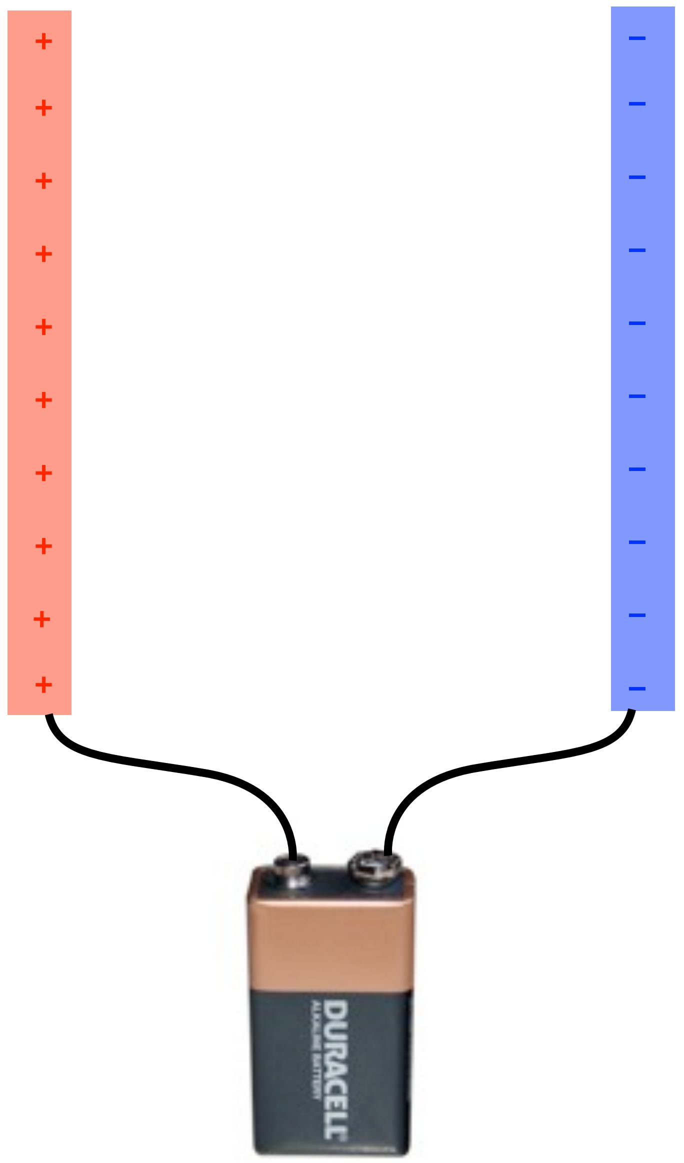


conservation of energy using potential

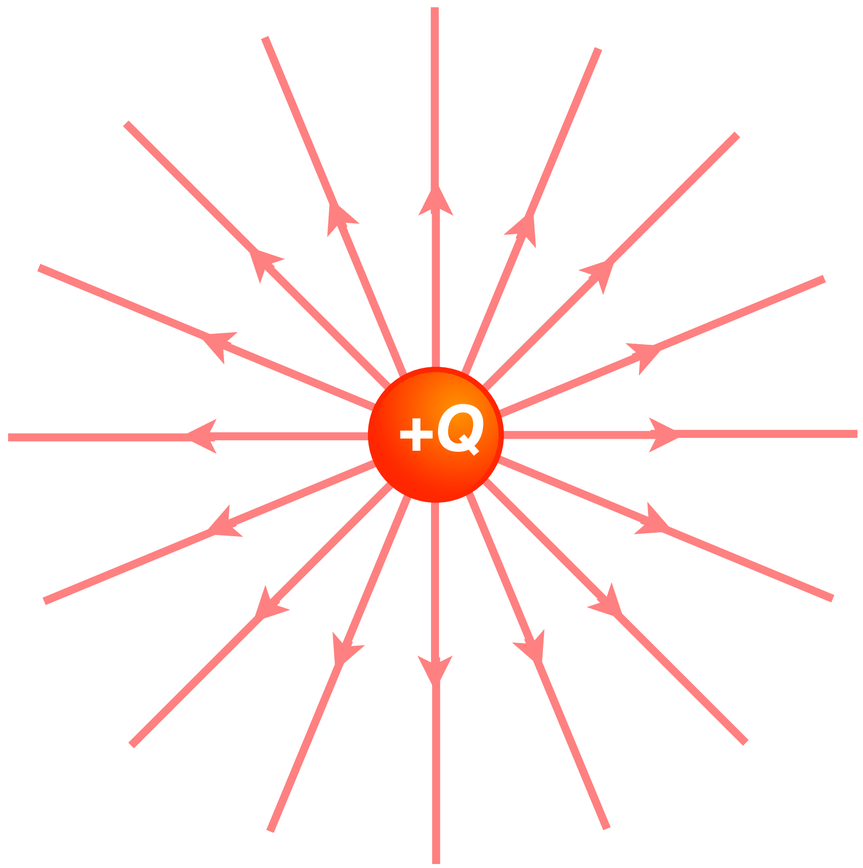
A 9 V battery is connected across two large parallel plates that are separated by 9.0 mm of air, creating a potential difference of 9.0 V. An electron is released from rest at the negative plate - how fast is it moving just before it hits the positive plate ?

$$K_a + U_a = K_b + U_b$$

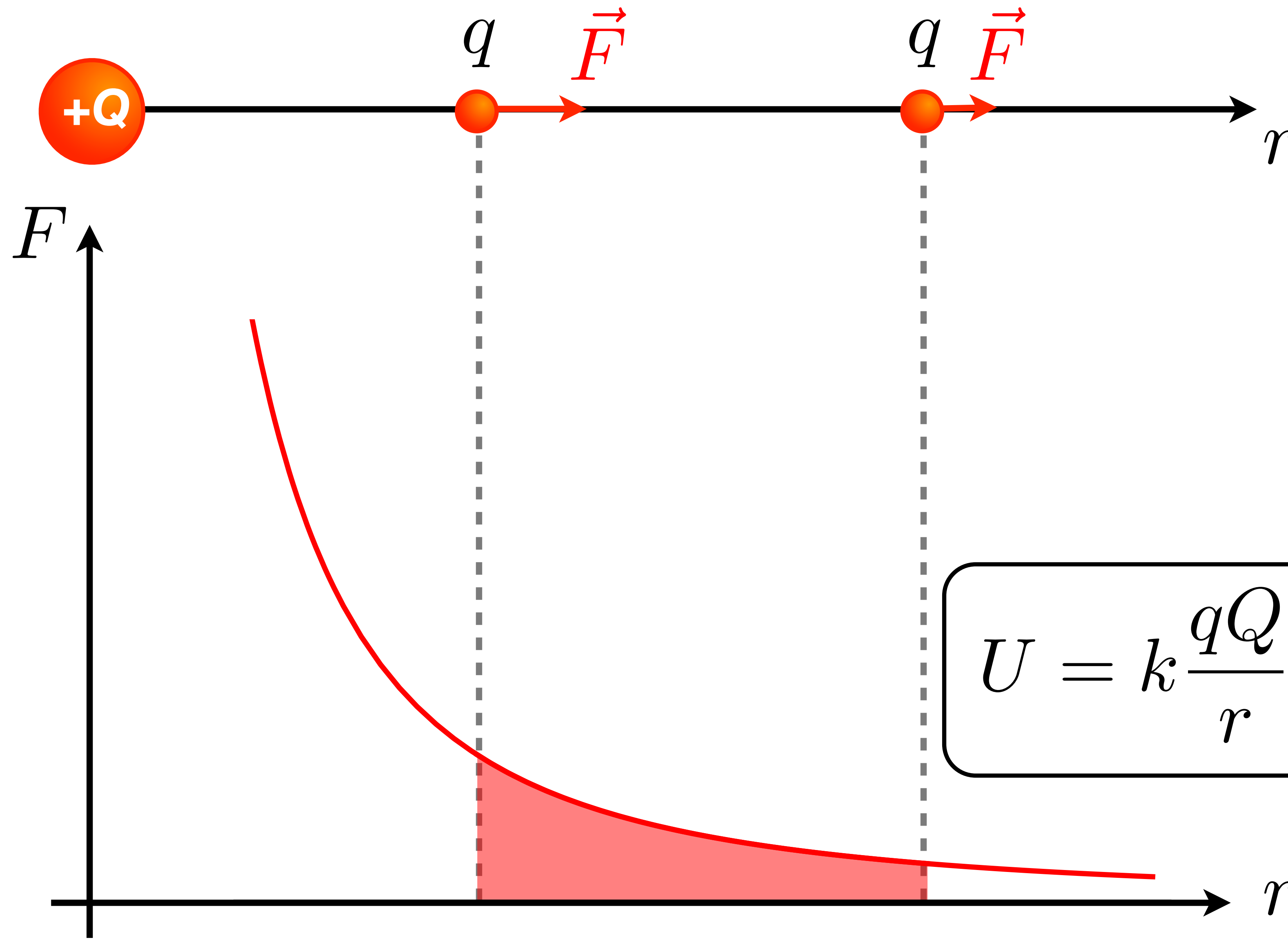
$$U = qV$$



potential energy between point charges



$$F = k\frac{|qQ|}{r^2}$$



$$U = k\frac{qQ}{r}$$

potential from a point charge

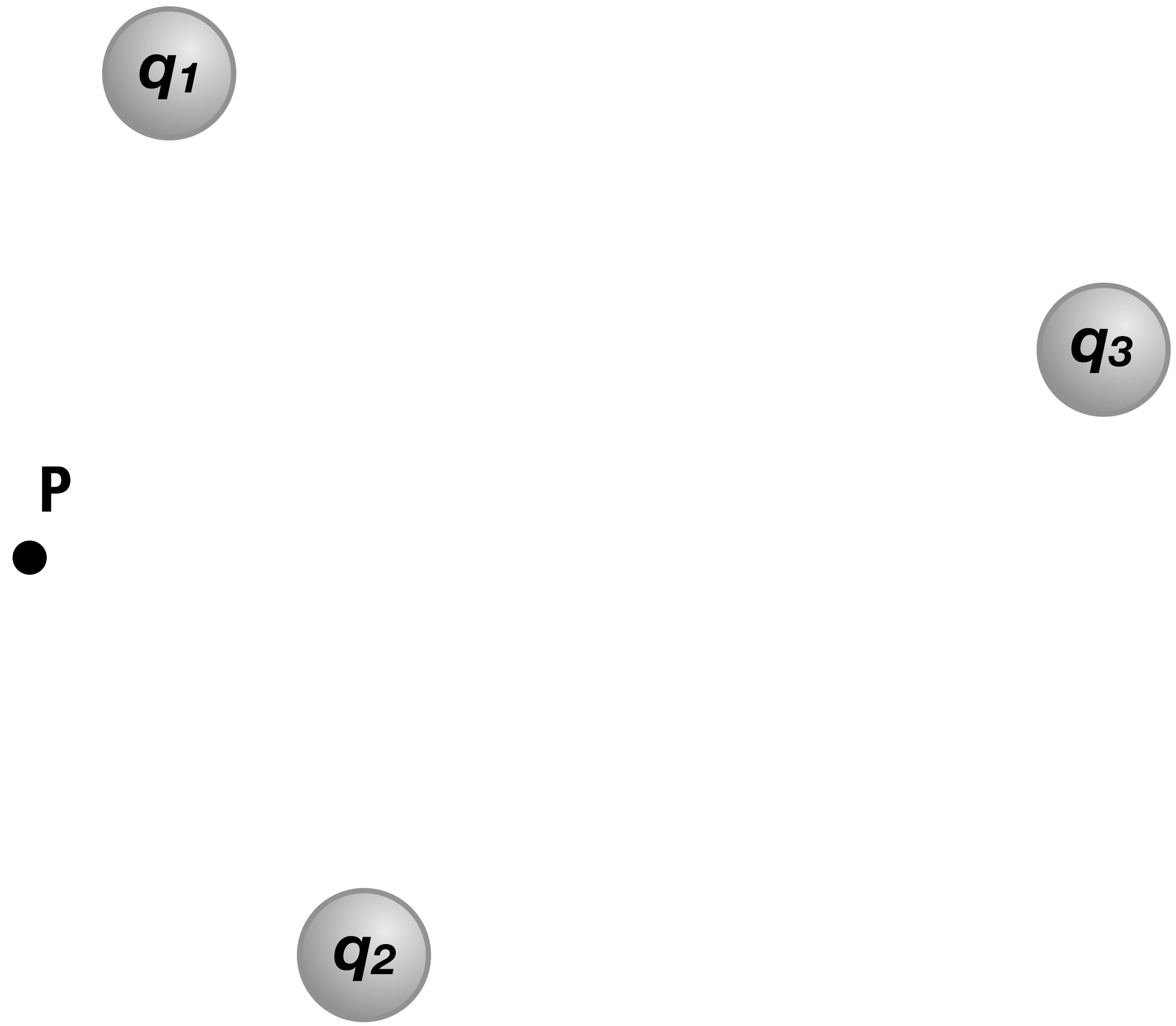
depend only on distance
from the charge Q

$$F = k\frac{|qQ|}{r^2} \longrightarrow \vec{E} = \frac{\vec{F}}{q} \longrightarrow E = k\frac{|Q|}{r^2}$$

$$U = k\frac{qQ}{r} \longrightarrow V = \frac{U}{q} \longrightarrow V = k\frac{Q}{r}$$

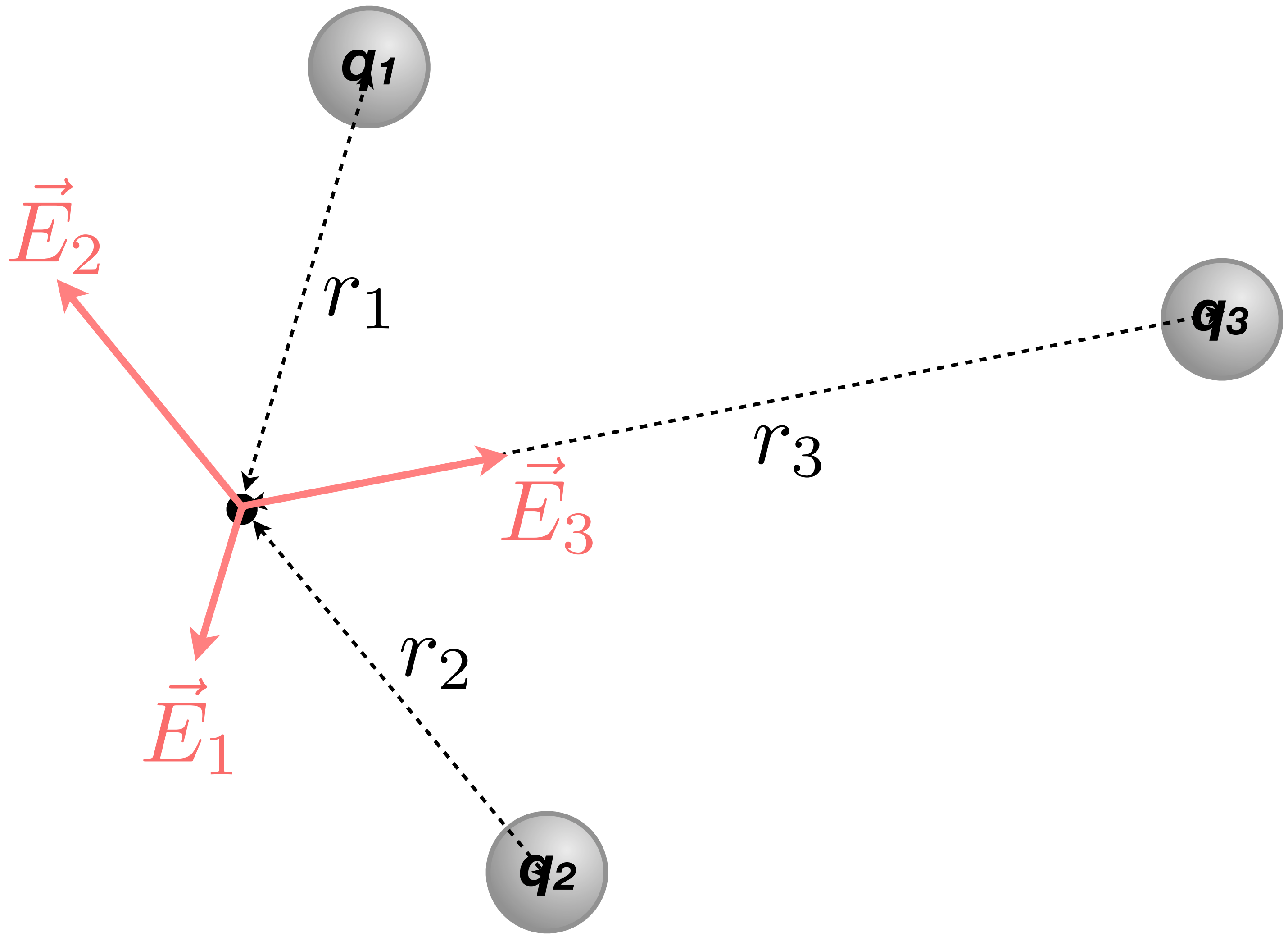
arbitrarily choose V=0
infinitely far from the charge

