Two Examples of Seiberg Duality in Gauge Theories With Less Than Four Supercharges

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Introduction

Seiberg duality (Seiberg 1994) is a highly non-trivial result about strongly coupled $\mathcal{N} = 1$ SQCD.

When $\frac{3}{2}N_c < N_f < 3N_c$ it states that:

An 'electric' $SU(N_c)$ theory with N_f quarks

and

A 'magnetic' $SU(N_f - N_c)$ theory with N_f 'dual' quarks and a 'meson', interacting via a superpotential $W = Mq\tilde{q}$,

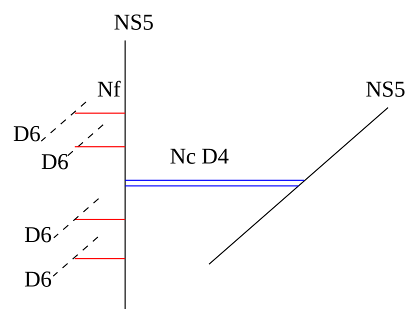
flow to the same IR fixed point.

The origin of the duality is mysterious and the result is surprising. In particular, in ordinary QCD we do not expect a description of the IR physics in terms of a dual gauge theory, but by a sigma model of mesons (a chiral Lagrangian).

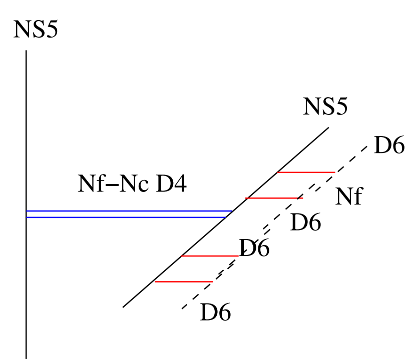
Introduction 2

The duality is supported by the matching of moduli spaces, 't Hooft anomaly matching of global symmetries and by string theory.

The realization of the 'electric' $U(N_c)$ theory in type IIA is via the following Hanany-Witten brane configuration



By swapping the NS5 branes, the Hanany-Witten effect leads to the $U(N_f - N_c)$ magnetic theory (Elitzur, Giveon, Kutasov, 1997)



Motivation

In this talk I will present two examples of Seiberg duality in theories with less than two supercharges.

The first example is in a 3d Yang-Mills Chern-Simons theory with two supercharges ($\mathcal{N} = 1$ SUSY in 3d) (A.A., Amit Giveon, Dan Israel, Vasilis Niarchos, 2009).

The other example is in a non-supersymmetric theory in 4d (A.A., Dan Israel, Gregory Moraitis, Vasilis Niarchos, 2008).

I believe that the first example can teach us about the *origin* of Seiberg duality.

The second example teaches us about the possibility (and limitation) of extending Seiberg duality to QCD.

$\mathcal{N} = 1$ Domain Walls in SYM in 4d

The first example is concerned with domain walls in 4d SYM. In this model the $U(1)$ R-symmetry is broken to Z_{2N} by the anomaly and then spontaneously to Z_2.

When a discrete symmetry, such as Z_{2N} is spontaneously broken, there exists domain walls which interpolate between the various vacua of the theory.

The domain walls are (2+1) dimensional objects, localised at, say $x_3 = z$. The "fundamental" wall interpolates between neighbouring vacua, whereas a k-wall "skips" k vacua.

We can think about the k-wall as a bound state of k elementary walls.

Domain Walls Tension

The tension of a k-wall is given (Dvali and Shifman, 1996) by the difference between the values of the gluino condensate in the given vacua

$$T_k = \frac{N}{8\pi^2} |\langle\lambda\lambda\rangle_{l+k} - \langle\lambda\lambda\rangle_l| = \frac{3N^2}{2\pi^2}\Lambda^3 \sin(\pi\frac{k}{N})$$

Note that when k is kept fixed and $N \to \infty$, $T_k \sim kT_1$ and also $T_1 \sim N\Lambda^3$.

