Mechanism Design with Allocative, Informational and Learning Constraints

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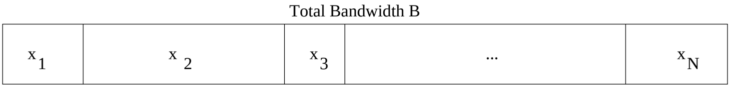
Joint work with Ph.D. student Abhinav Sinha (Graduating in September)

Allocation problems in networks

- A number of interesting problems in networks involve optimization of a social utility function (sum of agents' utilities) under resource constraints, privacy constraints, and strategic behavior by agents

- Examples include:
  - power production, distribution, consumption on the smart grid,
  - Bandwidth allocation to cellular service providers
  - unicast, multi-rate multicast service on the Internet
  - advertisement on social networks
  - economies with public or local public goods (e.g., investment on clean air or cyber-security)

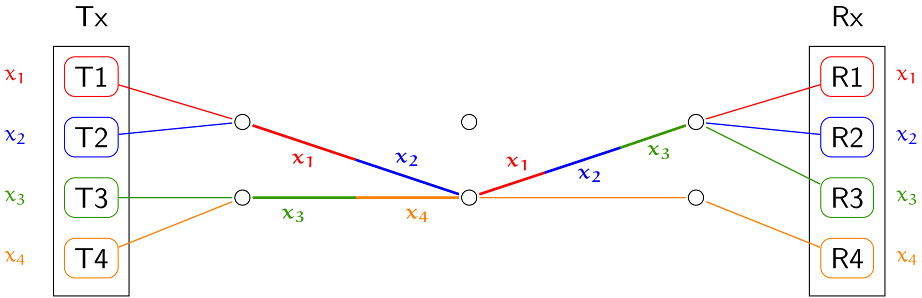
Spectrum allocation



$$\max_{x \in \mathbb{R}_+^N} \sum_{i \in \mathcal{N}} v_i(x_i)$$
$$\text{s.t.} \quad \sum_{i \in \mathcal{N}} x_i \leqslant B$$

- private consumption goods: utilities $v_i(\cdot)$ depend only on their own allocation

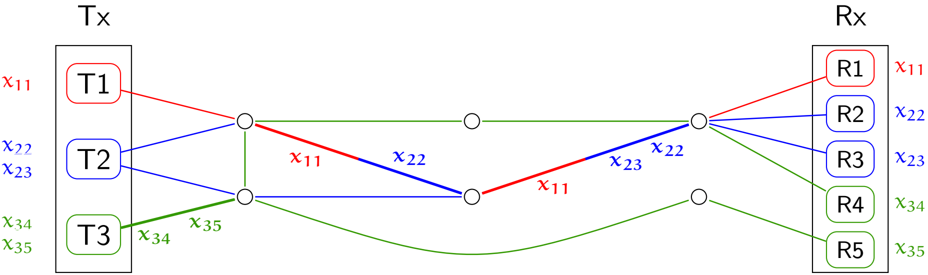
Unicast service on the Internet



$$\max_{x \in \mathbb{R}_+^N} \sum_{i \in \mathcal{N}} v_i(x_i)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{N}^l} x_i \leqslant c^l \quad \forall\, l \in \mathcal{L}$$

Multi-rate multicast service on the Internet

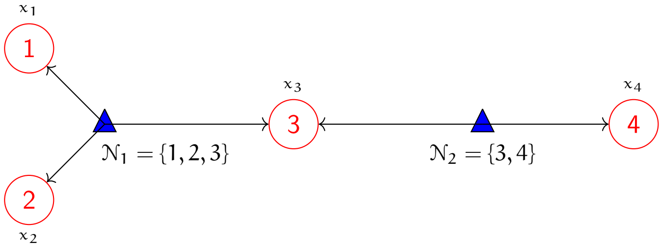


$$\max_{x \in \mathbb{R}_+^N} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{G}_k} v_{ki}(x_{ki})$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}^l} \max_{i \in \mathcal{G}_k^l} \{x_{ki}\} \leqslant c^l \quad \forall\, l \in \mathcal{L}$$

Power allocation in wireless networks

- Each "agent" is a (transmitter, receiver) pair.



$$\max_{x} \sum_{i \in \mathcal{N}} v_i(\{x_i\}_{i \in \mathcal{N}_{k(i)}})$$
$$\text{s.t.} \quad x \in \mathbb{R}_+^N$$

- The vector of transmission powers $x = (x_1, \ldots, x_N)$ is a public (or local public) good

NUM and Salient Features

- Maximize sum of utilities subject to network constraints

$$\max_{x} \sum_{i \in \mathcal{N}} v_i(x) \quad \text{s.t.} \quad x \in \mathcal{X}.$$