a set of point charges



at the point **P** there is an electric field **E** and an electric potential V

electric field from a set of point charges

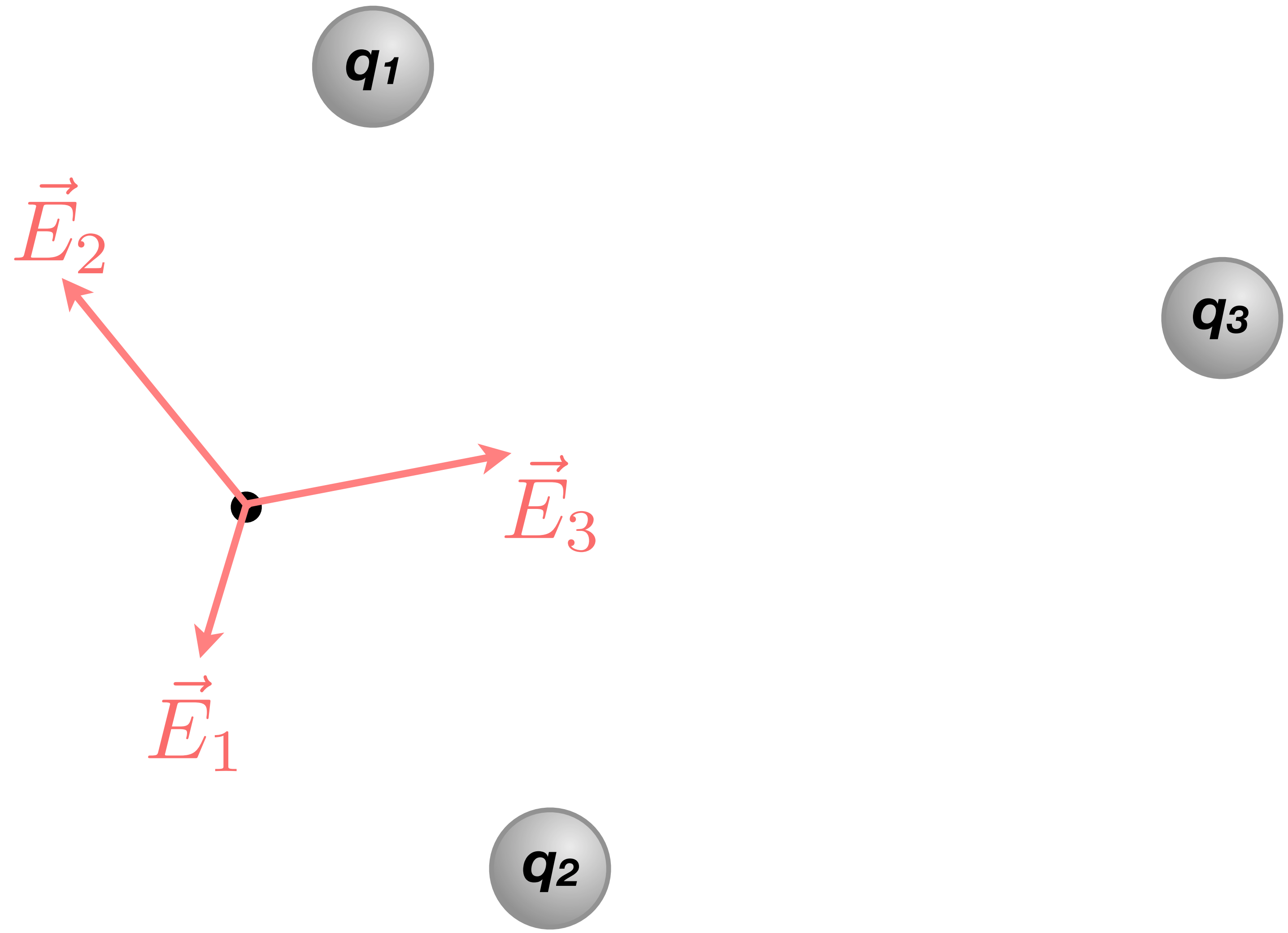


$$E_1 = k\frac{|q_1|}{r_1^2}$$

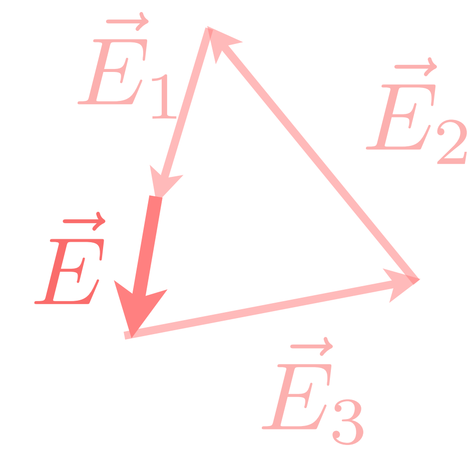
$$E_2 = k\frac{|q_2|}{r_2^2}$$

$$E_3 = k\frac{|q_3|}{r_3^2}$$

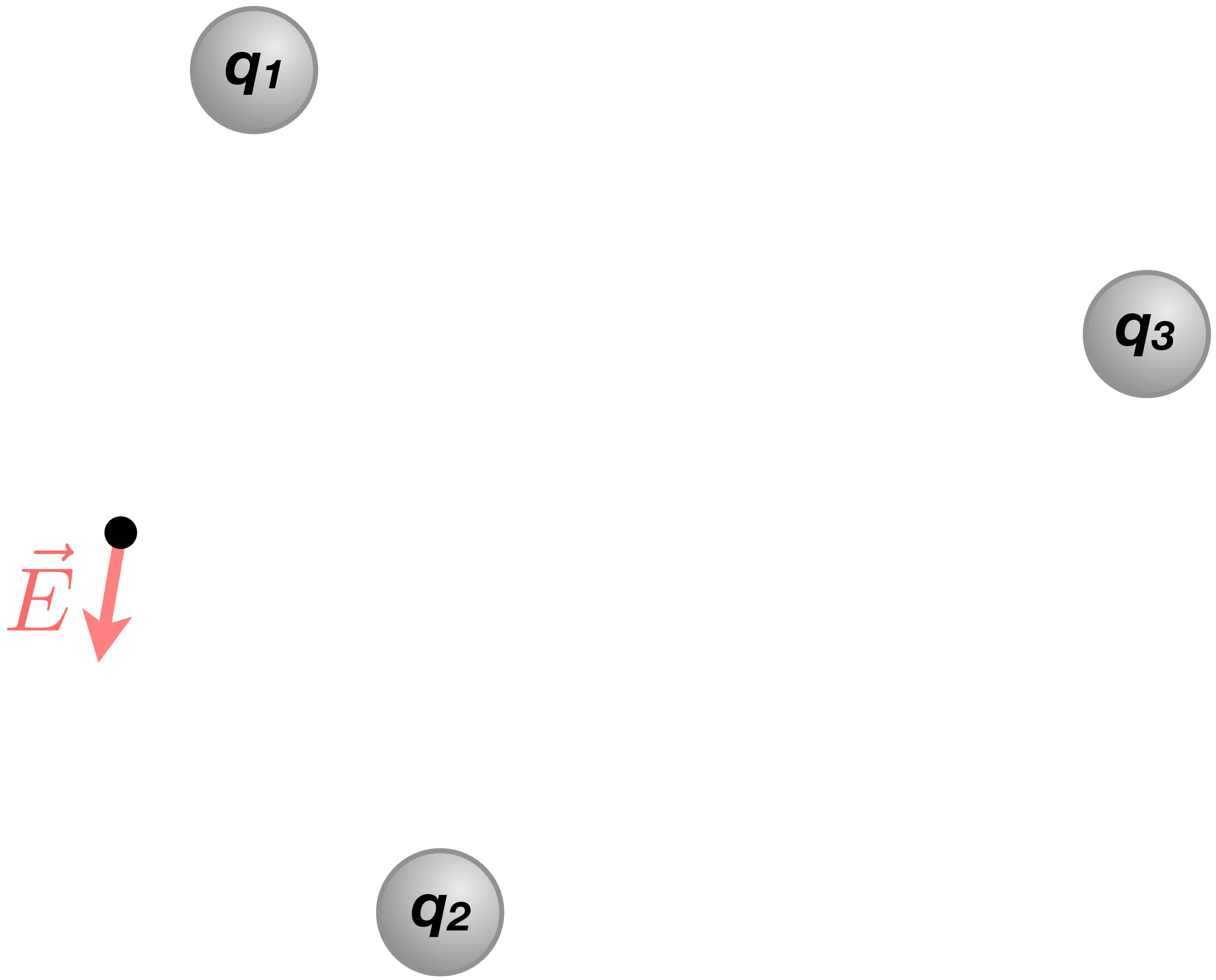
electric field from a set of point charges



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

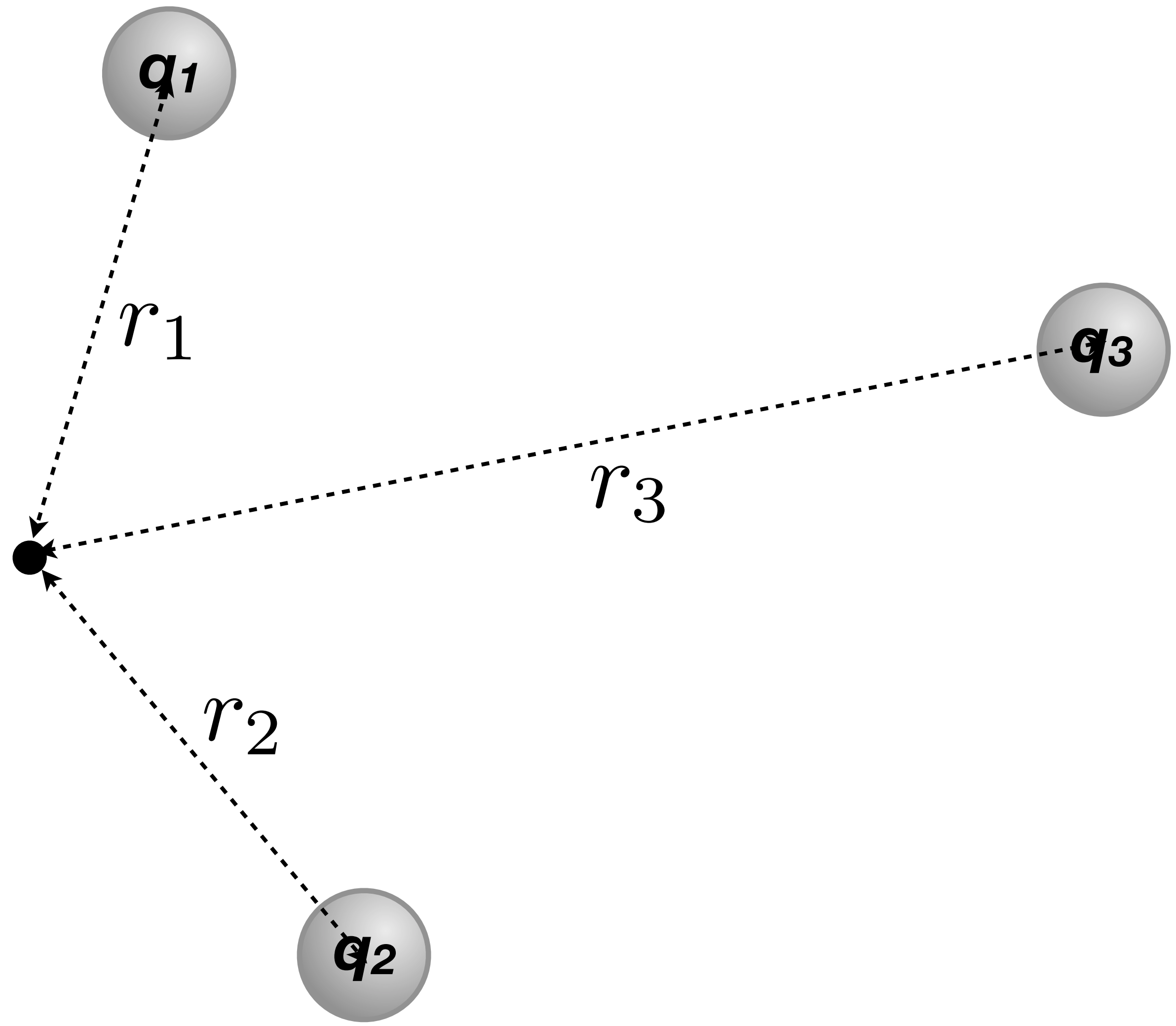


electric field from a set of point charges



have to do vector addition
- **difficult !**

electric potential from a set of point charges



$$V_1 = k\frac{q_1}{r_1}$$

$$V_2 = k\frac{q_2}{r_2}$$

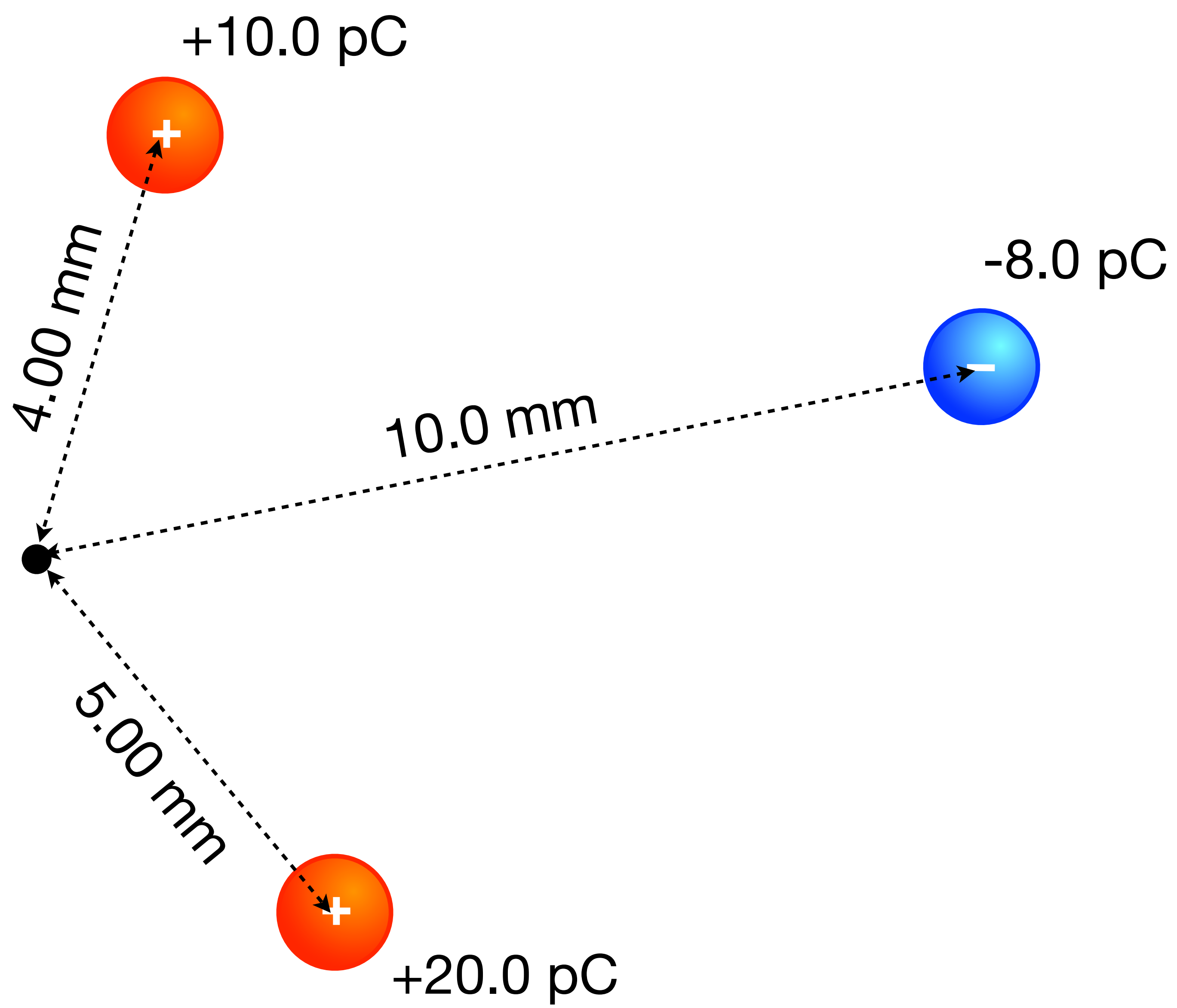
$$V_3 = k\frac{q_3}{r_3}$$

$$V = V_1 + V_2 + V_3$$

just scalar addition
- **easy !**

for example

find the electric potential

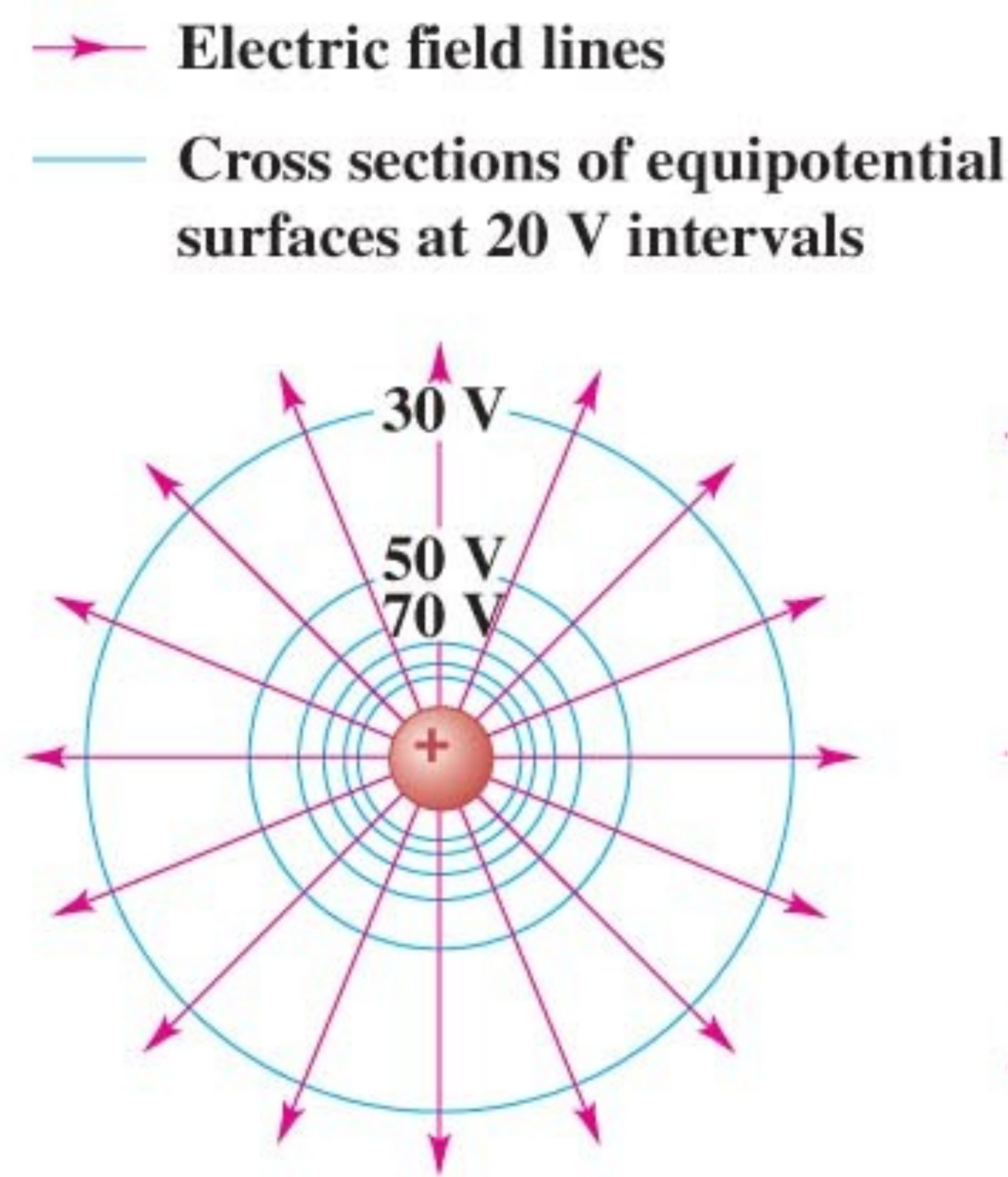


what potential energy would a charge of 2.0 nC have at this position ?

