A Memory-Efficient Tree Edit Distance Algorithm

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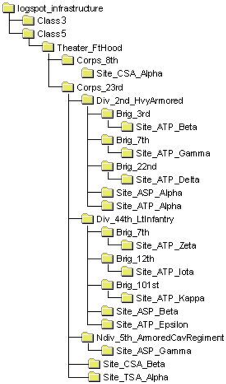
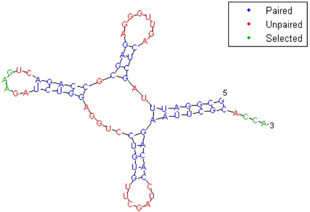
DEXA 2014

Tree-structured data

- hierarchical data is often modelled as trees
- an interesting query computes the similarity between trees



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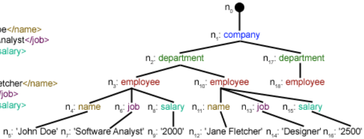

Tree Edit Distance (TED)

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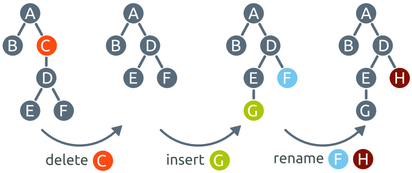
- TED is the minimum number of edit operations to transform one tree into another



Recursive solution for TED

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- GOAL of TED algorithms - minimize #subproblems

State of the art in TED

Algorithm		Time	Space	Comments
Tai	1979	$O(n^6)$	$O(n^6)$	first algorithm
Zhang&Shasha	1989	$O(n^4)$	$O(n^2)$	efficient for balanced trees $O(n^2 \log^2 n)$
Klein	1998	$O(n^3 \log n)$	$O(n^3 \log n)$	bad space complexity
Demaine et al.	2009	$O(n^3)$	$O(n^2)$	worst case is frequent
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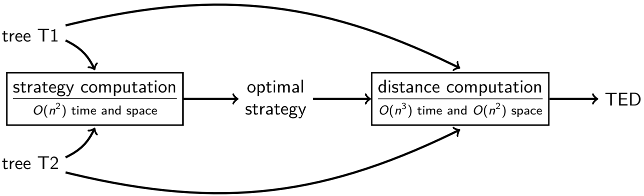
- RTED uses the optimal strategy
- unfortunately, RTED has a memory problem

RTED algorithm



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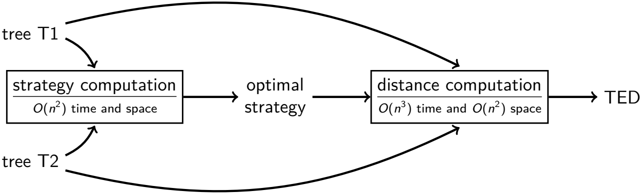

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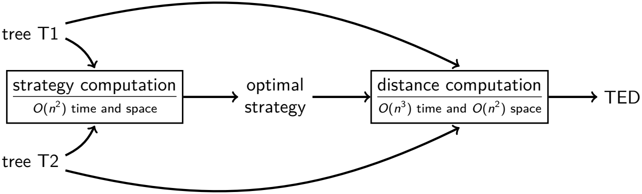

- overhead of strategy computation is very low
- #subproblems is minimal compared to previous solutions

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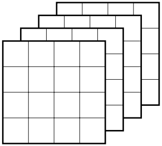
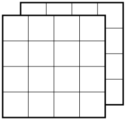


strategy - $4n^2$



distance - $2n^2$

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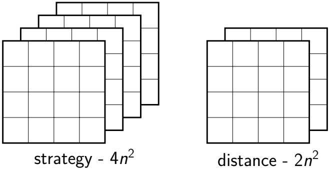

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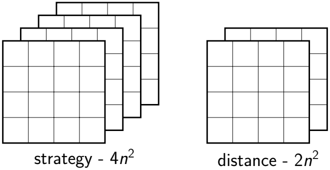
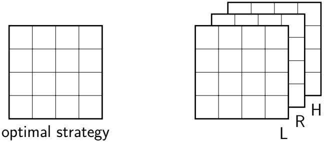


- strategy may use 2x the memory of distance computation: $4n^2$ vs $2n^2$
- strategy computation is a memory bottleneck

Memory allocation in RTED strategy

4 matrices of quadratic size are used:

- 1 for optimal strategy
- 3 for intermediate results



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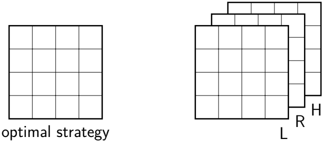

strategy computation (high-level description):

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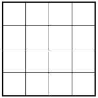
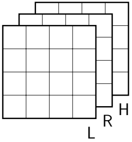


optimal strategy



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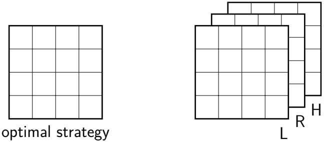

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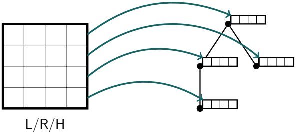
- observation: after a node is processed, its row is not needed any more



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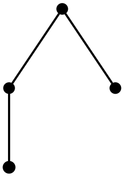

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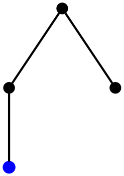

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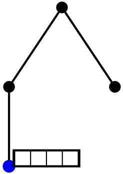

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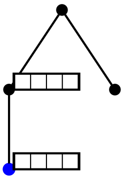

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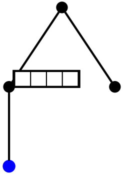