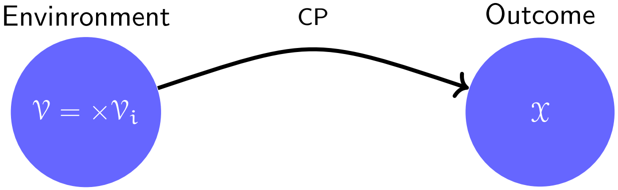
- Linear constraints - common knowledge.
- $v_i(\cdot)$ - Private information and known only to agent i
- Designer can impose taxes and quasi-linear utilities

$$u_i = v_i(x) - t_i.$$

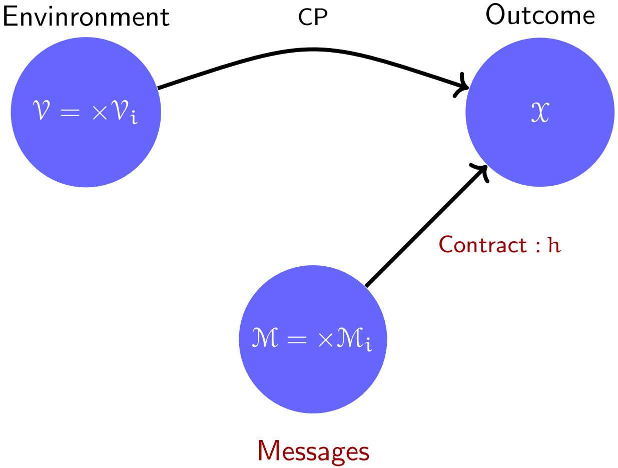
Informal Framework of Mechanism Design

- Designer wishes to allocate "optimally".
- Agents are strategic and...
- Possess private information relevant to optimal allocation.
- Designer wishes to cover this Informational gap
  - Design Message Space and Contract.
  - Agents then announce a selected message and receive outcomes based on the contract.

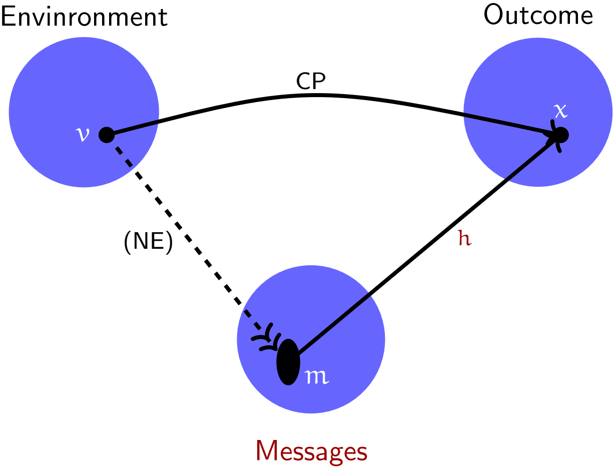
Hurwicz-Reiter Model



Hurwicz-Reiter Model



Hurwicz-Reiter Model



Design $\mathcal{M}$ & $h : \mathcal{M} \to \mathcal{X}$ to replicate CP mapping at NE.

Motivating Question

The motivating question, in this work, is whether it is possible to design Nash implementation mechanisms that posses **certain properties...**

Motivating Design Principles

(a) Reduction in communication overhead for NUM problems

- Models with "large" type spaces - Private info. $v_i : \mathbb{R}^A \to \mathbb{R}$.

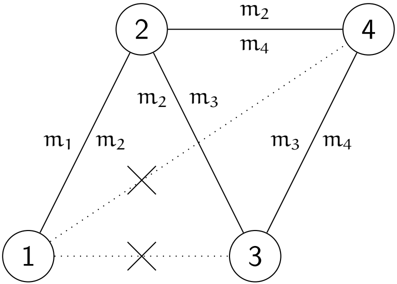
(b) Systematic Design to deal with Allocative constraints

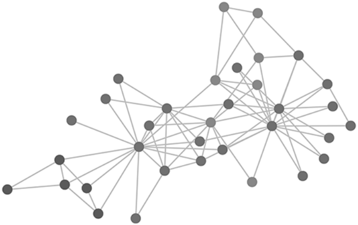
- Integrate different network constraints under one design umbrella.
- Special focus: Can off-equilibrium feasibility be accommodated?

Motivating Design Principles (contd)

(c) Informational constraints on messaging

- What if not all agents can talk to a common designer?
- What if they don't want to? (privacy)





(Not to be confused with Unicast or Multicast links or dependence of utility on local allocation only.)

Motivating Design Principles (contd)

(d) Learning of Nash equilibrium

- Solution concept of NE and convergence of a learning dynamic go together for models with stable environments
- Design for two Off-equilibrium properties
- Off-equilibrium Feasibility
- Convergence of a "large" class of Learning algorithms.