equipotential diagrams

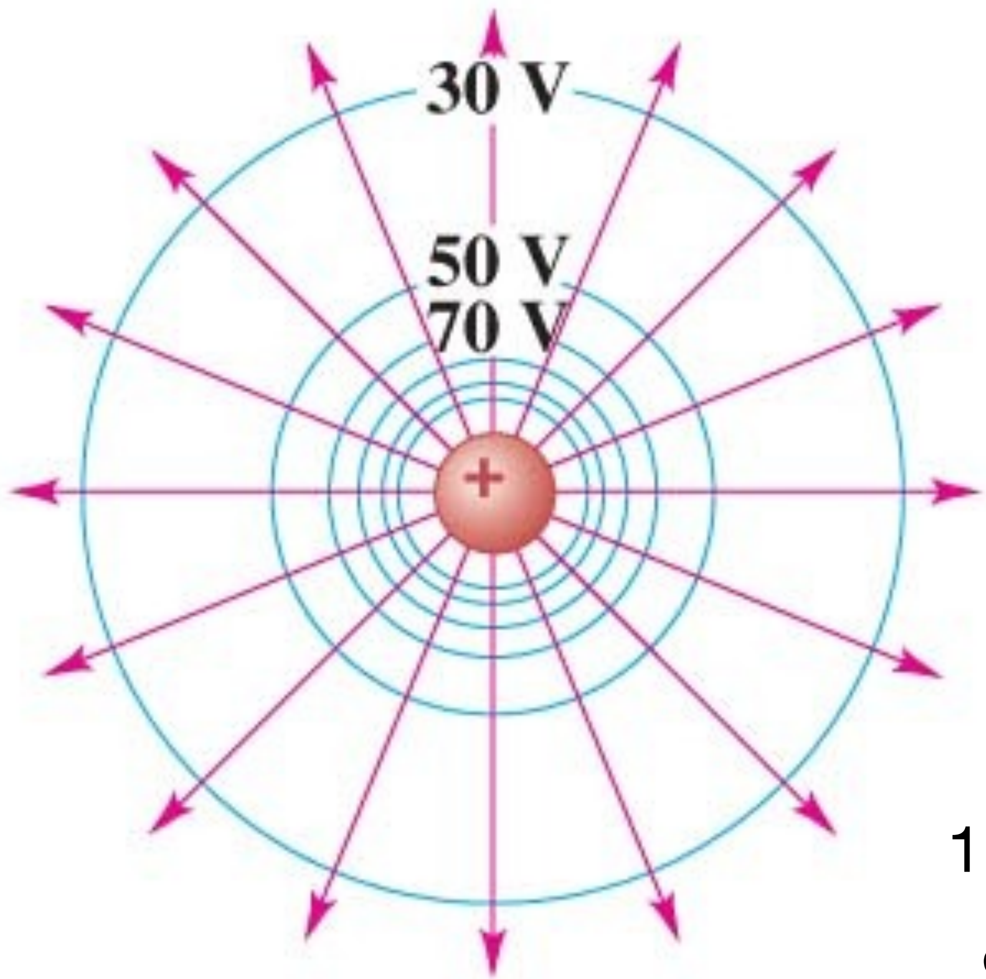
equipotentials are defined as the surfaces on which the *potential takes a constant value* - hence different equipotentials never intersect

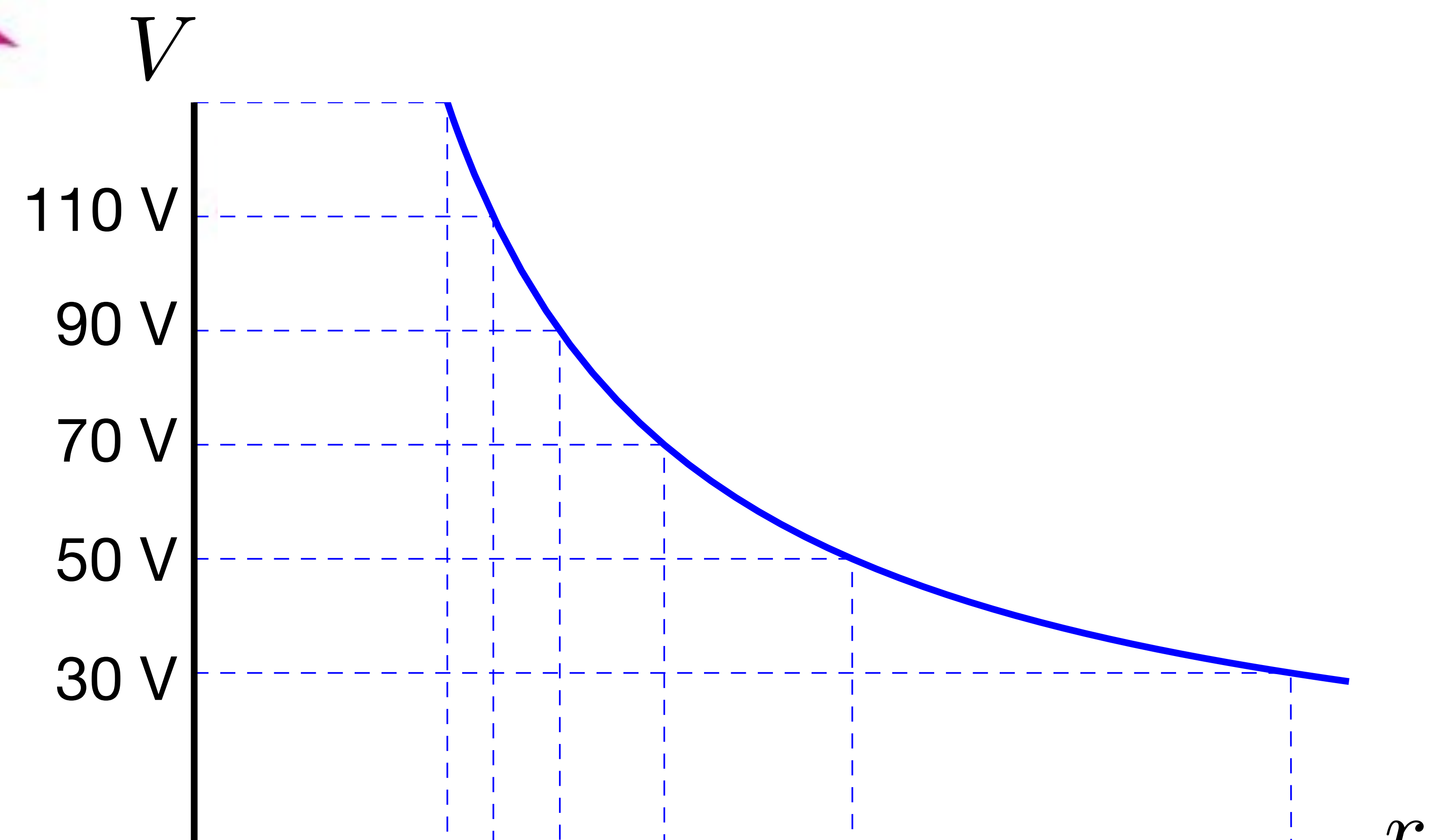
usually draw them with equal potential separations