It means that in the large-N limit the k-wall becomes a collection of k non-interacting fundamental domain walls.

The tension of each fundamental domain wall is proportional to N. This is surprising since solitons in a theory with adjoint matter should carry a tension $\sim N^2$.

In fact, every quantity (at least in perturbation theory, or semiclassically) should depend on N^2. This observation led to a bold conjecture by Witten ...

Domain Walls as D-branes

In 1998 Witten argued that the $\mathcal{N}=1$ SYM domain walls are *QCD D-branes*.

Witten noticed that the tension of the fundamental domain wall is $T_1 \sim N$ (at large-N). By using the relation between field theory and string theory $g_{string} \sim \frac{1}{N}$, we observe that the domain wall tension matches the expectation from a D-brane.

Moreover, Witten argued that the QCD-string (flux tube) can end on the domain wall, in the same way that the open-string ends on a D-brane.

In addition (A.A. and Shifman, 2003) we showed how domain walls interact via an exchange of glueballs. In fact, at large-N there is a cancellation between the attraction due to an exchange of even-parity glueballs and the repulsion due to an exchange of odd-parity glueballs. This is similar to the cancellation of the interaction between parallel D-branes.

The Acharya-Vafa theory

It is well known in string theory, that when Dp-branes coincide there appears a $(p+1)$ dimensional field theory "on the branes".

If domain walls are the "QCD D-branes", does something similar happen on the domain wall ?

It was argued by Acharya and Vafa that this is indeed the case (Acharya and Vafa, 2001). The field theory on the k wall was argued to be a $U(k)$ gauge theory which contains a level N Chern-Simons term

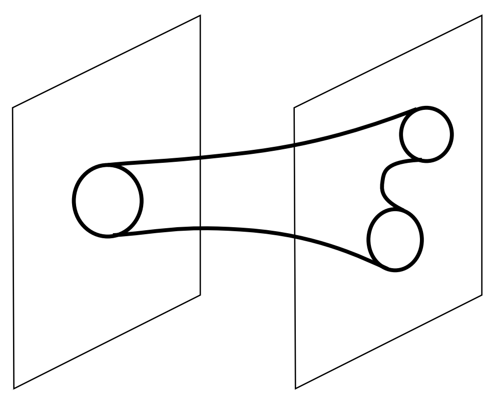
$$\mathcal{L}_{\mathrm{Acharya-Vafa}} = \frac{1}{2g^2}\mathrm{tr}\left(-\frac{1}{2}F_{mn}^2 - (D_i\phi)^2 \right.$$
$$\left. + N\epsilon_{ijk}(A^i\partial^j A^k + \frac{1}{3}A^i A^j A^k) + \text{fermions}\right)$$

The Potential between Domain-Walls

By using the Acharya-Vafa theory, it is possible to calculate the force between domain walls (A.A. and Hollowood, 2005, 2006).

As in standard D-branes physics, the vev of the scalar, $x \equiv \langle \phi \rangle$, parametrises the distance between domain walls.

A Coleman-Weinberg effective potential for ϕ, is interpreted as the potential between a pair of parallel domain walls

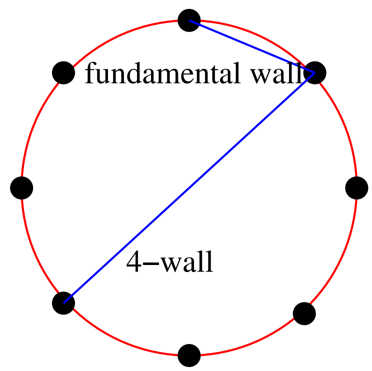


It turns out that the one-loop effective potential vanishes and we had to perform a two-loops calculation. The result is

$$V(x) \sim \frac{1}{N} \frac{x^2}{1+x^2} \, .$$

Seiberg duality in 3d Acharya-Vafa Theory

The k wall and the $N-k$ anti-wall are the same objects. Both interpolate between the same vacua



Domain Walls in SU(8)
N=1 Super Yang−Mills

Therefore the gauge theory on the k wall must be equivalent to the theory on the $N-k$ wall, hence the IR of