(e) Designing beyond sum of utilities - fairness

- What about mechanism design for other objectives?, e.g.,

$$\max_{x \in \mathcal{X}} \sum_{i \in \mathcal{N}} \log\left(v_i(x)\right) \qquad \text{or} \qquad \max_{x \in \mathcal{X}} \left(\min_{i \in \mathcal{N}} \left(v_i(x)\right)\right).$$

Overview

(a) Reduced communication overhead

(b) Systematic Design — **Part 1:** Multirate/Multicast

(c) Informational constraints on messaging

(d) Learning of NE — **Part 2:** Distributed M.D. and Learning

(e) Designing beyond SoU — Fairness (not in this talk)

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Part 1: Multirate/Multicast

Part 2: Distributed M.D. and Learning

Fairness (not in this talk)

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Part 1: Multirate/Multicast

Part 2: Distributed M.D. and Learning

Fairness (not in this talk)

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Overview

1. Systematic Design for Allocative constraints: Multi-rate/multicast transmission

2. Distributed Mechanisms with Learning Guarantees

Relevant Literature

- Earliest works by [Groves, Ledyard 1977] for Lindahl allocation and [Hurwicz 1979] for Walrasian allocation. Also [Tian 1989, Chen 2002] for Lindahl allocation.
- Recent works motivated by [Kelly, Maulloo, Tan 1998] - Competitive equilibria, only for price-takers.
- Two classifications for recent works: based on *constraint set*, based on *full/partial* implementation.

Single-link Unicast

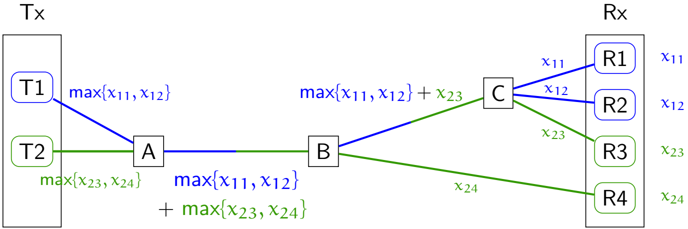
- [Yang, Hajek 2006b], [Maheshwaran, Başar 2004] - full implementation.

Relevant Literature (contd)

More General Constraint set

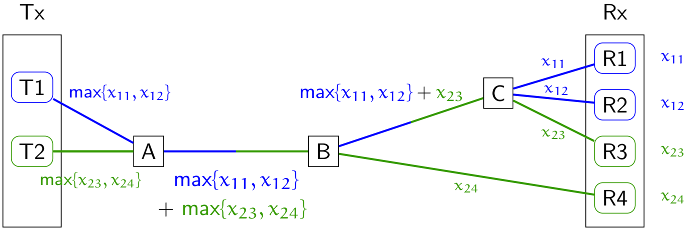
- [Yang, Hajek 2007], [Johari, Tsitsiklis 2009] - arbitrary convex constraint sets but with partial implementation.
  - Off-equilibrium Feasibility.
- [Jain, Walrand 2010] - Partial Implementation.
- [Kakhbod, Teneketzis 2012a,b] - Full implementation for general unicast and multicast.

The Multirate/Multicast Model



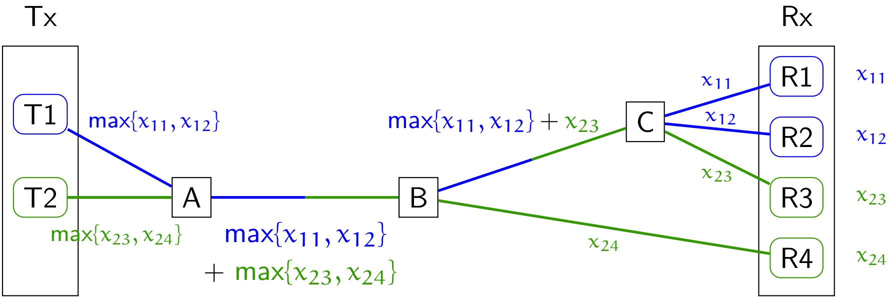
- Agent - a specific transmitter-receiver pair.
- **Multicast Group** - Group of agents requesting the same content. (e.g. same TV program).
- **Multi-rate** - Agents within the same multicast group can request different QoS (e.g. high/standard definition video).
  - Watching on mobile phone or on HDTV.

Notation



- Set of multicast groups $\mathcal{K} = \{1, \dots, K\}$.
- Agents indexed group-wise $(k, i) \in \mathcal{K} \times \mathcal{G}_k$ (called agent ki).
- Each agent uses a fixed route $\mathcal{L}_{ki} \subset \mathcal{L}$.
- Set of groups active on link $l \in \mathcal{L}$ denoted by $\mathcal{K}^l \subset \mathcal{K}$.
- Agents from group k that are active on link l are identified by $\mathcal{G}_k^l \subset \mathcal{G}_k$.

Centralized Problem



Agents

- T1-R1
- T1-R2
- T2-R3
- T2-R4

$$\max_{x \in \mathbb{R}_+^N} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{G}_k} v_{ki}(x_{ki}) \qquad (\mathrm{CP_{MM}})$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}^l} \max_{i \in \mathcal{G}_k^l} \{x_{ki}\} \leqslant c^l \quad \forall\, l \in \mathcal{L}$$

Public and Private goods

Two levels of interaction in the Multi-rate Multicast problem

1. Contest between groups through the highest rate at each link.
2. No contest within a group, since only maximum gets charged.

- Due to the $\max\{\cdot\}$, there's a possible case for free-riding.