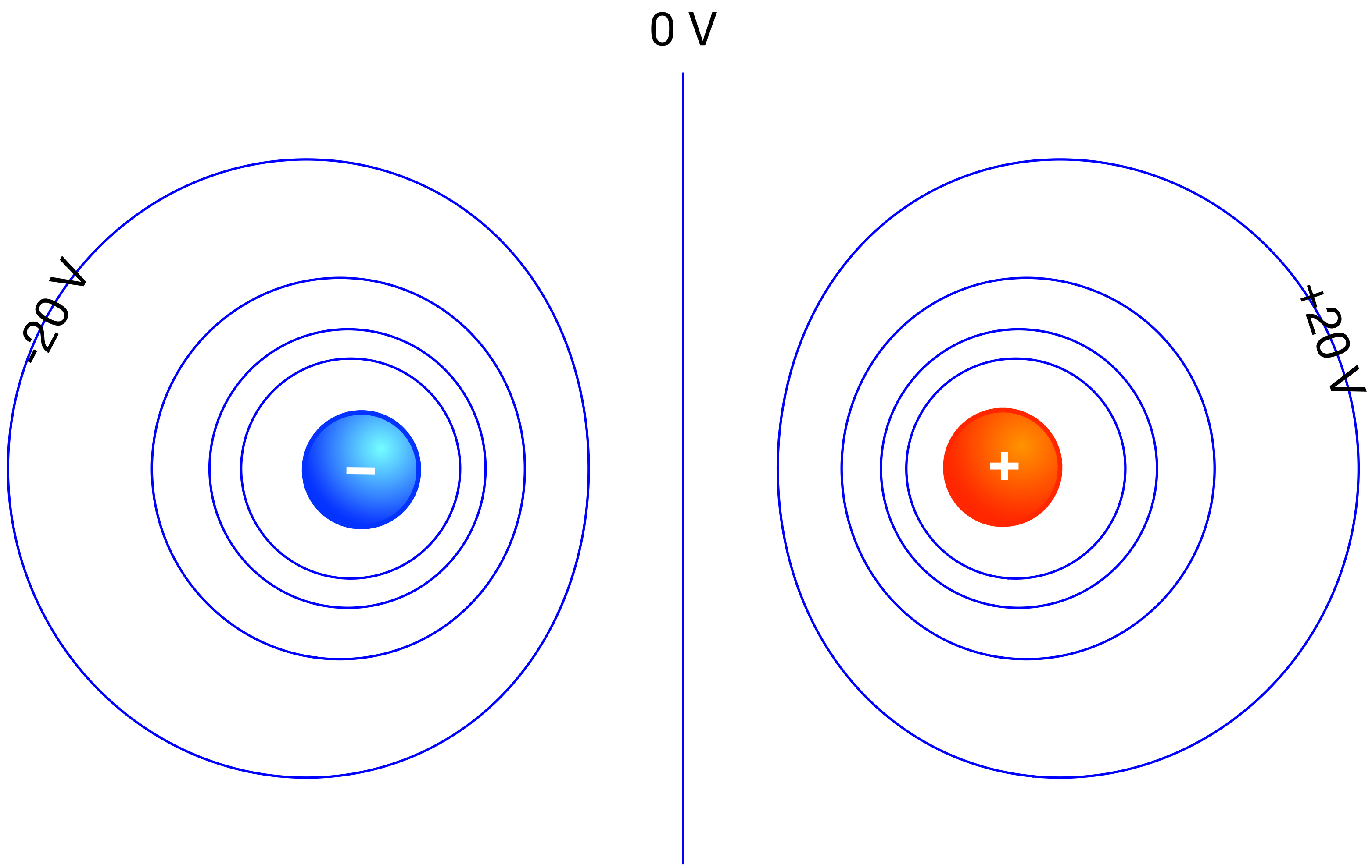
(a) A single positive charge

equipotentials for a point charge

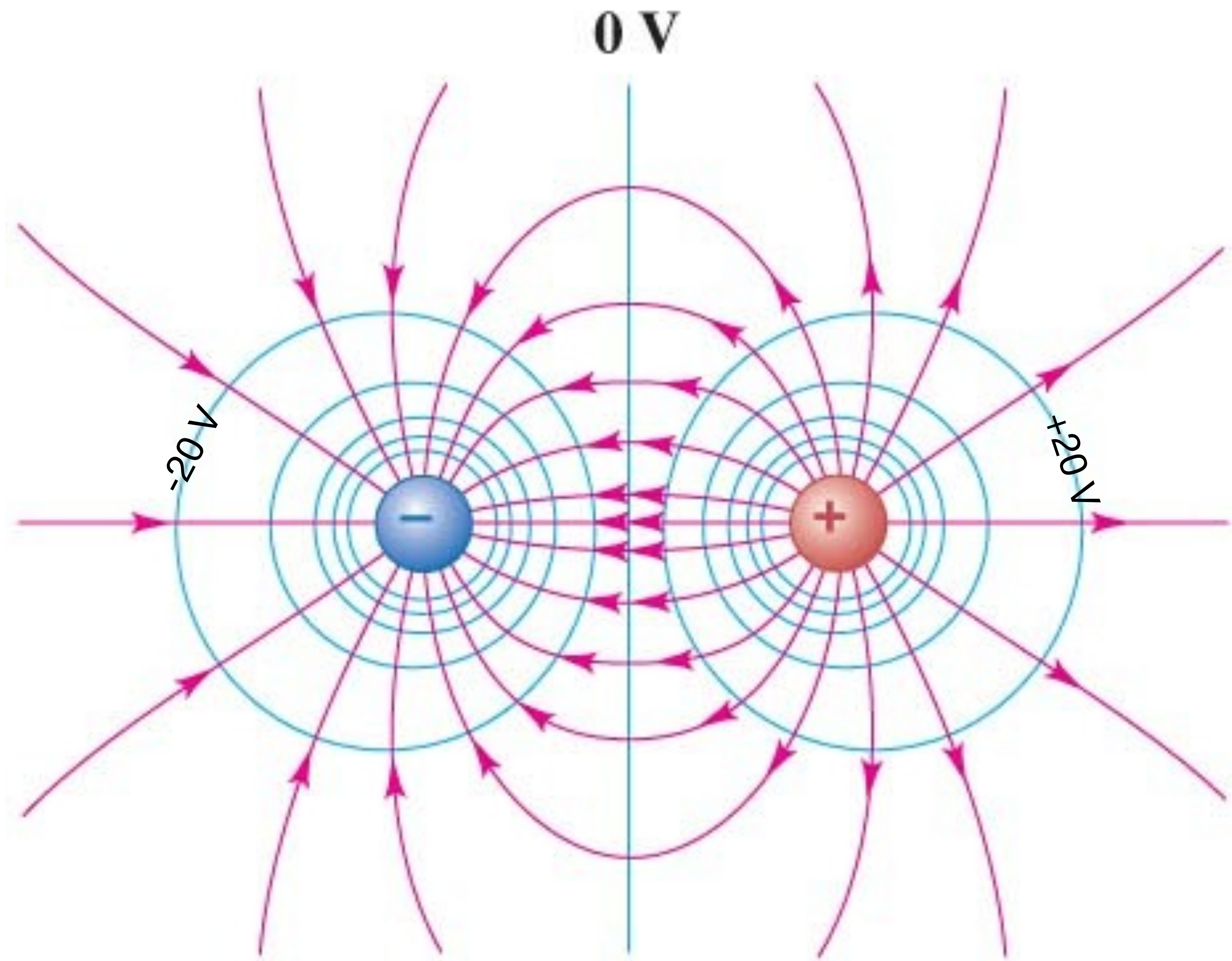




equipotentials from a dipole



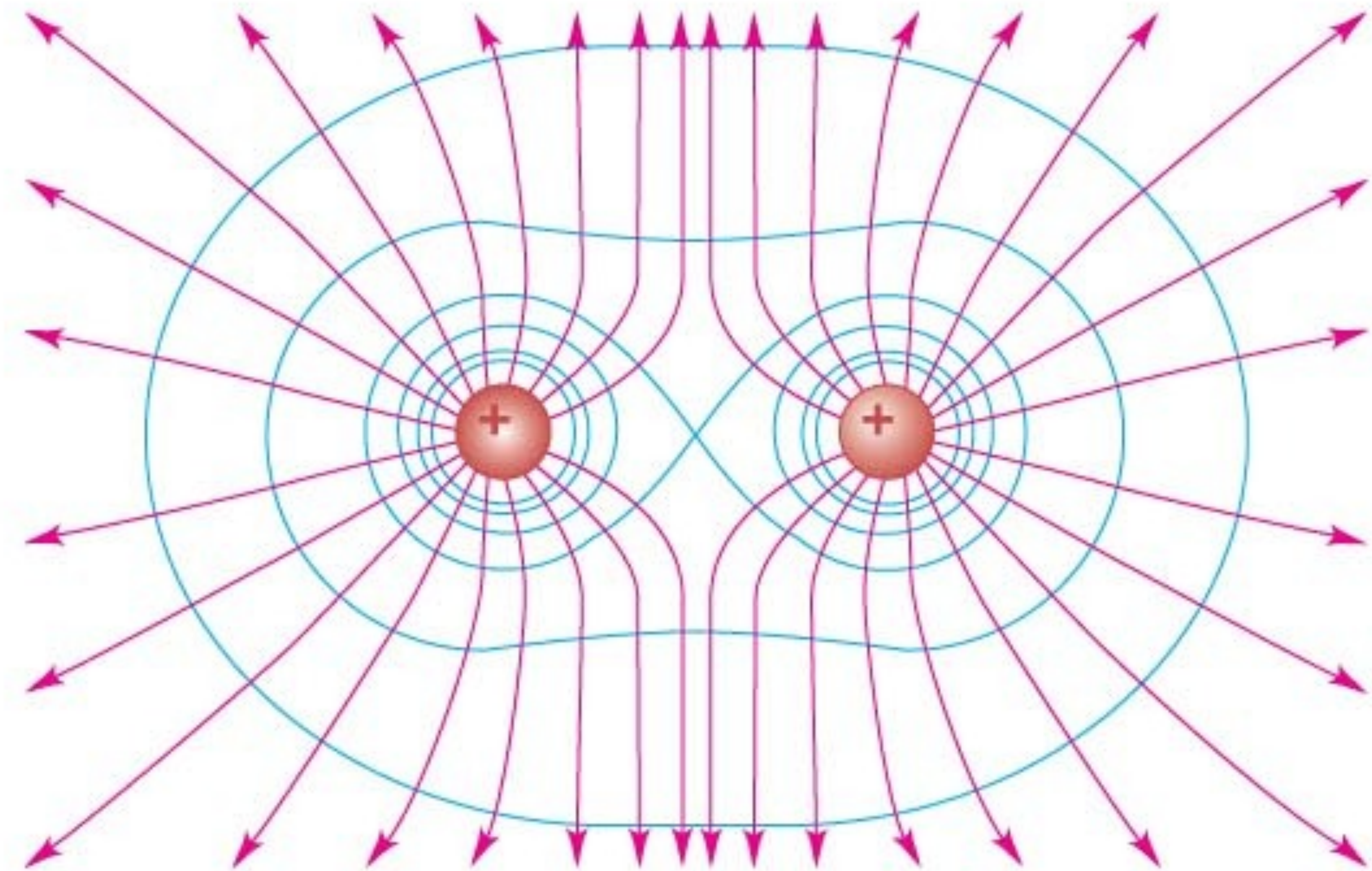
equipotentials from a dipole



(b) An electric dipole

notice that the field lines are always perpendicular to the equipotentials

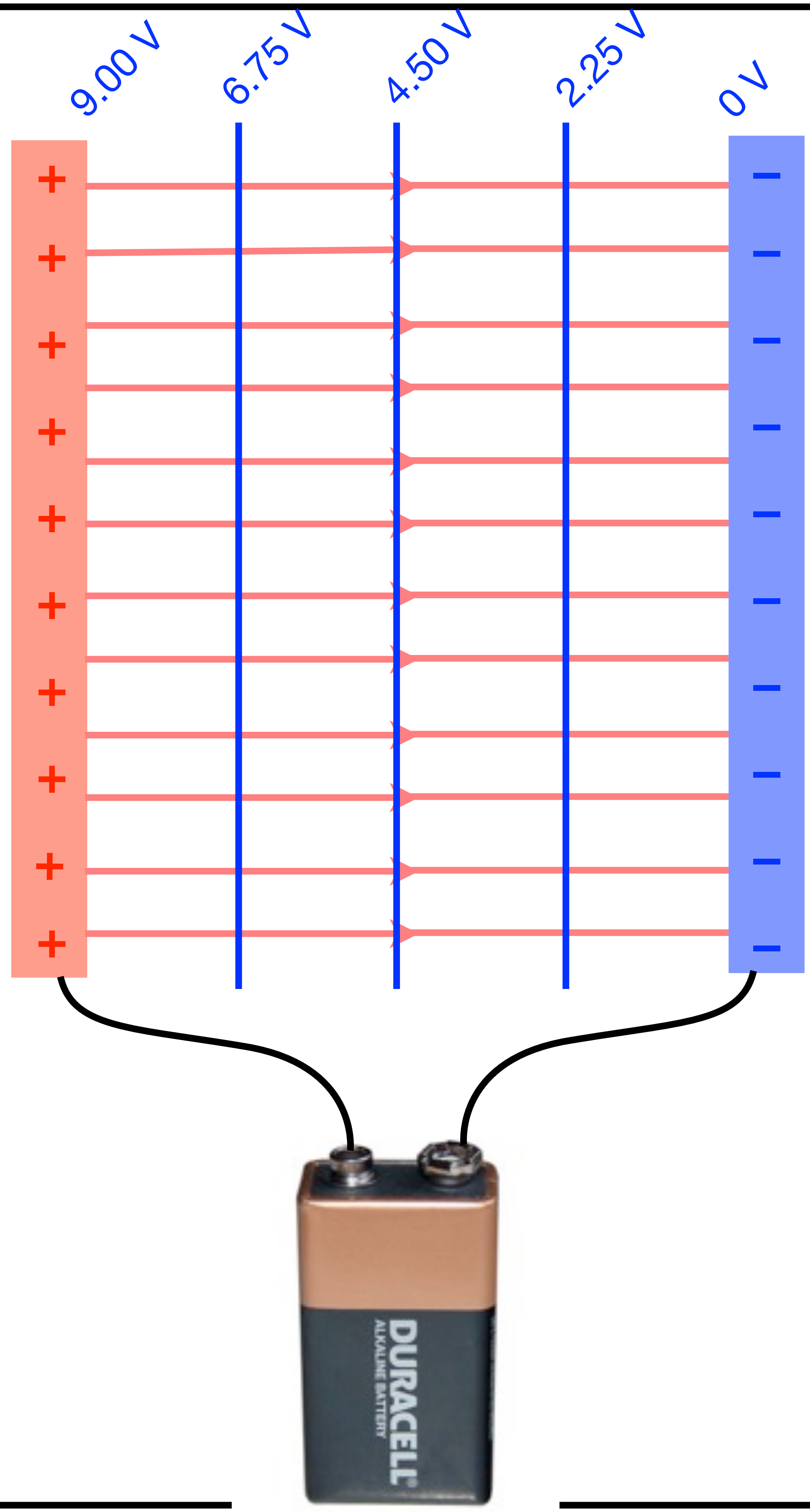
equipotentials from two equal point charges



(c) Two equal positive charges

notice that the field lines are always perpendicular to the equipotentials

a capacitor



notice that the field lines are always perpendicular to the equipotentials

equipotentials and field lines

we can use some logical deduction to see that
electric fields must be perpendicular to equipotentials

➜ we can move a test charge along an equipotential without changing potential

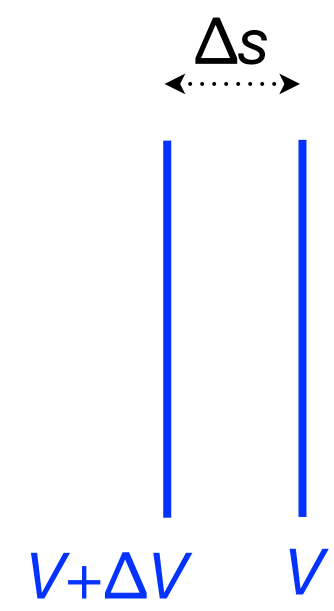
➜ hence the potential energy does not change

➜ thus no work is done

➜ if the ***E***-field had a component parallel to the equipotential we would do work

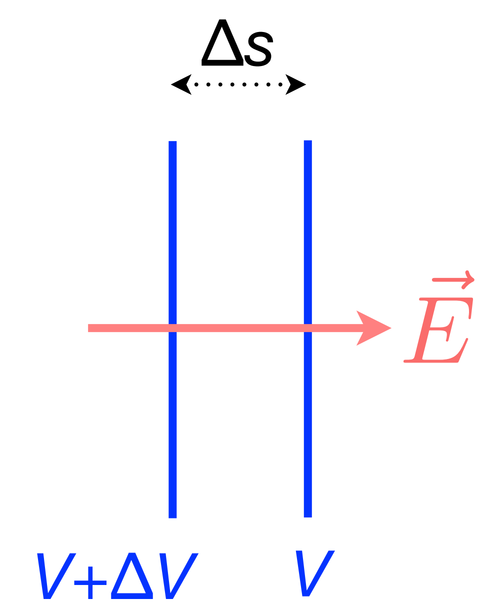
➜ hence there can be no component of ***E*** parallel to an equipotential

electric field as the gradient of the potential



consider two adjacent equipotential surfaces separated by a small distance, Δ*s*

potential difference between the surfaces is Δ*V*



for a small distance, the ***E***-field is approximately constant, so the work done per unit charge in moving from one surface to the other is *E* Δ*s*

this equals the change in potential, -Δ*V*

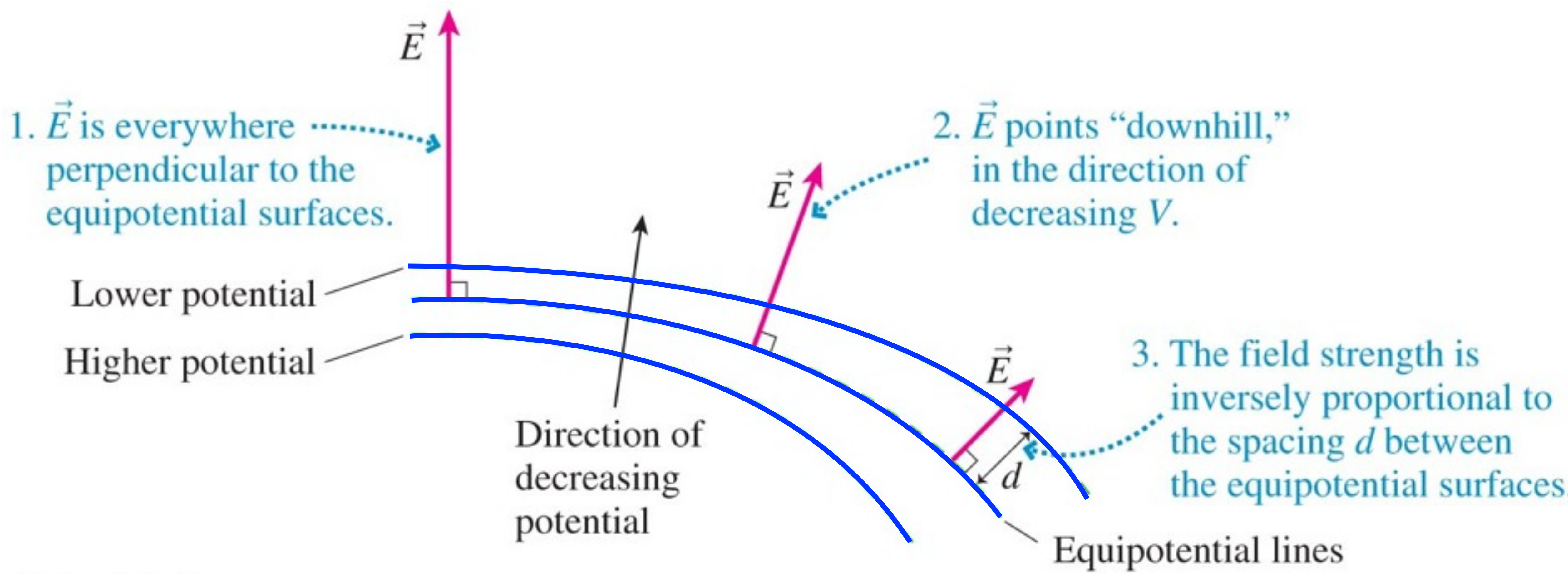
hence we can express the ***E***-field as a potential gradient

$$E = -\frac{\Delta V}{\Delta s}$$

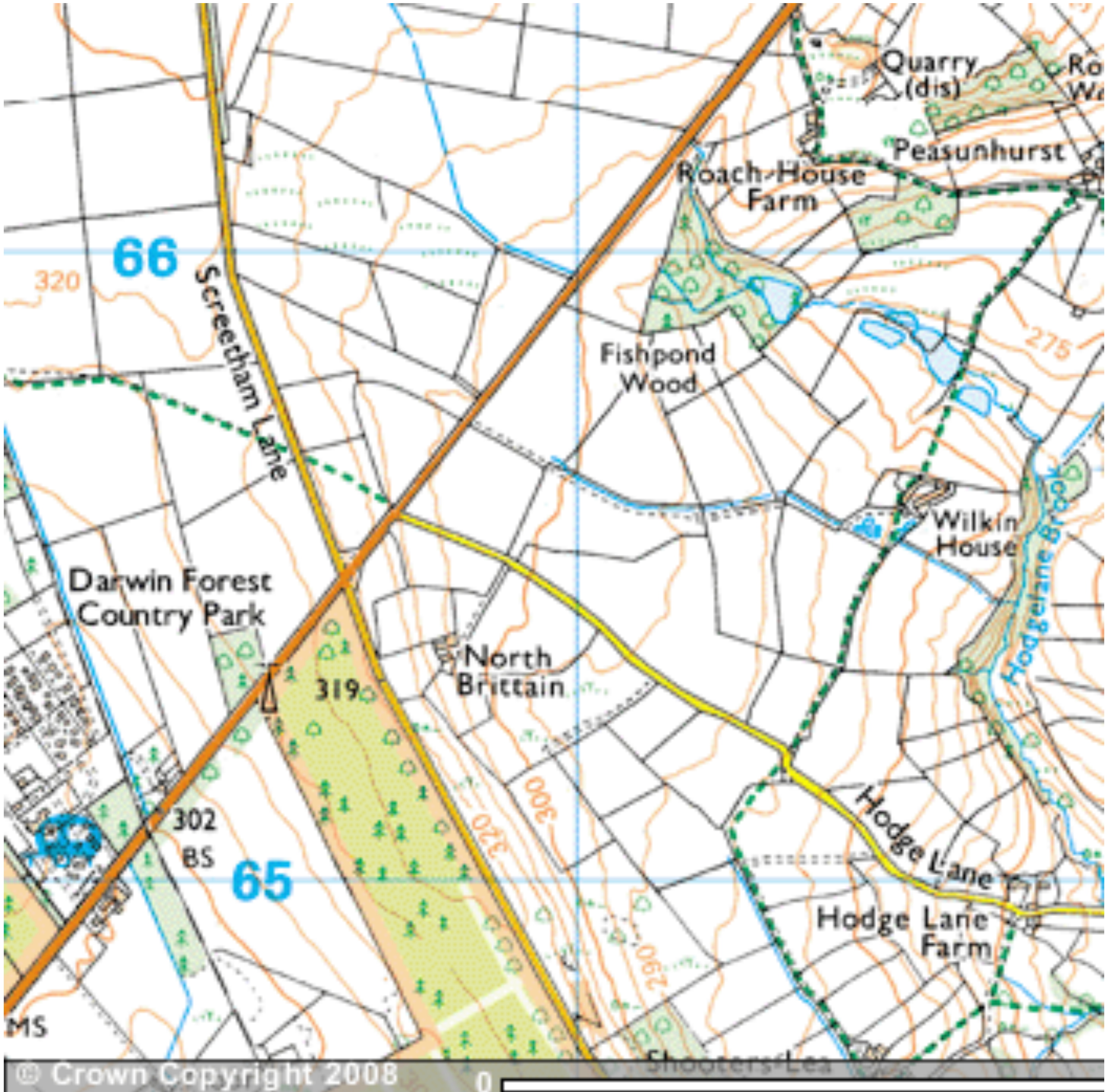
"electric fields point downhill"