A 3d $U(k)$ *gauge theory with a level* N *CS term*

$=$

A 3d $U(N-k)$ *gauge theory with a level* N *CS term*

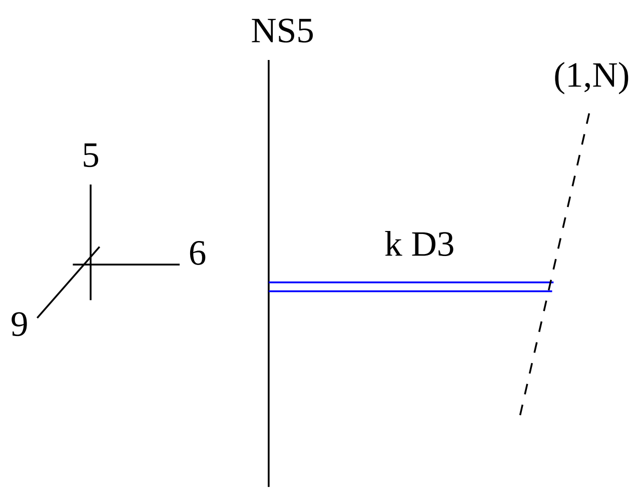
This is Seiberg duality in 3d !

Seiberg duality in 3d Acharya-Vafa Theory

Consider the following brane configuration in type IIB

$$
\begin{aligned}
k\ D3 \quad &: \quad 0126 \\
NS5 \quad &: \quad 012345 \\
(1,N) \quad &: \quad 01238 \begin{bmatrix} 5 \\ 9 \end{bmatrix}_{-\pi/2-\theta}
\end{aligned}
$$

with $\tan\theta = g_{st}N$



This brane configuration gives rise to the Acharya-Vafa Lagrangian.

Seiberg duality and Branes

By swapping the NS5 brane and the (1,N) fivebrane, we exchange the $U(k)$ YM-CS theory by the $U(N-k)$ theory.

The process includes a subtlety:

The two fivebranes are parallel along x_3, namely the D3 branes can slide along this direction. Usually, it means that there is no brane creation.

Notice however that this is a *classical* picture: as I've explained, in the quantum theory ϕ is a pseudo-modulus. Therefore in the quantum theory the D3 branes will not be able to move freely along the x_3 direction. Instead they will form about state at $\phi = 0$.

Lessons from 3d Seiberg duality

The outcome matches the field theory expectation:

Due to the CS mass, in the far IR we end-up with a topological field theory. It is a level $N-k$ $SU(k)$ Chern-Simons theory (note the shift $N \to N-k$). By level-rank duality it is equivalnet to a level k $SU(N-k)$ CS theory.

The brane picture makes a prediction: due to the s-rule (only one D3 brane can end on a D5 brane), the number of possible configurations is

$$\frac{N!}{k!(N-k)!}$$

It matches the field theory calculation of the number of possible k-walls in the $SU(N)$ theory.

In addition we learnt an interesting lesson: Seiberg duality in a 3d theory is a consequence of charge conjugation invariance in 4d.

Orientifold Planar Equivalence

The second example concerns the large-N equivalence between supersymmetric and non-supersymmetric gauge theories.

Together with Mikhail Shifman and Gabriele Veneziano, we argued a while ago, that exact non-perturbative results can be copied at large-N from a susy gauge theory to a non-supersymmetric gauge theory that lives on type 0' brane configuration.

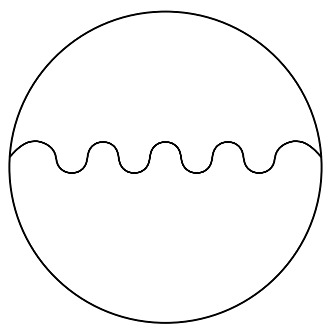
The prime example is the large-N equivalence between $\mathcal{N} = 1$ SYM and a gauge theory with a Dirac fermion in the two-index antisymmetric representation.