Systematic Mechanism Design

$$\max_{x,s} \sum_{ki \in \mathcal{N}} v_{ki}(x_{ki})$$

$$\text{s.t.} \quad x_{ki} \geq 0 \qquad \forall \; ki \in \mathcal{N} \qquad (C_1)$$

$$\text{and} \quad \sum_{k \in \mathcal{K}^l} s_k^l \leq c^l \quad \forall \; l \in \mathcal{L} \qquad (C_2)$$

$$\text{and} \quad x_{ki} \leq s_k^l \quad \forall \, i \in \mathcal{G}_k^l, \forall \, k, l \quad (C_3)$$

- s_k^l - proxy for $\max_{i \in \mathcal{G}_k^l} \{x_{ki}\}$.

KKT - Necessary and Sufficient

Assuming utilities are strictly concave (and monotonic) and differentiable
There exist

- Primal variables (x^*, s^*) and
- Dual variables $(\lambda_l^*)_{l \in \mathcal{L}}$ and $({\mu_{ki}^l}^*)_{ki \in \mathcal{N}, l \in \mathcal{L}_{ki}}$

such that:

1. Primal Feasibility – (x^*, s^*) satisfy multicast constraints.
2. Dual Feasibility – $\lambda^*, \mu^* \geqslant 0$.

KKT - Necessary and Sufficient

Assuming utilities are strictly concave (and monotonic) and differentiable

3. Complimentary Slackness –

$$\lambda_l^* \Big(\sum_{k \in \mathcal{K}^l} {s_k^l}^* - c^l \Big) = 0 \quad \forall \;\; l \in \mathcal{L}$$

$$ {\mu_{ki}^l}^* \left(x_{ki}^* - {s_k^l}^* \right) = 0 \quad \forall \;\; ki \in \mathcal{N}, \;\; l \in \mathcal{L}_{ki}$$

4. Stationarity –

$$v'_{ki}(x_{ki}^*) = \sum_{l \in \mathcal{L}_{ki}} {\mu_{ki}^l}^* \quad \forall \;\; ki \in \mathcal{N} \quad \text{if} \quad x_{ki}^* > 0$$

$$\lambda_l^* = \sum_{i \in \mathcal{G}_k^l} {\mu_{ki}^l}^* \quad \forall \;\; k \in \mathcal{K}^l, \;\; l \in \mathcal{L}$$

Mechanism - Message Space

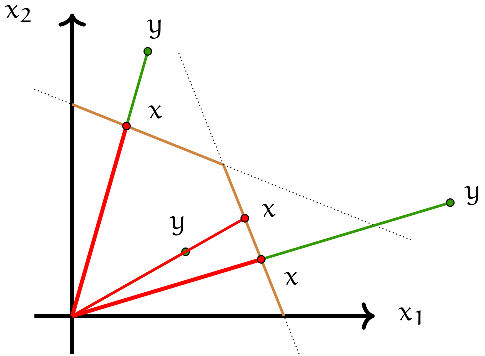
- $\mathcal{M}_{ki} = \mathbb{R}_+ \times \mathbb{R}_+^{2L_{ki}}$ where $m_{ki} = \left(y_{ki}, (p^l_{ki}, q^l_{ki})_{l \in \mathcal{L}_{ki}}\right)$.
- y_{ki} – agent ki's demand for allocation.
- p^l_{ki} – individual price for ki. (proxy for μ^l_{ki})
- Best not to have agents pay at prices quoted by themself.
- So q^l_{ki} used in place of p^l_{ke} for the "next" agent on link l.

p^l_{kj}	q^l_{kj}		
	p^l_{ki}	q^l_{ki}	
		p^l_{ke}	q^l_{ke}

Mechanism: Proportional Allocation

- A new allocation scheme ..... that guarantees off-equilibrium feasibility!

Demand y is translated into allocation x using a scaling factor, i.e.,
$\widehat{x}_{ki}(m) = r\, y_{ki}$ with $r = \min_{l \in \mathcal{L}} \{r^l(m)\}$.



Mechanism: Taxes

- Tax for any agent ki is the sum of taxes t^{l}_{ki} over route $\mathcal{L}_{ki}$.

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Consider agents $kj \rightsquigarrow ki \rightsquigarrow ke$ from $\mathcal{G}^l_k$.

Mechanism: Taxes

Consider agents $kj \rightsquigarrow ki \rightsquigarrow ke$ from $\mathcal{G}_k^l$.

$$t_{ki}^l = x_{ki} q_{kj}^l + (q_{ki}^l - p_{ke}^l)^2 + q_{kj}^l \left(p_{ki}^l - q_{kj}^l\right) \left(s_k^l - x_{ki}\right)$$
$$+ (w_k^l - \bar{w}_{-k}^l)^2 + \bar{w}_{-k}^l \left(w_k^l - \bar{w}_{-k}^l\right) \left(c^l - \sum_{k' \in \mathcal{K}^l} s_{k'}^l\right)$$

Mechanism: Taxes

Consider agents $kj \rightsquigarrow ki \rightsquigarrow ke$ from $\mathcal{G}_k^l$.

$$x_{ki} q_{kj}^l$$

Payment for x_{ki}

Mechanism: Taxes

Consider agents $kj \rightsquigarrow ki \rightsquigarrow ke$ from $\mathcal{G}_k^l$.