electric field as the gradient of the potential

$$E = -\frac{\Delta V}{\Delta s}$$

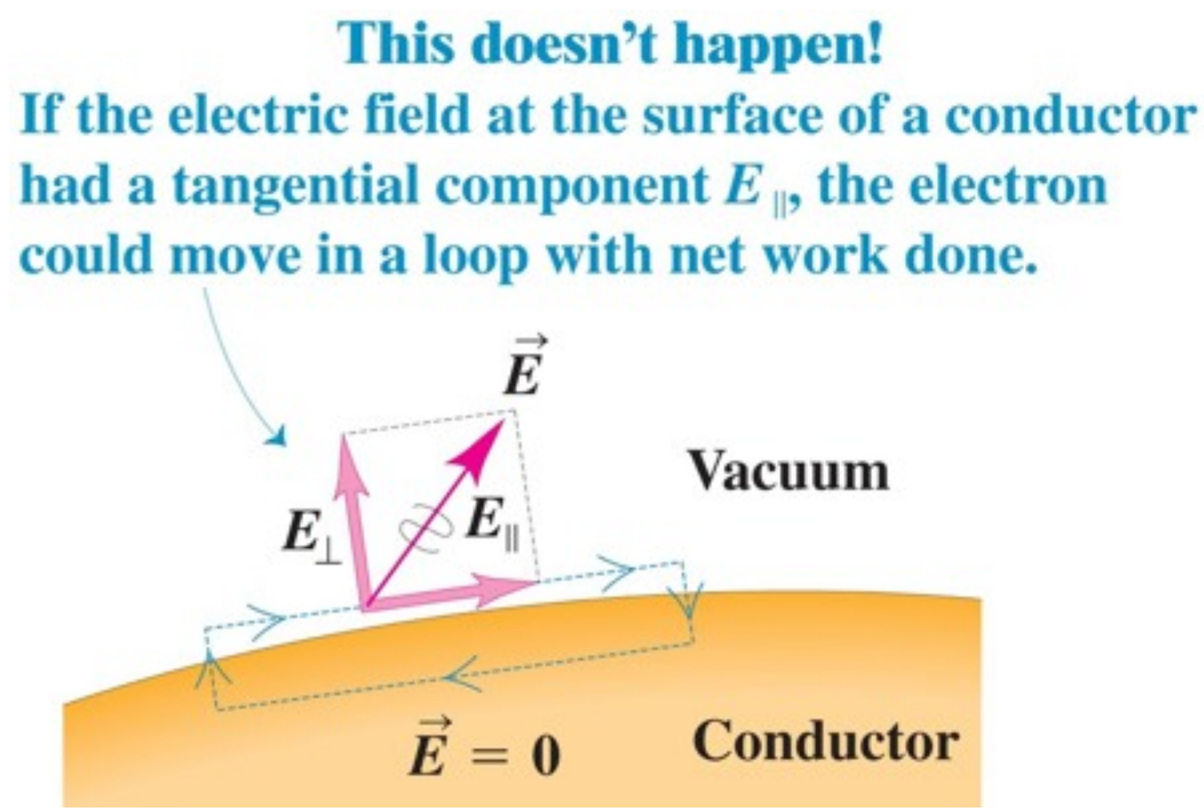


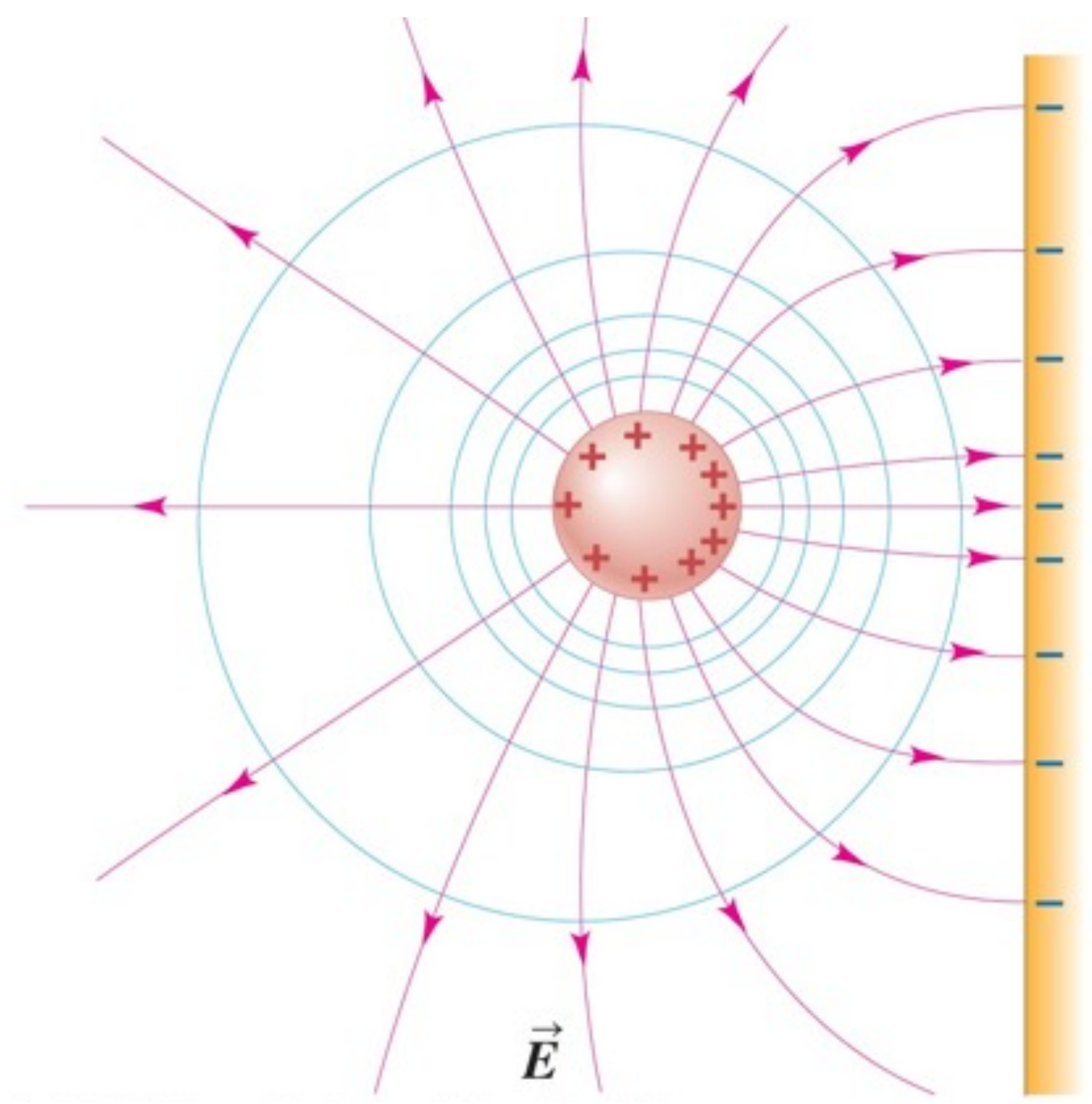
topological maps



electric fields at the surface of conductors

electric fields meet the surface of conductors at right angles



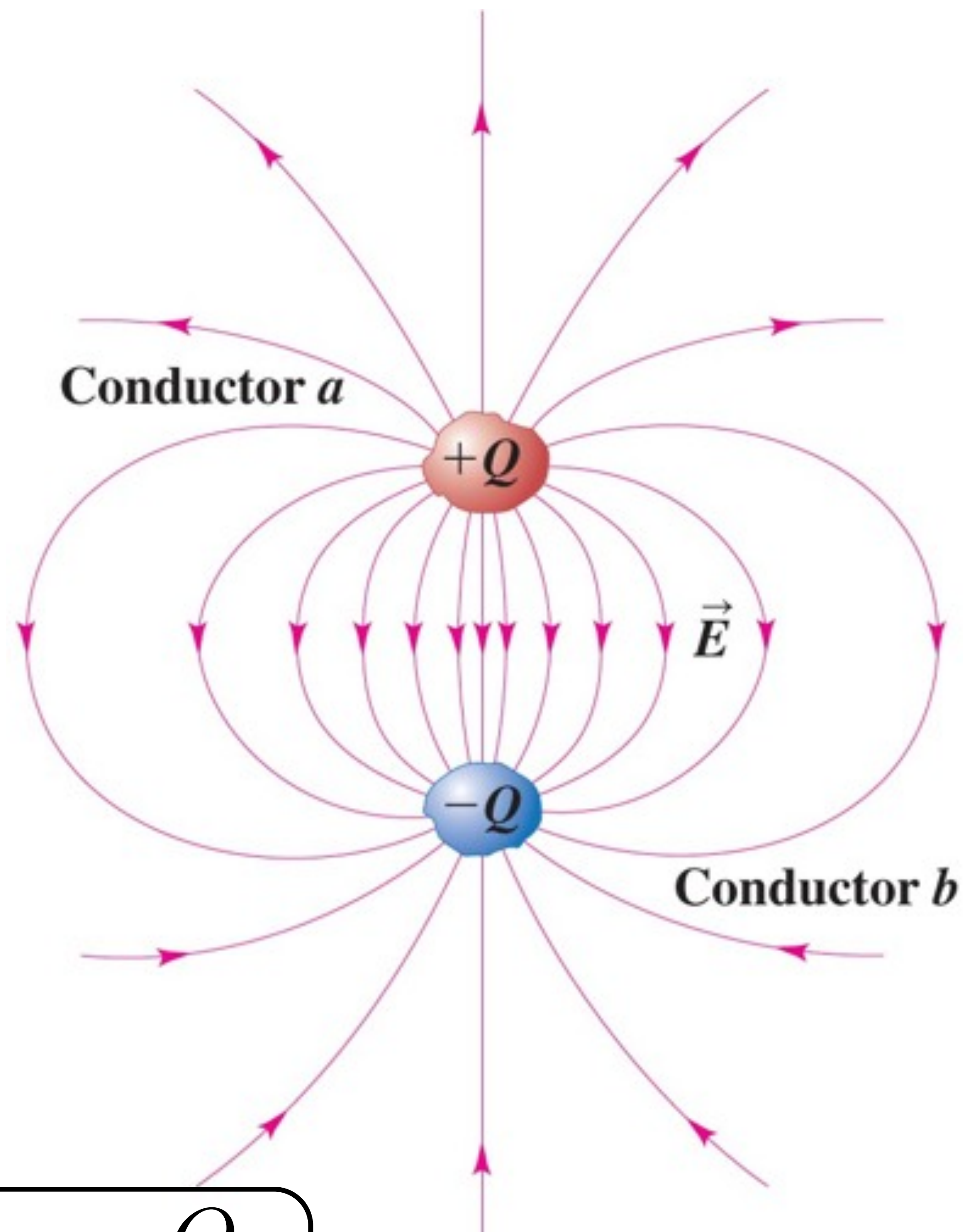


➜ the electric field in a conductor is zero
➜ means the potential can't have a gradient
➜ potential in a conductor is constant

capacitors & capacitance

consider two conductors, separated in space, carrying equal and opposite charge

➜ this is a capacitor
➜ electric fields will fill the space between the conductors
➜ a potential difference will be set up between the conductors
➜ electrostatic energy is stored in the fields



the potential difference between *a* and *b* is proportional to the charge *Q*

$$V_{ab} \propto Q$$

the constant of proportionality that tells us "*how much charge do I need per unit potential*" is called the capacitance, *C*

$$C = \frac{Q}{V_{ab}}$$

units are farads

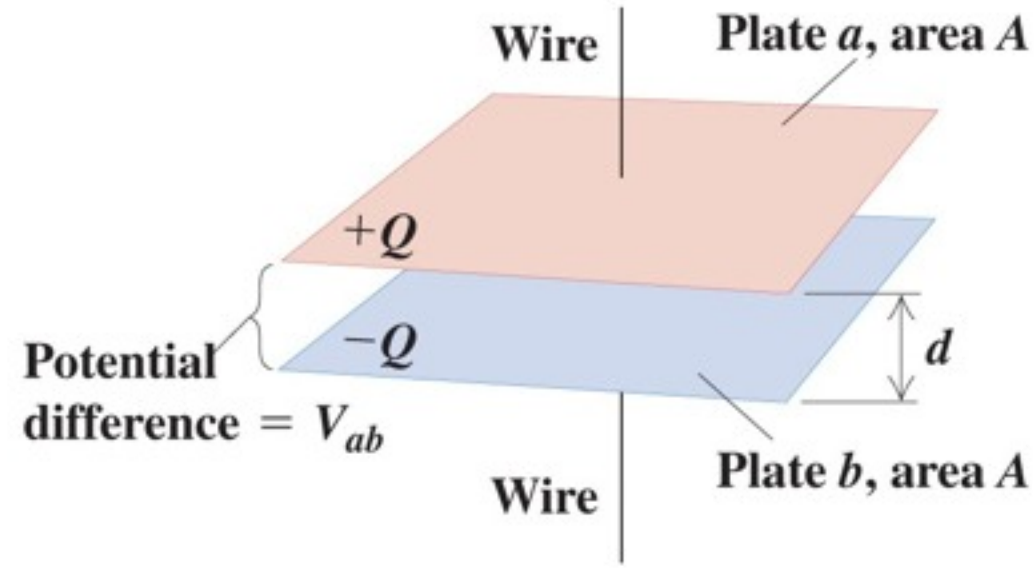
$$1\,\mathrm{F} = 1\,\frac{C}{V}$$

parallel plate capacitors

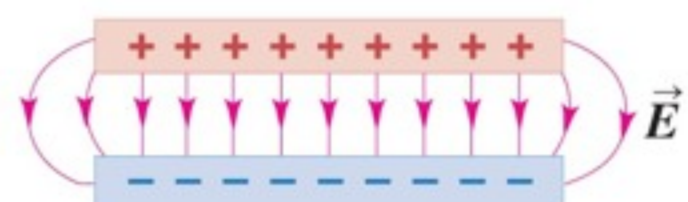
two parallel metal plates, of area *A*, separated by a distance *d*

we can show that the electric field between large plates is uniform and of magnitude

$$E = \frac{Q}{\epsilon_0 A}$$



(a) A basic parallel-plate capacitor



(b) Electric field due to a parallel-plate capacitor

what's this ε_0 thing ?

$$E = \frac{Q}{\epsilon_0 A}$$

it's a property of the vacuum of empty space that tell us how strong electric fields should be

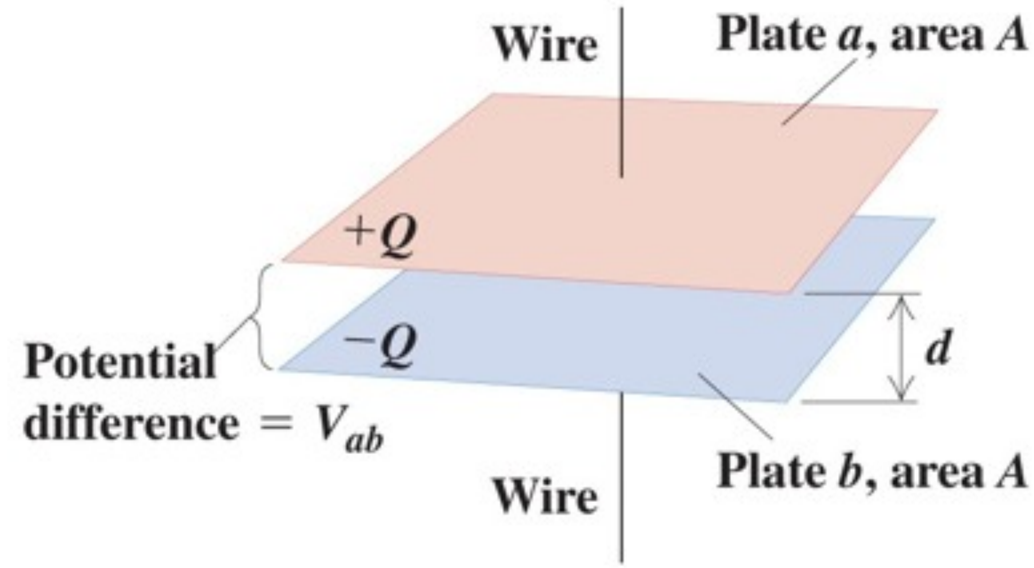
$$\epsilon_0 = 8.854 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{Nm}^2}$$

it was in Coulomb's law, but we hid it in the constant *k*

$$k = \frac{1}{4\pi\epsilon_0}$$

parallel plate capacitors

two parallel metal plates, of area *A*, separated by a distance *d*



(a) A basic parallel-plate capacitor

we can show that the electric field between large plates is uniform and of magnitude

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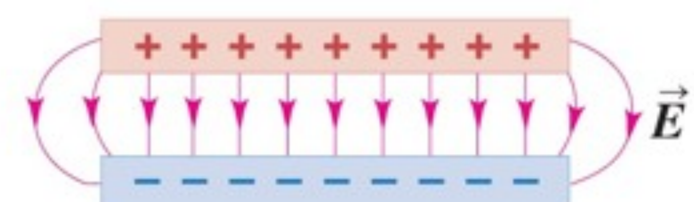


(b) Electric field due to a parallel-plate capacitor

since it's uniform the potential difference must be

$$V = Ed = \frac{Qd}{\epsilon_0 A}$$

hence

$$\frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

so the capacitance is $C = \dfrac{\epsilon_0 A}{d}$

which depends only on the geometry of the capacitor

parallel plate capacitors

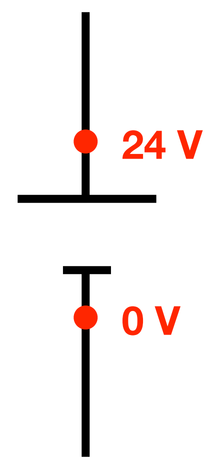
$$C = \frac{Q}{V} \qquad C = \frac{\epsilon_0 A}{d}$$

circuit diagrams and ‘rules’

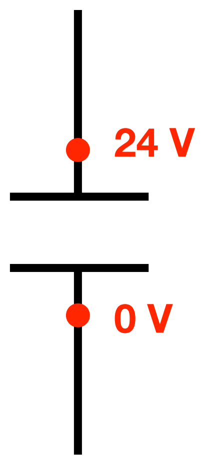
‘wires’ are treated as being resistance-less,
they act as equipotentials

potential changes occur when electrical components are attached to the wires

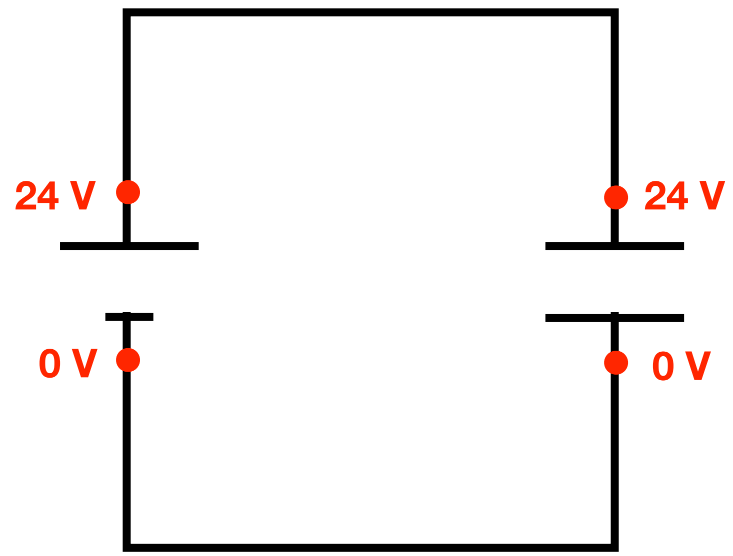
e.g. a battery
- keeps two wires at fixed potential difference



e.g. a capacitor
- potential drop

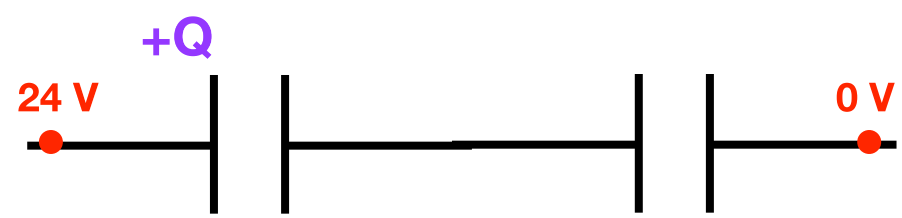


so we can build a legitimate *circuit* out of these two components and wires

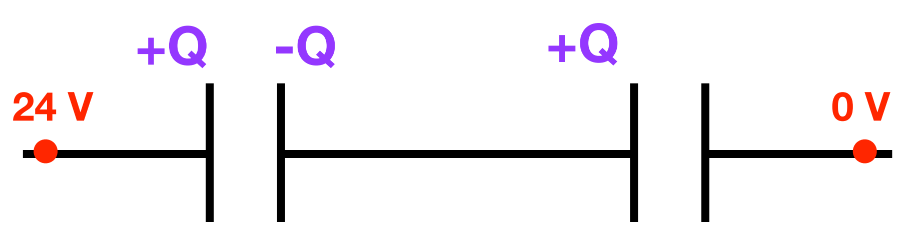


circuit diagrams and ‘rules’

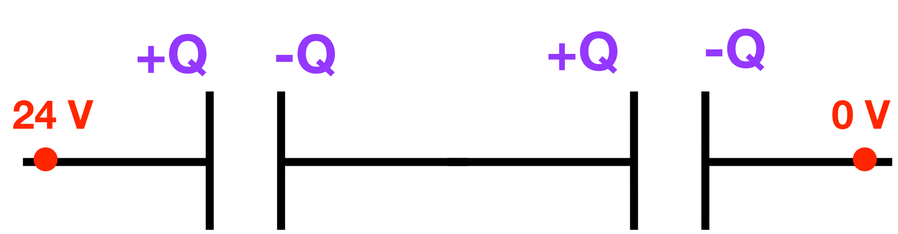
net charge doesn’t accumulate, a circuit starts with total charge of zero and always has total charge of zero



positive charge build up on one plate pushes positive charge off the other plate, leaving negative charge behind



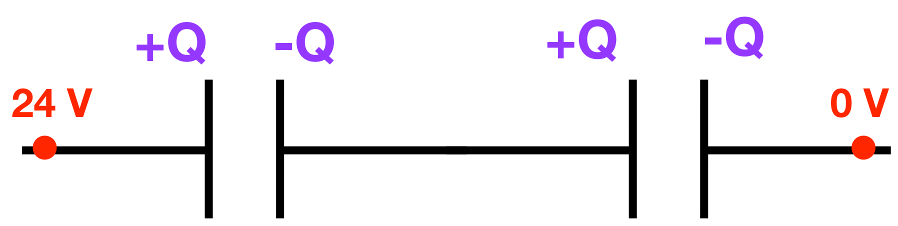
positive charges pushed off end up on the next plate



same `push-off’ occurs here

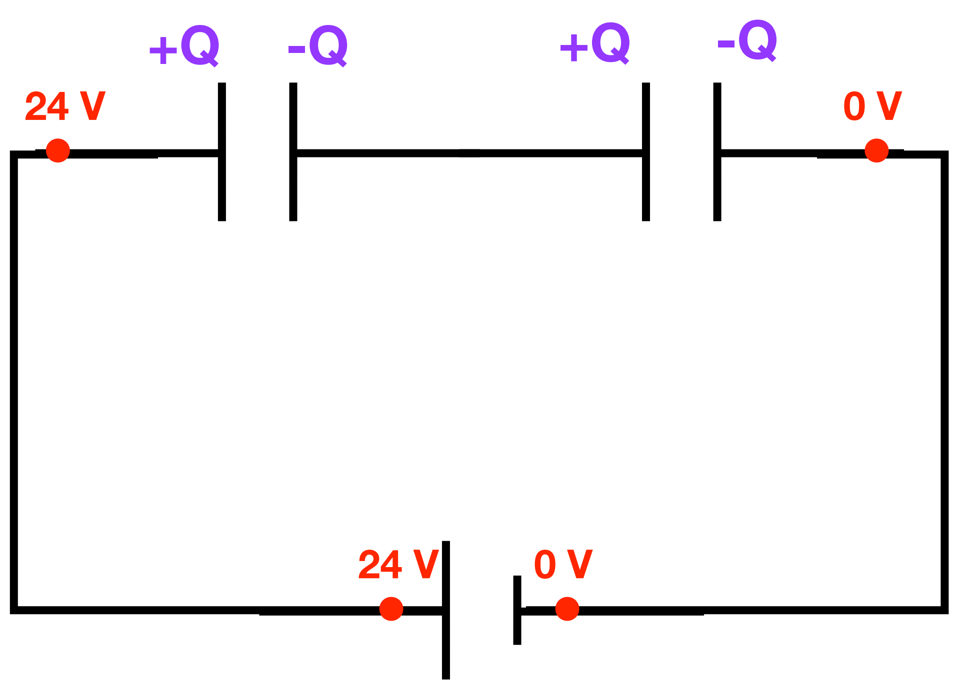
circuit diagrams and ‘rules’

net charge doesn’t accumulate, a circuit starts with total charge of zero and always has total charge of zero



but where do the ‘pushed-off’ positive charges go ?