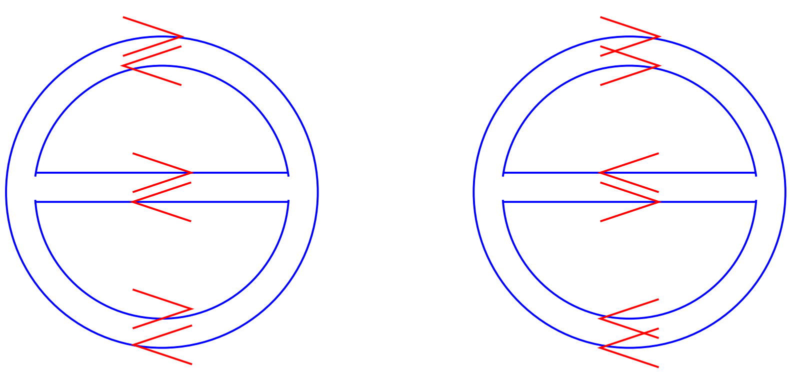
Orientifold Planar Equivalence

From the field theory point of view the reason for the equivalence is that the the planar graphs of the susy and the non-susy theories are in one-to-one correspondence.

Adjoint fermion   Antisymmetric fermion

Let us demonstrate the equivalence in a simple example





N=1 SYM   Orientifold

Orientifold Planar Equivalence

The full non-perturbative equivalence (A.A., Shifman and Veneziano, 2003,2004),(Unsal and Yaffe,2006) requires an unbroken charge conjugation symmetry.

The field theory lives on a brane configuration of type 0' string theory: a string theory whose closed string sector is the bosonic truncation of type II string theory.

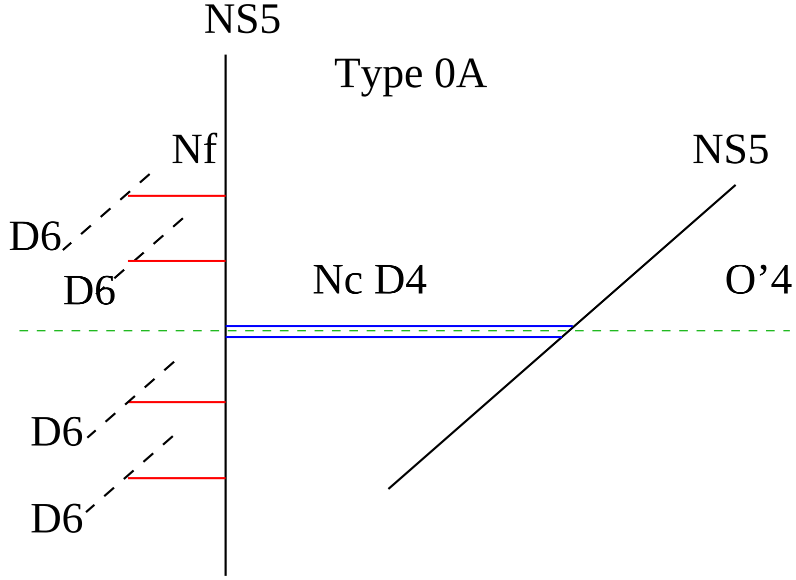
Now, let us think about the AdS/CFT as a modern version of the master field idea: a classical theory that controls the large-N gauge theory.

If two gauge theories are controlled by the same master field (same bosonic fields, same interactions), then the two gauge theories should be equivalent at large-N.

Non-Susy 'orientifold daughter' of SQCD

What is the orientifold daughter of SQCD ?

It is the theory that lives on the following brane configuration of type 0 string theory



The theory is a non-supersymmetric $U(N_c)$ gauge theory with an antisymmetric Dirac fermion and an extra 'matter multiplet'.