$$(q_{ki}^l - p_{ke}^l)^2$$

Equal Prices (individual)

p_{kj}^l	q_{kj}^l		
	p_{ki}^l	q_{ki}^l	
		p_{ke}^l	q_{ke}^l

Mechanism: Taxes

Consider agents $kj \rightsquigarrow ki \rightsquigarrow ke$ from $\mathcal{G}_k^l$.

$$q_{kj}^l \left(p_{ki}^l - q_{kj}^l\right) \left(s_k^l - x_{ki}\right)$$

Complimentary Slackness (individual)

Mechanism: Taxes

Consider agents $kj \rightsquigarrow ki \rightsquigarrow ke$ from $\mathcal{G}_k^l$.

$$(w_k^l - \bar{w}_{-k}^l)^2$$

Equal Prices (group)

- Define $w_k^l = \sum_{i \in \mathcal{G}_k^l} p_{ki}^l$ and $\bar{w}_{-k}^l = \frac{1}{K^l - 1} \sum_{k' \in \mathcal{K}^l \setminus \{k\}} w_{k'}^l$.

Mechanism: Taxes

Consider agents $kj \rightsquigarrow ki \rightsquigarrow ke$ from $\mathcal{G}_k^l$.

$$\bar{w}_{-k}^{l}\left(w_k^l - \bar{w}_{-k}^{l}\right)\left(c^l - \sum_{k' \in \mathcal{K}^l} s_{k'}^{l}\right)$$

Complimentary Slackness (group)

Summary of Results

Theorem (Full Implementation+IR+WBB/SBB)

At any Nash equilibrium m^* *of the induced game,*

- *The allocation* $\hat{x}(m^*)$ *is the unique solution to* (CP_{MM})*.*
- *Individual Rationality is satisfied for all agents.*
- *Weak Budget Balance* $\sum_{ki \in \mathcal{N}} t_{ki}(m^*) \geqslant 0$*.*
- *Strong Budget Balance* $\sum_{ki \in \mathcal{N}} t_{ki}(m^*) = 0$*, with an augmented message space.*

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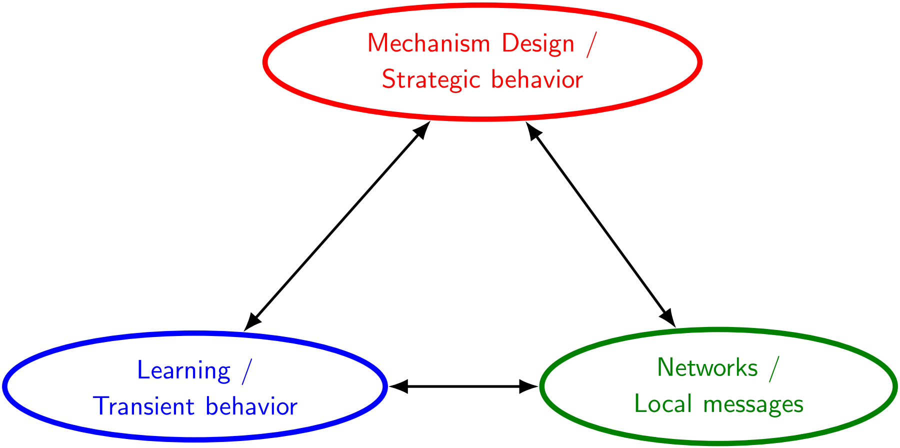
Basic idea behind proof of Implementation

- F.O.C. for NE gives KKT as necessary conditions.
- Existence using S.O.C. and invertibility.

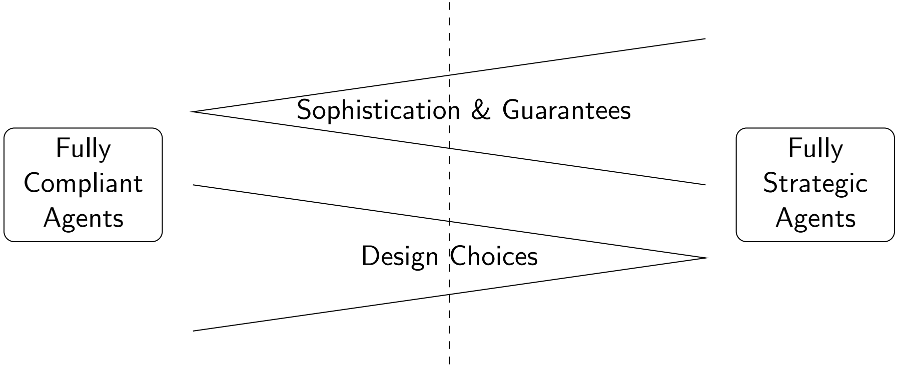
Overview

1. Systematic Design for Allocative constraints: Multi-rate/multicast transmission
2. Distributed Mechanisms with Learning Guarantees