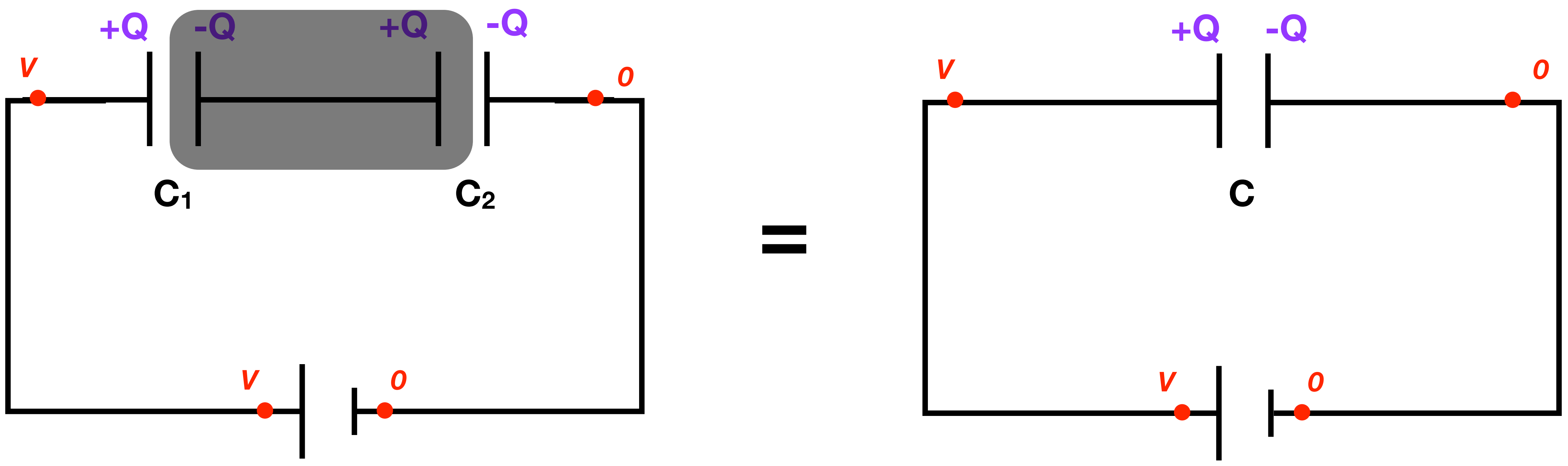
remember we have to make a circuit !



they moved all the way around the circuit and formed the first set of positive charges !

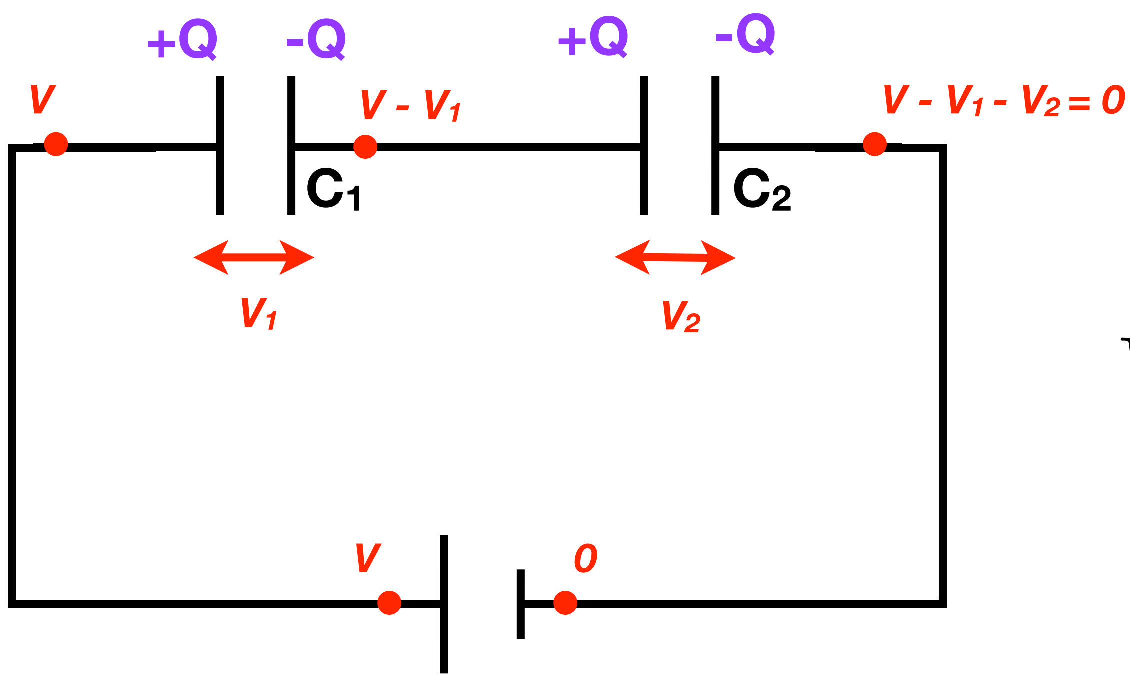
capacitors in series

imagine removing the `inner' plates and the wire joining them :



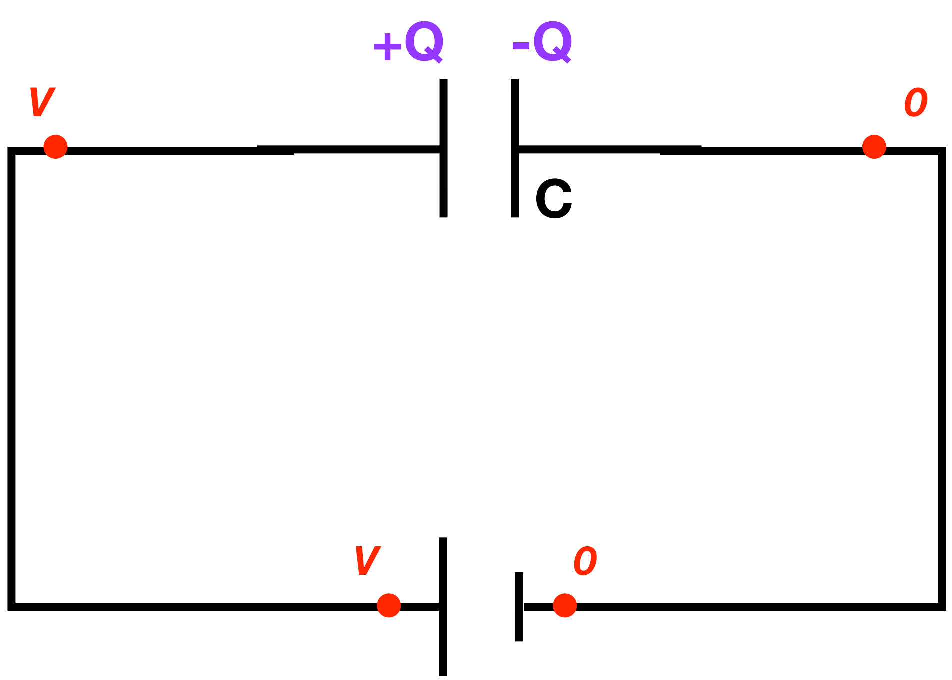
capacitors in series

we can derive a formula for C in terms of C_1 and C_2 :



$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2}$$

$$V = V_1 + V_2 \qquad V = \frac{Q}{C_1} + \frac{Q}{C_2}$$



$$V = \frac{Q}{C} \qquad \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

capacitors in series - using the formula

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

many students use this formula wrongly

it is **NOT** the same as $C = C_1 + C_2$

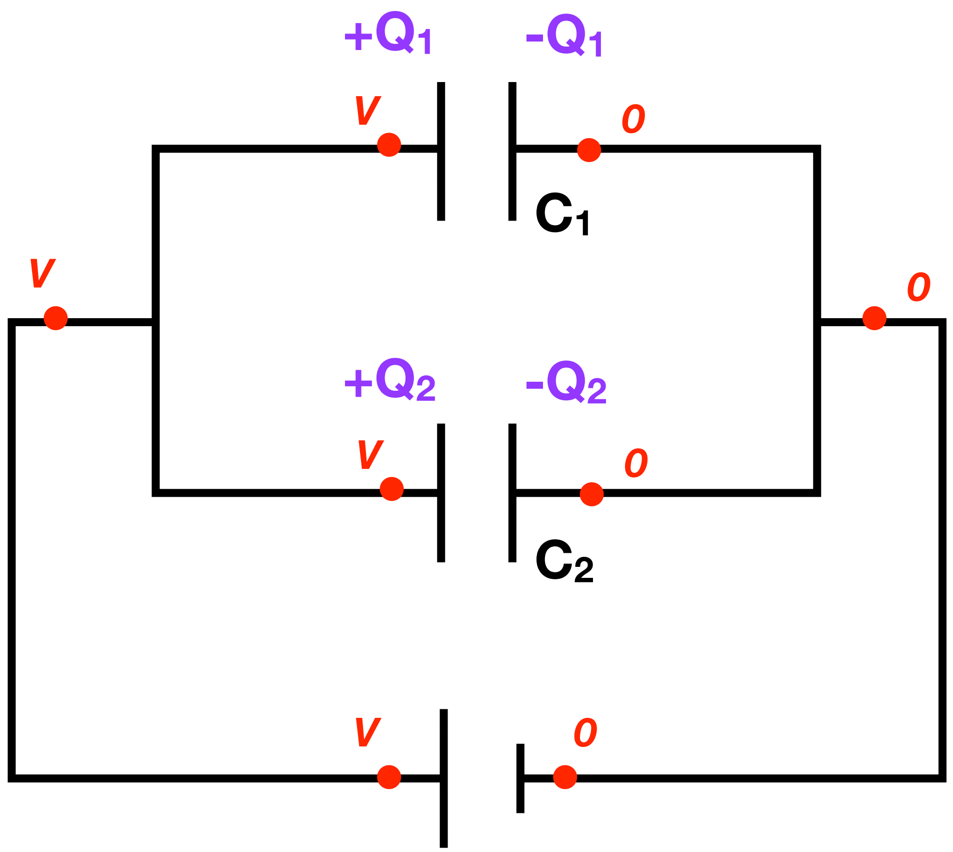
e.g. $C_1 = 1$ F, $C_2 = 1$ F,

then $C_1 + C_2 = 2$ F

but $\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1\,\mathrm{F}} + \frac{1}{1\,\mathrm{F}} = 2\,\mathrm{F}^{-1}$

so $\frac{1}{C} = 2\,\mathrm{F}^{-1}$ and hence $C = \frac{1}{2}\,\mathrm{F}$

capacitors in parallel

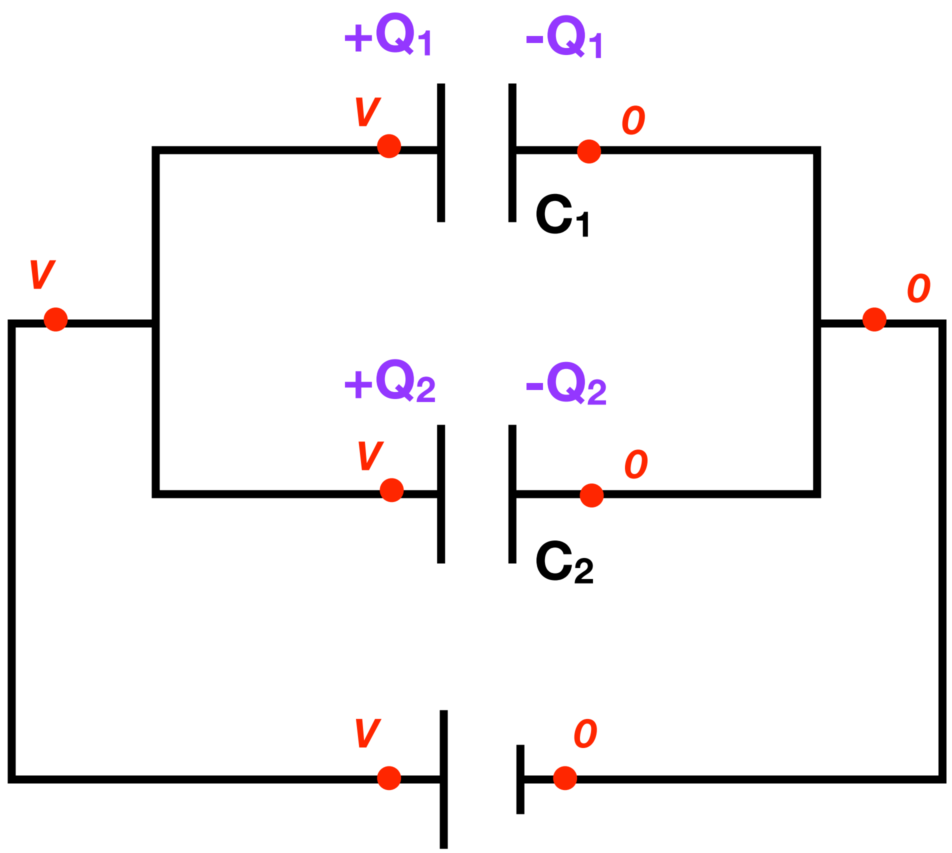


$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

total charge on the left-hand plates $$Q = Q_1 + Q_2$$

capacitors in parallel



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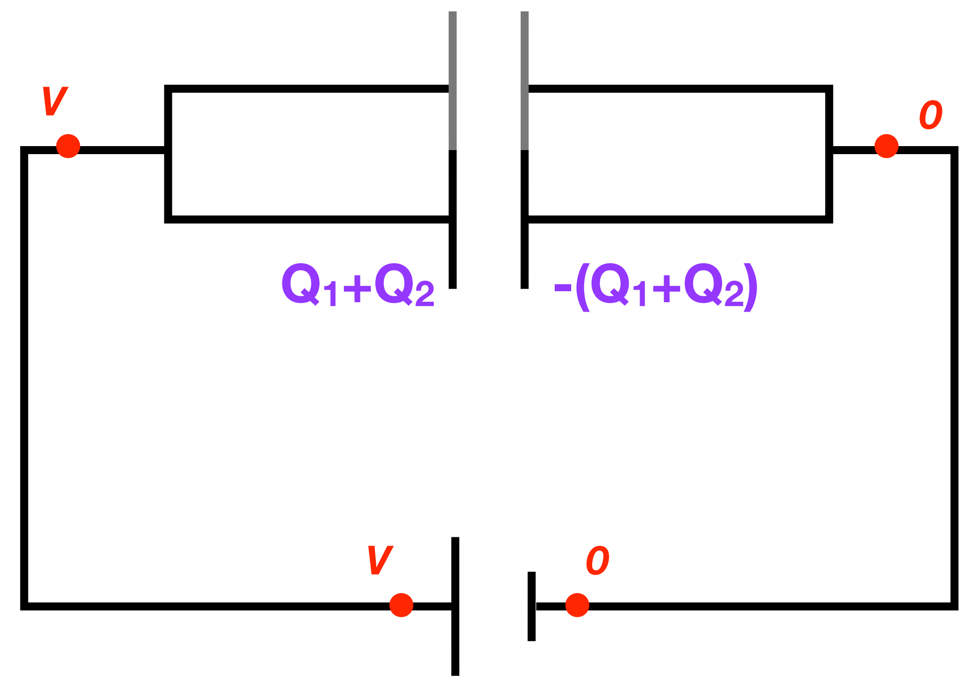
suppose we joined the plates together



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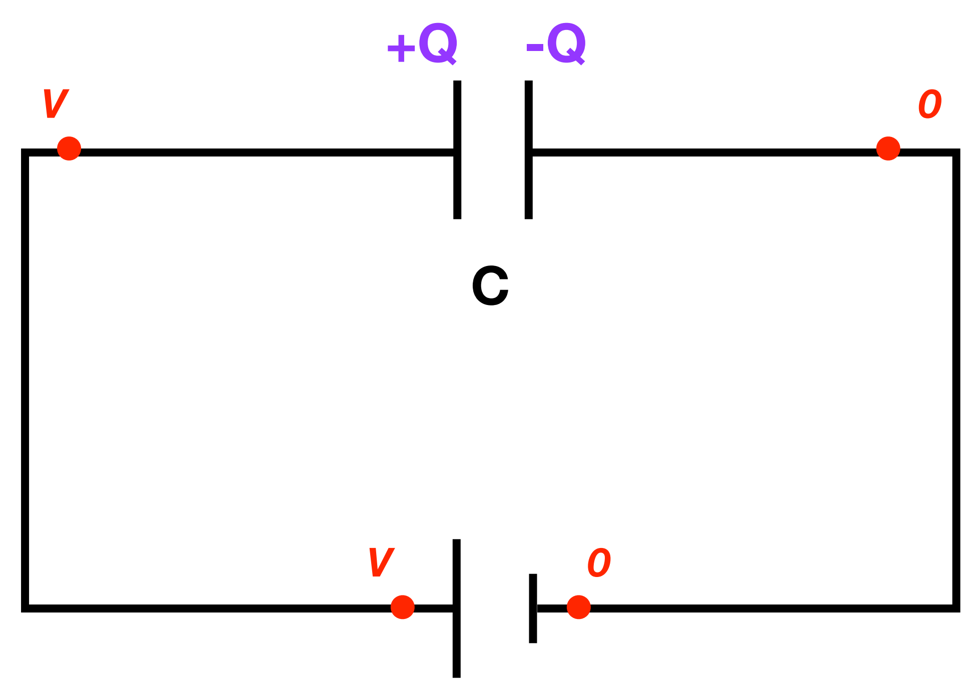
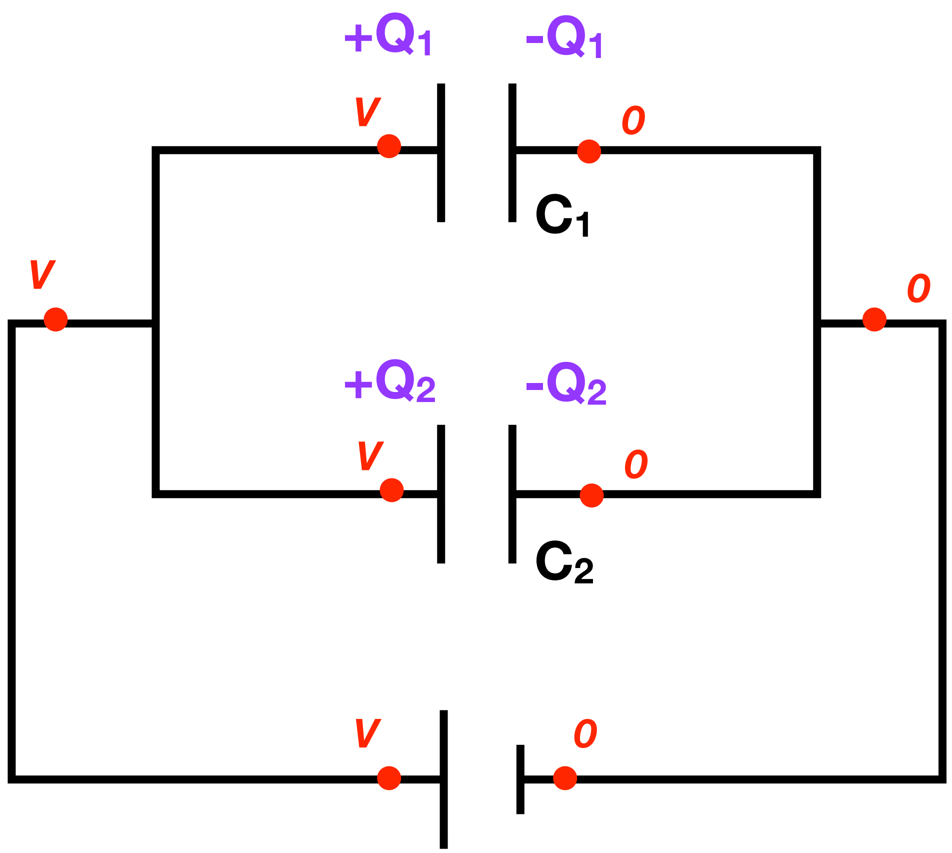