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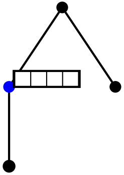

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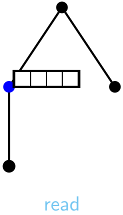

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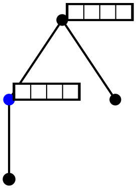

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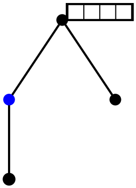

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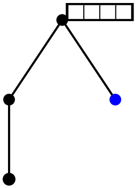

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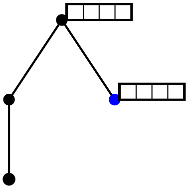

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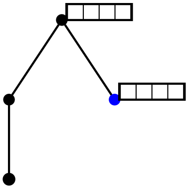

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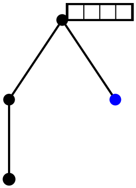

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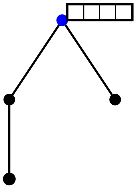

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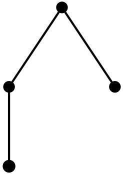
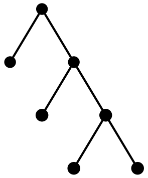
- What is the max #rows?



Upper bound for the number of rows

calculate max #rows for postorder:



Upper bound for the number of rows

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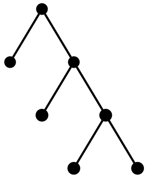
- (1) only leftmost-child leaf nodes add 1 to max #rows



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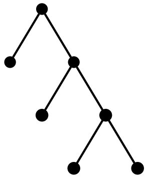

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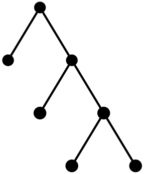
- there are maximum $n/2$ leftmost-child leaf nodes



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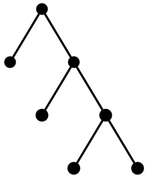
calculate max #rows for postorder:

- (1) only leftmost-child leaf nodes add 1 to max #rows
- (2) other nodes do not affect max #rows or reduce it by 1
- there are maximum $n/2$ leftmost-child leaf nodes



right-to-left postorder

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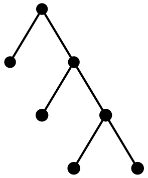

right-to-left postorder

- max #rows = #rightmost-child leaf nodes

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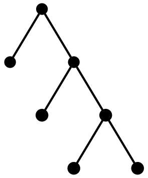

right-to-left postorder

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rule: if #leftmost-child leaves $\leq$ #rightmost-child leaves

- use postorder and right-to-left postorder otherwise

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right-to-left postorder

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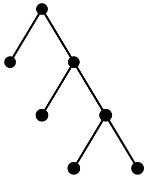
rule: if #leftmost-child leaves $\leq$ #rightmost-child leaves

- use postorder and right-to-left postorder otherwise
- max #rows = $n/3$



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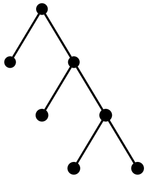

right-to-left postorder

- max #rows = #rightmost-child leaf nodes

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- max #rows $= n/3$

	strategy	**L/R/H**	**total**
old (RTED)	n^2	$3n^2$	$4n^2$
new	n^2	$3n\frac{n}{3} = n^2$	$2n^2$

Experiments

dataset	#trees	old (RTED)		new	
		#rows	memory(MB)	#rows	memory(MB)
TreeBank-200	50				
TreeBank-400	10				
SwissProt-1000	20				
SwissProt-2000	10				
TreeFam-400	50				
TreeFam-1000	20				

Experiments

dataset	#trees	old (RTED)		new	
		#rows	memory(MB)	#rows	memory(MB)
TreeBank-200	50	195.4			
TreeBank-400	10	371.3			
SwissProt-1000	20	987.5			
SwissProt-2000	10	1960.1			
TreeFam-400	50	402.6			
TreeFam-1000	20	981.9			

Experiments

dataset	#trees	old (RTED)		new	
		#rows	memory(MB)	#rows	memory(MB)
TreeBank-200	50	195.4		6.0	
TreeBank-400	10	371.3		6.3	
SwissProt-1000	20	987.5		3.0	
SwissProt-2000	10	1960.1		3.0	
TreeFam-400	50	402.6		12.3	
TreeFam-1000	20	981.9		14.1	

Experiments

dataset	#trees	old (RTED)		new	
		#rows	memory(MB)	#rows	memory(MB)
TreeBank-200	50	195.4	1.10	6.0	
TreeBank-400	10	371.3	2.86	6.3	
SwissProt-1000	20	987.5	16.64	3.0	
SwissProt-2000	10	1960.1	63.20	3.0	
TreeFam-400	50	402.6	3.33	12.3	
TreeFam-1000	20	981.9	16.73	14.1	

Experiments

dataset	#trees	old (RTED)		new	
		#rows	**memory(MB)**	**#rows**	**memory(MB)**
TreeBank-200	50	195.4	1.10	6.0	0.72
TreeBank-400	10	371.3	2.86	6.3	1.30
SwissProt-1000	20	987.5	16.64	3.0	5.13
SwissProt-2000	10	1960.1	63.20	3.0	17.80
TreeFam-400	50	402.6	3.33	12.3	1.46
TreeFam-1000	20	981.9	16.73	14.1	5.58

Conclusion

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- early deallocation technique solves the memory problem

Future work

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  e.g., trees of 1.000.000 nodes may require 1TB and 100h

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- there is a need for other, better solutions

  e.g., efficient pruning of the search space