Mechanism Design, Learning and Networks



Learning and Modeling trade-offs



Good enough guarantee and
large enough design space

Mechanism Design, Learning and Networks

Well-known algorithms for finding roots and/or optimization

- Stochastic Approximation: [Robbins, Monro 1951]
- Gradient Descent: [Nesterov 1983]
- Simulated Annealing: [Khachaturyan et al 1979, Kirkpatrick et al 1983]
- Online algorithms: prediction with advice [Littlestone, Warmuth 1994, Vovk 1992]; exp3 [Auer et al 2002]

Learning /
Transient Behavior

Mechanism Design, Learning and Networks

Decentralized system

- Message passing is done locally
- Realistic and scalable architecture

Several well-known graph problems such as

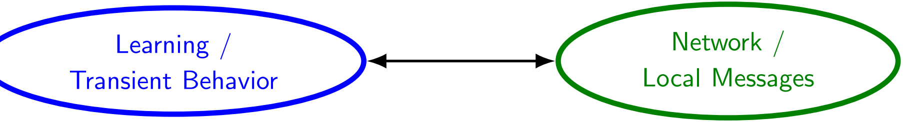
- Consensus on a Network [Fischer, Lynch, Paterson 1985]
- Byzantine Generals [Lamport, Shostak, Pease 1982]

Network /
Local Messages

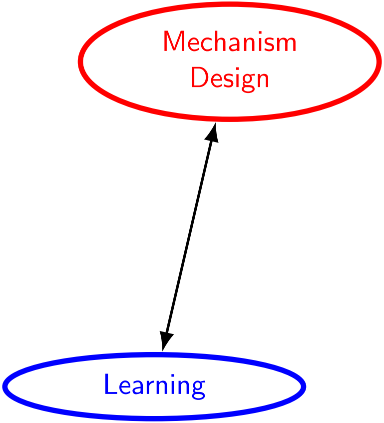
Learning + Networks

- State of the art design has accommodated at most two out of three requirements
- Learning on a network with **non-strategic agents**: distributed optimization

- Distributed Stochastic Gradient descent: [Tsitsiklis, Bertsekas 1986]
- Consensus and Optimization: [Nedic, Ozdaglar, Parrilo 2008]
- ADMM: [Boyd, Parikh, Chu, Peleato, Eckstein 2011].



Mechanism Design + Learning



- Mechanism Design with Learning guarantees in **Broadcast** environments (i.e., fully connected networks).
- Supermodularity has played a major role in providing learning guarantees in games. [Milgrom, Roberts 1990; Chen 2002]

Mechanism Design + Learning

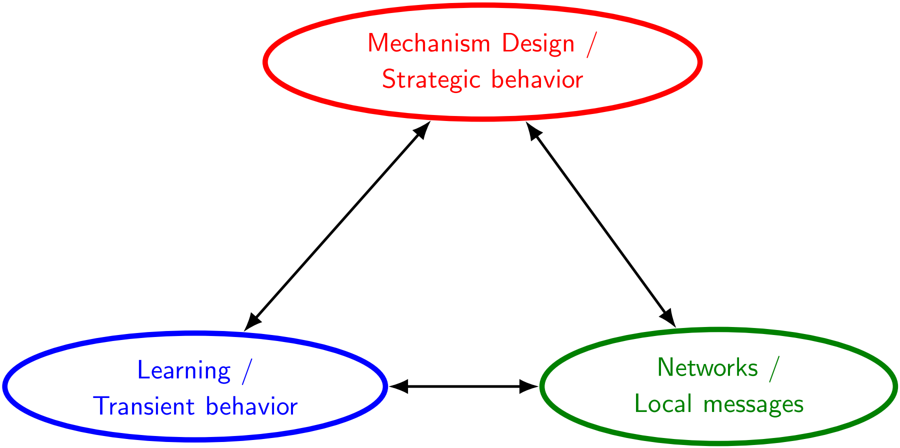
Theorem (Milgrom and Roberts 1990)

For a supermodular game i.e., compact action space and an increasing best-response, any learning strategy within the adaptive dynamics (AD) class converges to a point between two most extreme NE.

Limitations

- Requirement of a compact action space, and
- A region for the convergent point if multiple equilibria exist.
- Experimental results have shown that supermodularity does not result in fast convergence

Goal - Design simultaneously for all three



Walrasian and Lindahl allocation

- Two centralized problems - private and public goods
- Private good

$$\max_{x \in \mathbb{R}^N} \sum_{i \in \mathcal{N}} v_i(x_i) \quad \text{s.t.} \quad \sum_{i \in \mathcal{N}} x_i = 0. \qquad \text{(Walrasian)}$$

- Public good

$$\max_{x \in \mathbb{R}} \sum_{i \in \mathcal{N}} v_i(x). \qquad \text{(Lindahl)}$$

Walrasian and Lindahl allocation

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- Public good

$$\max_{x \in \mathbb{R}} \sum_{i \in \mathcal{N}} v_i(x). \tag{Lindahl}$$

Environment assumption: Utilities $v_i : \mathbb{R} \to \mathbb{R}$ are assumed to be strictly concave with continuous second derivatives such that

$$v_i''(\cdot) \in \left(-\eta, -\frac{1}{\eta}\right).$$

Summarizing our goals

We aim to design, as before, reduced message space Nash implementation mechanisms such that...