capacitors in parallel

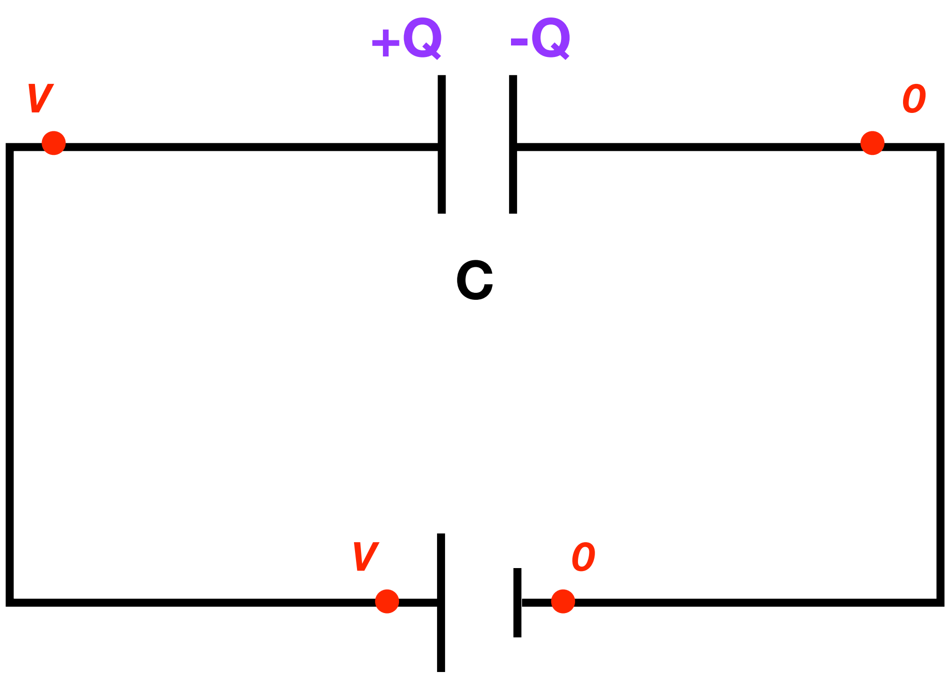


$$Q_1 = C_1 V \qquad Q_2 = C_2 V$$

total charge on the left-hand plates $\quad Q = Q_1 + Q_2$

$$Q = C_1 V + C_2 V$$

=



$$Q = CV$$

$$CV = C_1 V + C_2 V$$

$$\boxed{C = C_1 + C_2}$$

combining capacitors

Two capacitors, one with capacitance 12.0 nF and the other of 6.0 nF are connected to a potential difference of 18 V. Find the equivalent capacitance and find the charge and potential differences for each capacitor when the two capacitors are connected in
(a) series
(b) parallel

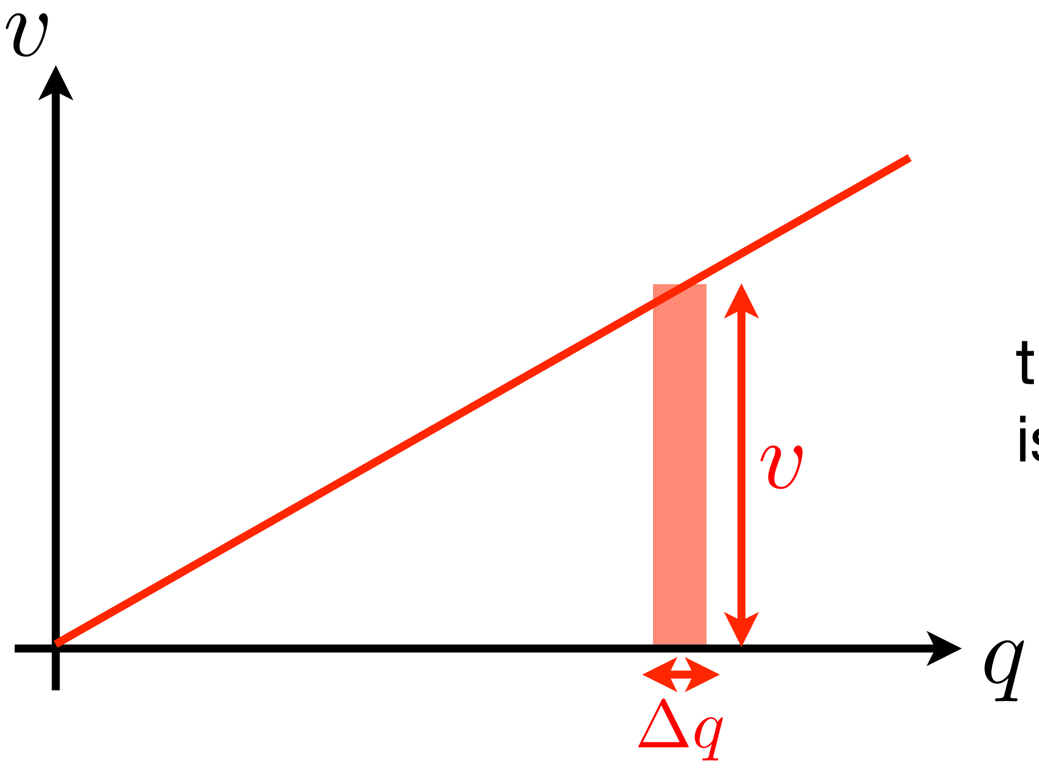
stored energy in a capacitor

getting the charges in place on the plates requires work - this work ends up as energy 'stored' in the electric fields

➜ consider charging up a capacitor from zero charge to a charge Q

➜ if at some time the charge is q, the potential is $v = q/C$

➜ to add another small amount of charge Δq , will need to do work of $\Delta W = v\, \Delta q$



the extra work needed
is the area of the rectangle

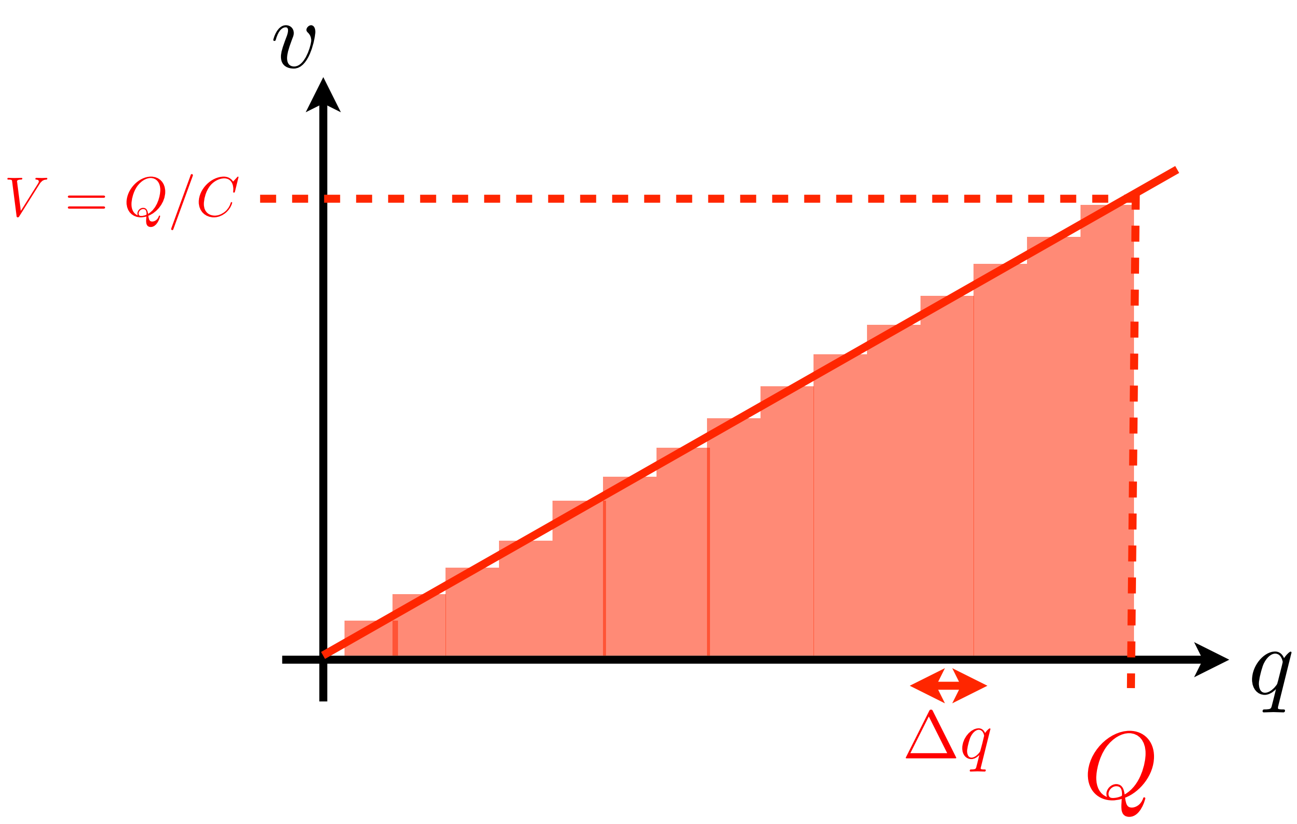
stored energy in a capacitor

getting the charges in place on the plates requires work - this work ends up as energy ‘stored’ in the electric fields

➜ consider charging up a capacitor from zero charge to a charge Q

➜ if at some time the charge is q, the potential is $v = q/C$

➜ to add another small amount of charge Δq , will need to do work of $\Delta W = v\, \Delta q$



the total work required is the area under the curve

$$W = \frac{1}{2} \times Q \times \frac{Q}{C} = \frac{Q^2}{2C}$$

$$W = \tfrac{1}{2} C V^2$$

energy stored in electric fields

capacitors store energy - this can be thought of as residing in the **field** between the plates

➜ define energy density as the energy per unit volume

$$u \equiv \frac{U}{\text{vol}}$$

➜ for a parallel plate capacitor

$$\left.\begin{aligned} U &= \tfrac{1}{2} C V^2 \\ C &= \frac{\epsilon_0 A}{d} \\ \text{vol} &= A d \end{aligned}\right\} \quad u = \tfrac{1}{2}\epsilon_0 \left(\tfrac{V}{d}\right)^2$$

$$E = \frac{V}{d}$$

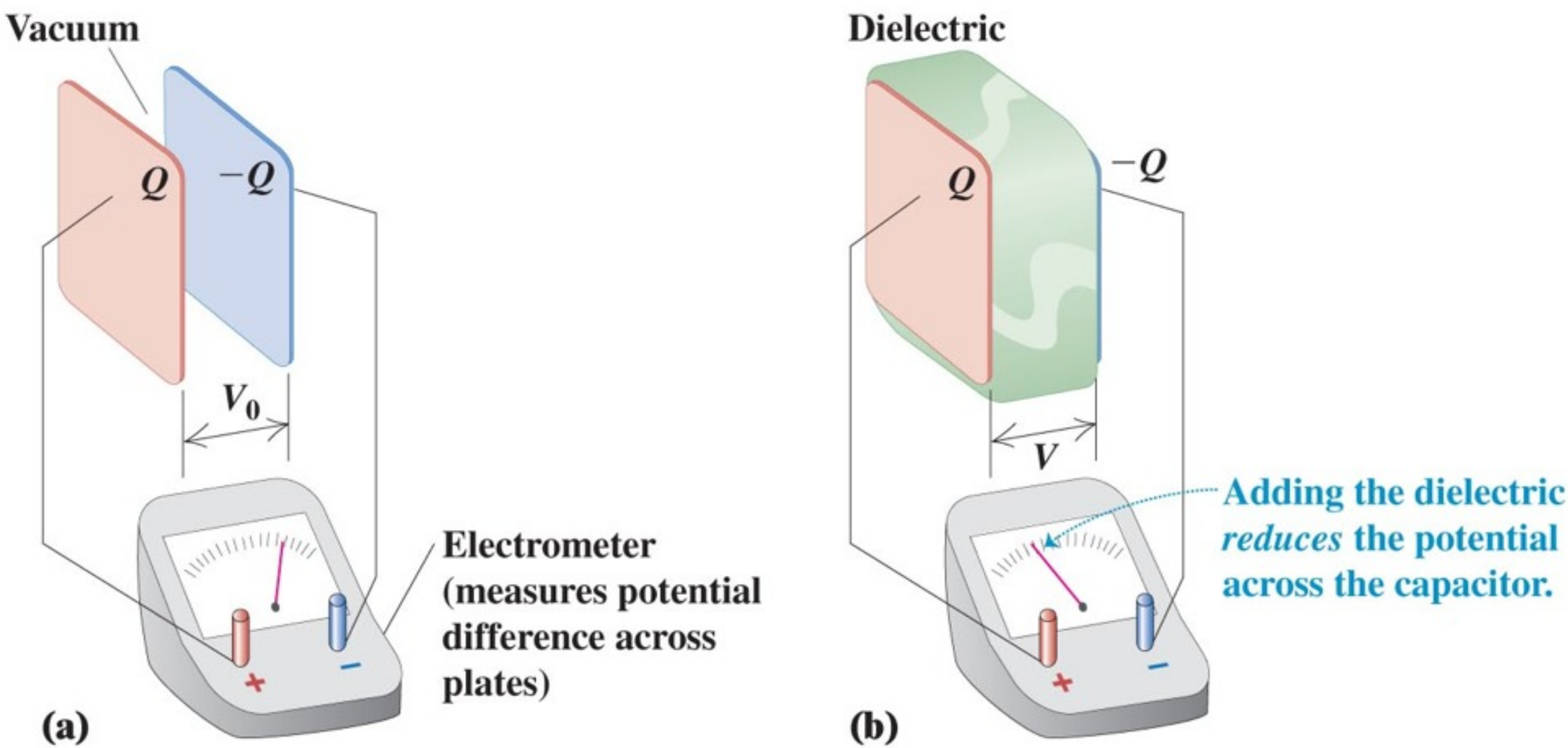
$$\boxed{u = \tfrac{1}{2}\epsilon_0 E^2}$$

this formula turns out to be true
for all electric field configurations

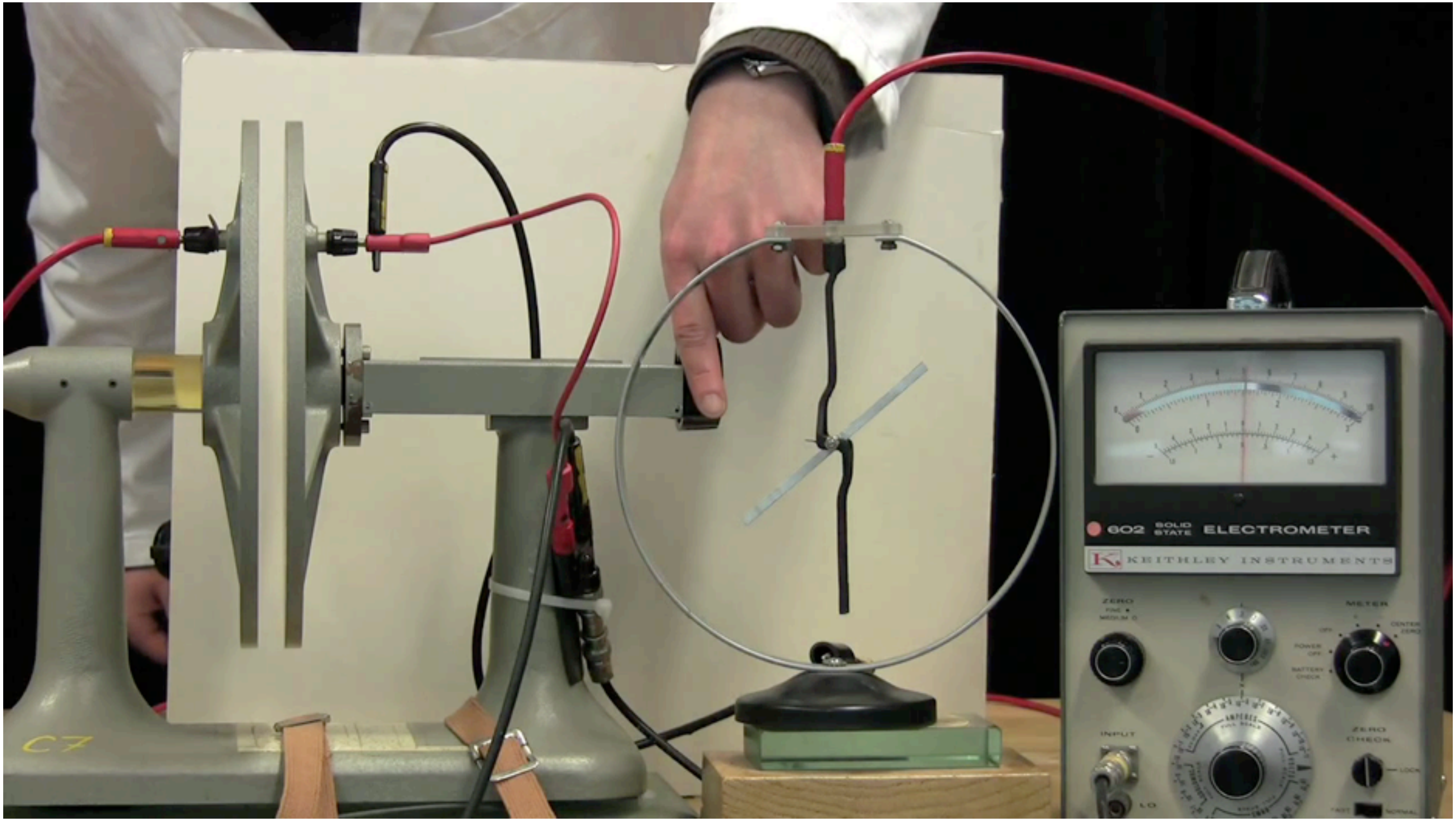
dielectrics

➜ so far we've assumed that the gap between the plates is filled with vacuum (or air)

➜ it doesn't have to be - suppose we place some nonconducting material in there



adding a dielectric



dielectrics

➜ the voltage changes - reflects a change in the capacitance

$$C_{\text{di.}} = K C_0$$

capacitance
with dielectric

capacitance
without dielectric

dielectric
constant

➜ if the voltage goes down (for fixed charge) when a dielectric is added, what can we say about K ?