The electric theory

OQCD-AS				
	$U(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_R$
A_μ	adjoint	$\bullet$	$\bullet$	0
λ	$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	$\bullet$	$\bullet$	1
$\tilde{\lambda}$	$\overline{\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}}$	$\bullet$	$\bullet$	1
Φ	$\overline{\Box}$	$\Box$	$\bullet$	$\frac{N_f - N_c + 2}{N_f}$
Ψ	$\Box$	$\Box$	$\bullet$	$\frac{-N_c + 2}{N_f}$
$\tilde{\Phi}$	$\Box$	$\bullet$	$\overline{\Box}$	$\frac{N_f - N_c + 2}{N_f}$
$\tilde{\Psi}$	$\overline{\Box}$	$\bullet$	$\overline{\Box}$	$\frac{-N_c + 2}{N_f}$

Table 1: *The non-susy electric theory.*

By swapping the NS5 branes we arrive at the magnetic theory.

The magnetic theory

OQCD-AS ($\tilde{N}_c = N_f - N_c + 4$)				
	$\mathrm{U}(\tilde{N}_c)$	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_R$
A_μ	adjoint	$\bullet$	$\bullet$	0
λ	$\begin{array}{c}\Box\\\Box\end{array}$	$\bullet$	$\bullet$	1
$\tilde{\lambda}$	$\overline{\begin{array}{c}\Box\\\Box\end{array}}$	$\bullet$	$\bullet$	1
ϕ	$\Box$	$\overline{\Box}$	$\bullet$	$\frac{N_c-2}{N_f}$
ψ	$\overline{\Box}$	$\overline{\Box}$	$\bullet$	$\frac{N_c-N_f-2}{N_f}$
$\tilde{\phi}$	$\overline{\Box}$	$\bullet$	$\Box$	$\frac{N_c-2}{N_f}$
$\tilde{\psi}$	$\Box$	$\bullet$	$\Box$	$\frac{N_c-N_f-2}{N_f}$
M	$\bullet$	$\Box$	$\overline{\Box}$	$\frac{2N_f-2N_c+4}{N_f}$
χ	$\bullet$	$\Box\Box$	$\bullet$	$\frac{N_f-2N_c+4}{N_f}$
$\tilde{\chi}$	$\bullet$	$\bullet$	$\overline{\Box\Box}$	$\frac{N_f-2N_c+4}{N_f}$

Table 2: *The matter content of the proposed magnetic description of OQCD-AS, with number of colours* $\tilde{N}_c = N_f - N_c + 4$.

Seiberg duality

Planar equivalence (provided that it holds) guarantees the existence of Seiberg duality between the two non-supersymmetric theories in the Veneziano limit ($N_c, N_f \to \infty$, N_f/N_c fixed).

The duality is also supported by string theory, even at finite N.

Moreover, the global anomalies match, at any N

$$SU(N_f)^3 = N_c$$

$$SU(N_f)^2 U(1)_R = \frac{-N_c^2 + 2N_c}{N_f}$$

$$U(1)_R = -N_c^2 + 3N_c$$

$$U(1)_R^3 = N_c \left(N_c - 1 - 2\frac{(N_c - 2)^3}{N_f^2} \right)$$

Seiberg duality

It is not clear whether the finite N duality holds.

If so, by requiring a vanishing beta function at the edges of the conformal window we obtain

$$\frac{3}{2}N_c - \frac{20}{3} \leq N_f \leq 3N_c + \frac{4}{3} \quad , \quad N_c \geq 5$$

Interestingly, for $SU(3)$ the antisymmetric fermion becomes equivalent to the fundamental, so the $SU(3)$ theory is QCD + scalars.

Unfortunately, the present duality does not hold for $SU(3)$.

Since the anomalies matching concern the fermions, it is not impossible that a more sophisticated version of the duality will hold even without the scalars, perhaps even for QCD.

Conclusions

I've presented two new examples of Seiberg duality.

The first can teach us about the origin of Seiberg duality:

Seiberg duality in a 3d setup originates from charge conjugation symmetry in 4d.

Is it possible that Seiberg duality in 4d is due to a simple symmetry in a higher dimensional theory ?

The other example concerns a non-supersymmetric QCD like theory.

It is possible that Seiberg duality exists for QCD as well ?