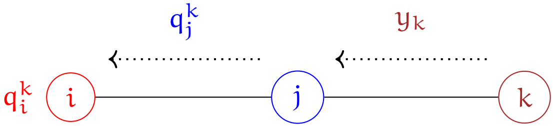
A. Allocation and Tax function depend only on neighborhood messages

$$\widehat{x}_i, \widehat{t}_i : \mathcal{M} \to \mathbb{R}, \quad \text{with} \quad \widehat{x}_i(m_i, m_{\mathcal{N}(i)}) \;\text{ and }\; \widehat{t}_i(m_i, m_{\mathcal{N}(i)})$$

B. There is guaranteed convergence, to the Nash equilibrium, when agents choose their learning strategies within a class $\mathcal{L}$ of learning strategies.

Walrasian Mechanism - Proxy via neighbor

- **Message**: $m_i = (y_i, q_i) \in \mathbb{R}^{N+1}$,
  - Demand $y_i \in \mathbb{R}$,
  - Proxy $q_i = (q_i^1, \ldots, q_i^N) \in \mathbb{R}^N$.
- Proxy q_i are included to collect demand y_j from non-neighbor agents.



Mechanism - Allocation and Tax

Allocation: $$\widehat{x}_i = y_i - \frac{1}{N-1}\left(\sum_{k \neq i} y_k\right).$$

Mechanism - Allocation and Tax

Allocation: $$\widehat{x}_i = y_i - \frac{1}{N-1}\left(\sum_{k \neq i} q^k_{n(i,k)}\right).$$

$n(i,k)$ neighbor of i closest to k,

Mechanism - Allocation and Tax

Allocation: $$\widehat{x}_i = y_i - \frac{1}{N-1}\left(\sum_{k \neq i} q^k_{n(i,k)}\right).$$

Tax: $\widehat{t}_i = \underbrace{\widehat{p}_i}_{\text{price}} \widehat{x}_i$

$$\widehat{p}_i = \left(y_i + \sum_{k \neq i} y_k\right)$$

Mechanism - Allocation and Tax

Allocation: $$\widehat{x}_i = y_i - \frac{1}{N-1}\left(\sum_{k\neq i} q^k_{n(i,k)}\right).$$

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Mechanism - Allocation and Tax

Allocation: $$\hat{x}_i = y_i - \frac{1}{N-1}\left(\sum_{k\neq i} q^k_{n(i,k)}\right).$$

Tax: $$\hat{t}_i = \underbrace{\hat{p}_i}_{\text{price}}\, \hat{x}_i + \sum_{\substack{k\in\mathcal{N}(i),\\ k=i}} \underbrace{\left(q^k_i - y_k\right)^2}_{\text{explicit duplication}} + \sum_{\substack{k\notin\mathcal{N}(i)\\ k\neq i}} \underbrace{\left(q^k_i - q^k_{n(i,k)}\right)^2}_{\text{duplication via message passing}},$$

$$\hat{p}_i = \left(q^i_{n(i,i)} + \sum_{k\neq i} q^k_{n(i,k)}\right).$$

Mechanism - Allocation and Tax

Allocation: $$\widehat{x}_i = y_i - \frac{1}{N-1}\left(\sum_{k\in\mathcal{N}(i)} \frac{q^k_{n(i,k)}}{\xi} + \sum_{\substack{k\notin\mathcal{N}(i)\\ k\neq i}} \frac{q^k_{n(i,k)}}{\xi^{d(i,k)-1}}\right).$$

$d(i,k)$ shortest distance to k

Tax: $$\widehat{t}_i = \underbrace{\widehat{p}_i}_{\text{price}}\, \widehat{x}_i + \sum_{\substack{k\in\mathcal{N}(i),\\ k=i}} \underbrace{\left(q^k_i - \xi y_k\right)^2}_{\text{explicit duplication}} + \sum_{\substack{k\notin\mathcal{N}(i)\\ k\neq i}} \underbrace{\left(q^k_i - \xi q^k_{n(i,k)}\right)^2}_{\text{duplication via message passing}},$$

$$\widehat{p}_i = \frac{1}{\delta}\left(\frac{q^i_{n(i,i)}}{\xi} + \sum_{k\in\mathcal{N}(i)} \frac{q^k_{n(i,k)}}{\xi} + \sum_{\substack{k\notin\mathcal{N}(i)\\ k\neq i}} \frac{q^k_{n(i,k)}}{\xi^{d(i,k)-1}}\right).$$

Main Result

Theorem

1. *The induced game has unique NE and the corresponding allocation is* x^*.
2. *Best-Response for the induced game is contractive and hence every learning dynamics in the ABR class, converges to the NE.*
3. *Budget Balance: the total tax paid at NE is zero,* $\hat{t}_1 + \cdots + \hat{t}_N = 0$.