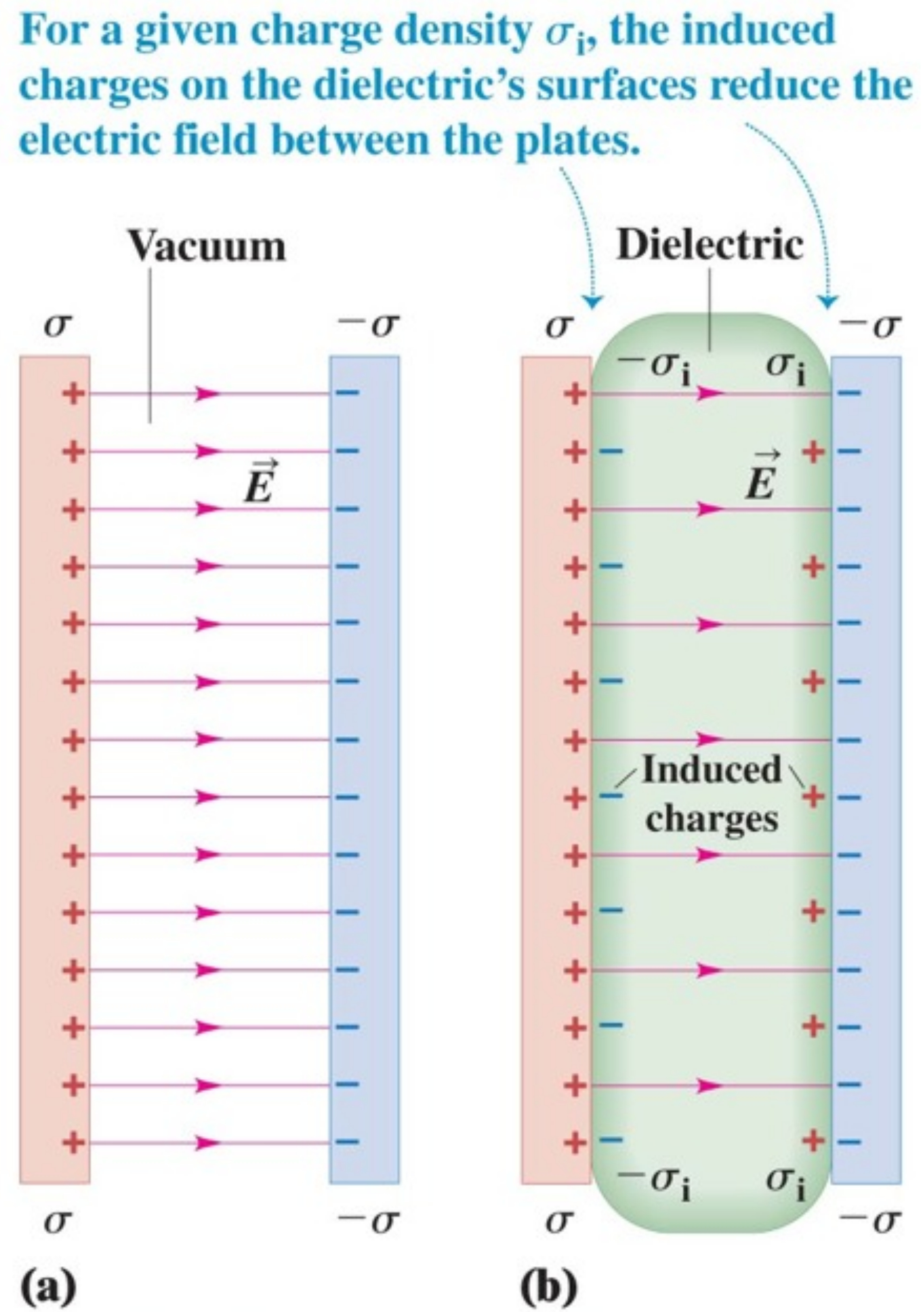
1. *K* is negative
2. *K* is less than 1
3. *K* is greater than 1
4. *K* is zero

dielectrics

➜ the voltage drop corresponds to a reduction of the electric field in the gap

$$E_{\text{di.}} = \frac{E_0}{K}$$

➜ the reason is induced charges on the surface of the dielectric



dielectrics

➜ the voltage drop corresponds to a reduction of the electric field in the gap

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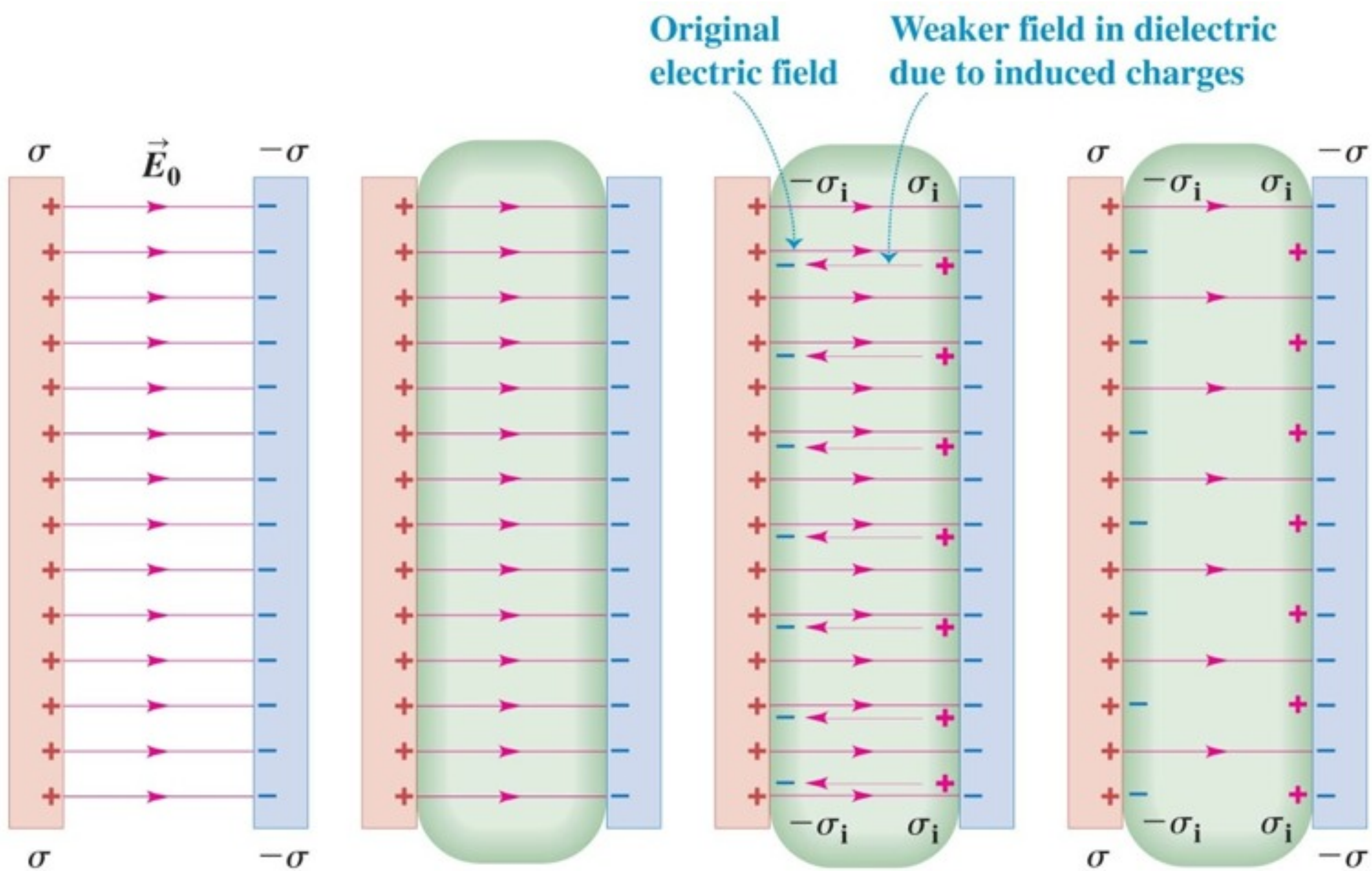
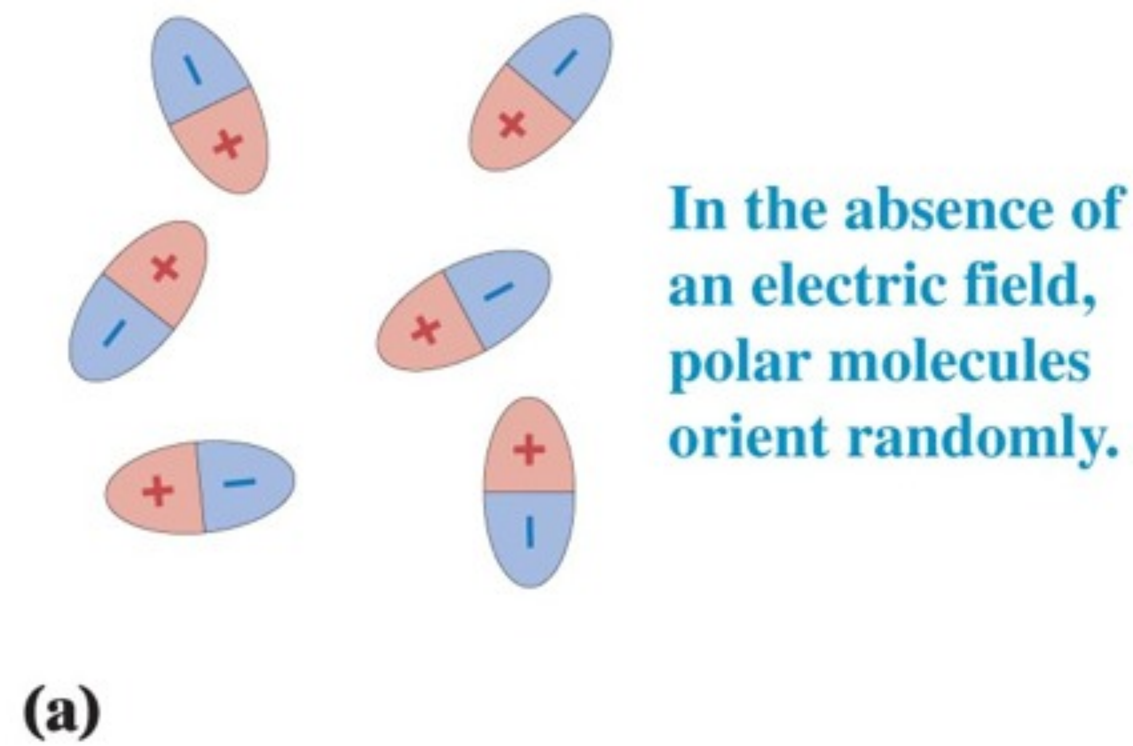
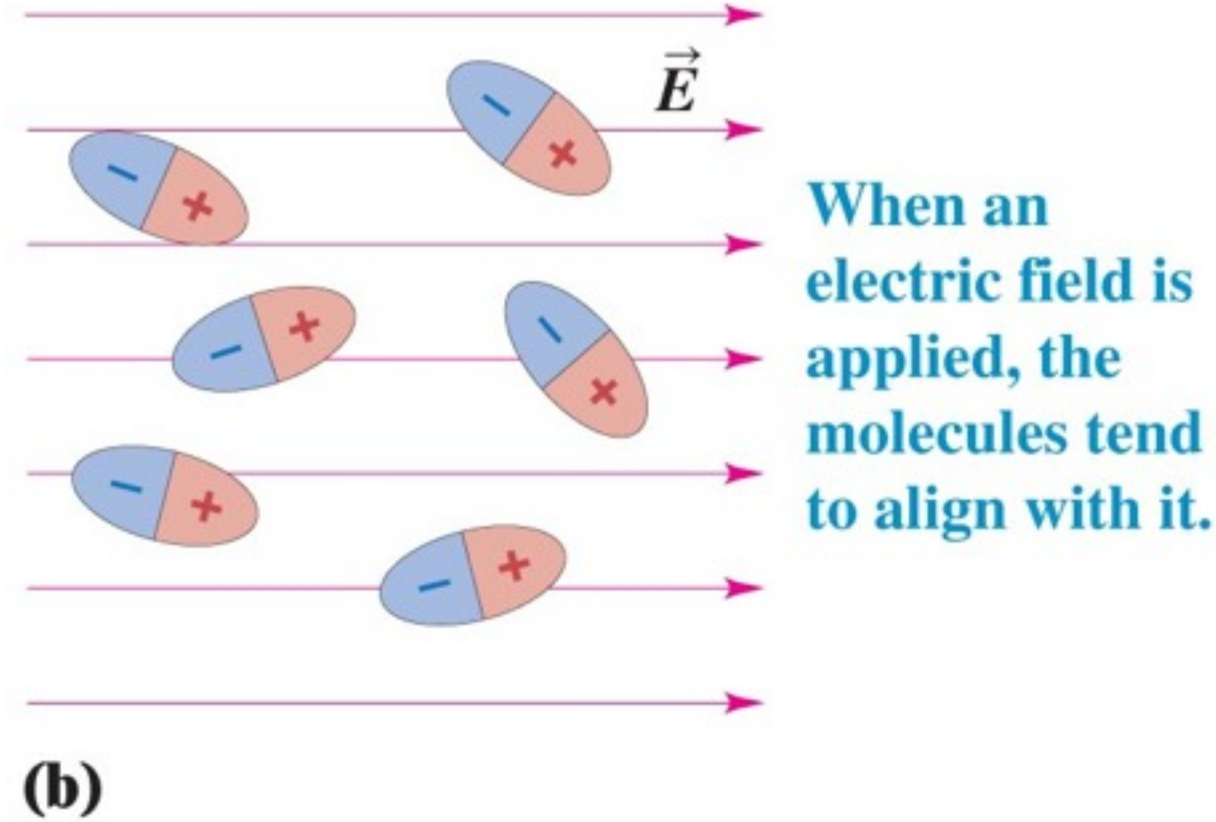


(a) No dielectric **(b)** Dielectric just inserted **(c)** Induced charges create electric field **(d)** Resultant field

dielectrics

➜ the induced charge is caused by the polarization of electric dipoles in the dielectric





dielectrics

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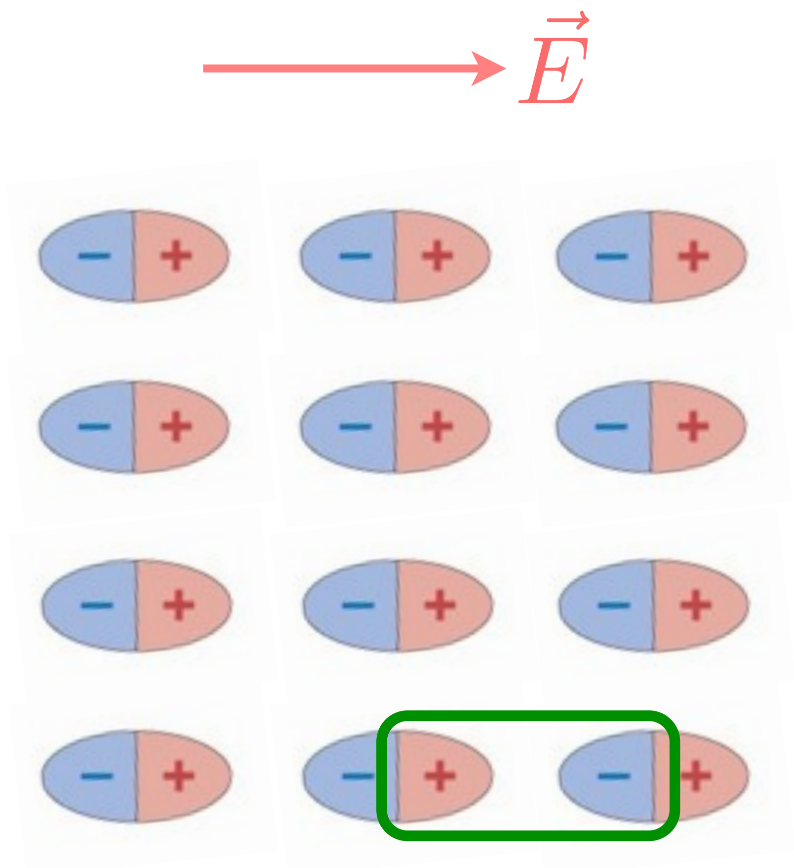
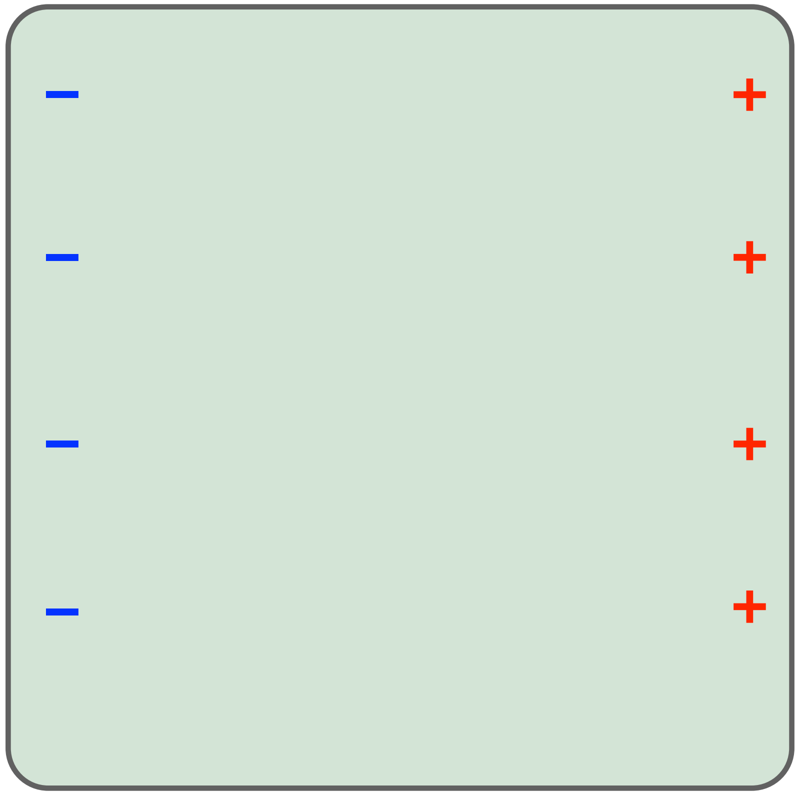


cancellation of
charges in the bulk



surface charge remains