- *Same results for Lindahl allocation.*

Sketch of the proof

- Efficient Nash equilibrium – arguments same as before.
- Contraction – verify $\|\nabla \mathrm{BR}\| < 1$.
  - Row-sum matrix norm.
- Explicitly write down best-response $(y_i, q_i) = \mathrm{BR}_i\,(y_{-i}, q_{-i})$.
  - Quadratic tax and Linear allocation make this easy.
- Show there exist parameters $\xi \in (0,1)$ and $\delta \in (0,\infty)$ for any given $\eta \in (1,\infty)$.

Contraction and ABR

Theorem (Healy and Mathevet (2012))

If a game is contractive i.e.,

$$\|\nabla \mathrm{BR}\| < 1,$$

then all ABR dynamics converge to the unique Nash equilibrium.

Contraction and ABR

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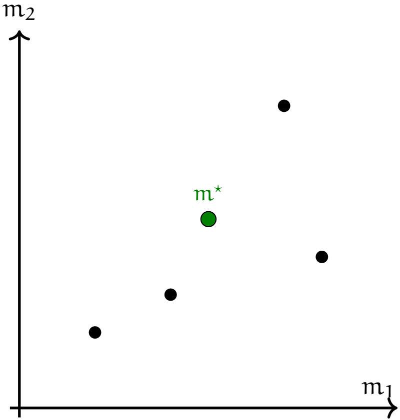
- Cournot Best-Response Dynamics.
- Best-Response to empirical distrib. from past k periods.
- Best-Response to any convex combination of past k periods.
- Fictitious Play (under concave utilities).

Adaptive Best-Response Dynamics (ABR)

"Asymptotically, the support of randomized actions must not be further than the best-response to the worst observed action in the finite past."



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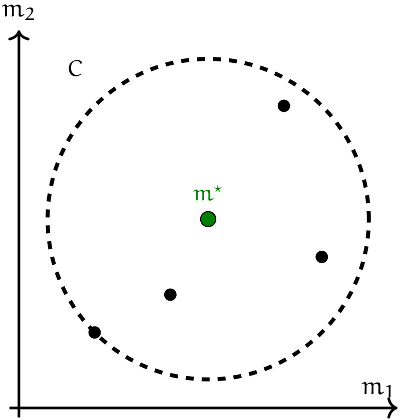

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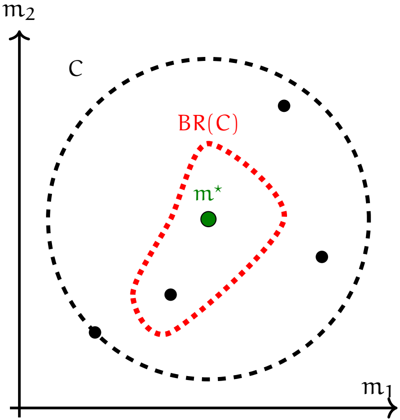

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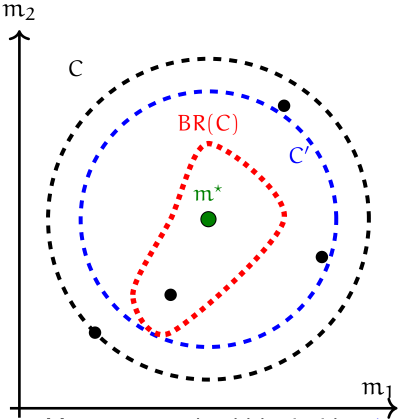
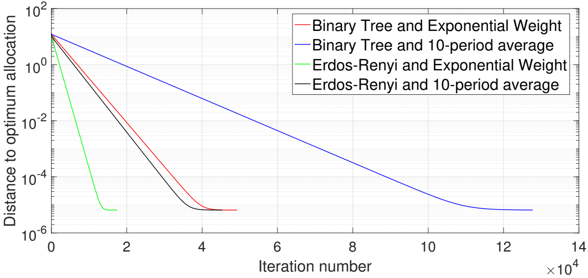


Message at t should be inside C'

Numerical example

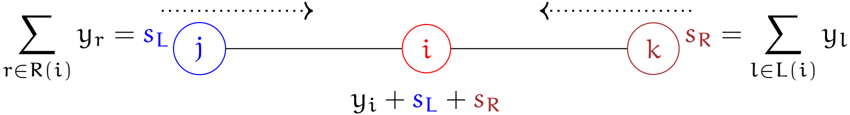
- $N = 31$, $\eta = 25$. Agents' utility function as quadratic $v_i(x) = \theta_i x_i^2 + \sigma_i x_i$.
- Two graphs: (1) full binary tree and (2) Erdős-Reńyi random graph where any two edges are connected with probability $p = 0.3$
- Two types of learning dynamics: (a) action taken is the best-response to an exponentially weighed average of past actions; (b) best-respond to the arithmetic mean of past 10 rounds



Conclusions and Future Research Directions

- We try to systematize the design of mechanisms for NUM problems with small message spaces, allocative constraints and off-equilibrium feasibility, informational constraints and provide learning guarantees.

- Can we further reduce average message space from $(N+1)$ to $(\overline{\deg}+1)$?



- Impossibility of Learning for "small and continuous" mechanisms.
- What about dynamic environments? (dynamic mechanism design/dynamic games/Perfect Bayesian Equilibria...)

Thank you.