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IFUSP/P-946

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Outubro/1991

MESONIC EXCITATIONS IN THE LINEAR SIGMA MODEL

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Abstract: A semiclassical version of the linear sigma model is studied in a variational approach. Specifically we study the bosonic spectroscopy for both bound states and resonances. We give a quantitative descrption of the pion and its two observed resonances.

1. Supported by CNPq.

2. Partially supported by CNPq.

3. Supported by JNICT , Portugal.

1. INTRODUCTION

One of the main open problems in the area of particle physics is the understanding of the properties of hadrons starting from the microscopic theory of strong interactions, namely quantum chromodynamics (QCD). It is well known that chiral symmetry — one of the main characteristics of QCD — plays a special role to describe the low energy spectra of hadrons. In this sense much effort has been made studying chiral effective meson and, quark-meson lagrangians [1,2,3] which incorporate the most relevant symmetries of QCD, and reproduce the features of that theory in the non perturbative phase. The weakness of these models is that they do not possess the color confinement and asymptotic freedom properties of QCD. Its applicability is therefore limited to those hadronic phenomena which do not depend sensitively on details of the confinement mechanism. However, for many important aspects of low energy hadron physics, the symmetries of QCD are probably as important as confinement. It is within this context that we shall discuss both the merits and the limitations of approaches based on effective chiral models.

When using such simplified models, however, one has to make sure that the results are not very sensitive to the high energy behaviour of the model which is known to deviate radically from the behaviour of QCD. As pointed out by Perry and Cohen et al. [4,5], the breakdown of such theories occur when the momentum scale greatly exceeds other mass scales in the problem. In such regions one cannot relly upon theoretical results which ignore the internal structure of nucleons and mesons.

As far as asymptotic freedom is concerned, it is well known that phenomenological models lacking such property will present instabilities at the one loop level [6,7].

Among the large variety of models currently used for the purpose of describing low lying hadronic properties, the NJL model [8] seems to give a good description of light meson spectra [9,10]. More recently, attempts have been made to include meson resonances into such descriptions. The first pion resonance has been successfully accounted for in the context of the NJL model [11,12] by including RPA continuum models (ref. [11]) or equivalently by examining both poles and cuts of the pion propagator (ref. [12]).

In this paper we study the mesonic sector of the linear sigma model [13]. We present a quantitative description of the pion and its two experimentally measured resonant states [14].

The lagrangean of the chiral-symmetric sigma model with linear breaking is known to be renormalizable [13]. The first renormalization scheme for the sigma model without nucleons has been formulated by B.W. Lee [15] and generalizations to include fermions have been proposed afterwards [16,17]. Explicit one loop renormalization of the model is given in ref. [18].

This important feature of the model guarantees that one can obtain quantitative results at the one loop level, which are finite. However, as pointed out at the beginning of this introduction, due to the lack of asymptotic freedom, one is not free from instabilites, even when working with the renormalized version of the model [19]. We shall show in what follows that if one regards the linear sigma model as an effective model in the same sense as the NJL (and therefore only valid within a restricted configuration space) it is possible to obtain an adequate quantitative description of the discussed mesons and mesonic excitations. This is the philosophy behind our theoretical scheme.

The purpose of the present work is twofold: Firstly to introduce an effective hamiltonian based on the original linear sigma model which contains a cutoff parameter and allows for a unified description of the mesonic properties (mesons and their resonances). Secondly to introduce a method to obtain a complete set of stationary states corresponding to small amplitude motion. The method stems from traditional many body techniques in which the continuum solutions are naturally included. Energy weighted sum rules are introduced and used to study the collectivity of calculated states. Although we shall be working within the context of a particular model, the method is quite general.

In section 2 we present the effective chiral sigma model which will be described semiclassically in the sense that fermions are treated quantum mechanically whereas meson fields are treated classically. The added counterterms and their significance will be discussed. We illustrate the method in sections 3 and 4 where bound states and continuum states (for the scalar and pseudoscalar fields) are obtained. In section 5 we present the main results for pion and their resonances with the help of the energy weighted sum rule.

2. THE EFFECTIVE CHIRAL MODEL

The σ model is a field theoretical model originally introduced by Gell - Mann and Lévy [13] as an example of a phenomenological model which realizes one important characteristic feature of QCD,

chiral symmetry and partial conservation of the axial current. It involves a fermionic iso-doublet field of zero bare mass interacting with a triplet of pseudoscalar pions $\vec{\Psi}$ and a scalar field σ.

In the present section we present a semiclassical realization of the above mentioned model, and introduce the corresponding effective hamiltonian describing a system of N fermions occupying either positive or negative energy states, interacting with the classical fields corresponding to $\vec{\Psi}$ and σ. Our effective hamiltonian is written as

$$H = \sum_{j=1}^{N}[\vec{p}_j.\vec{\alpha}_j + g\beta_j(\sigma(x_j) + i\gamma_5(j)\vec{\tau}_j.\vec{\Psi}(x_j))]$$
$$+\frac{1}{2}\int d^3x\,(\Pi_\sigma^2 + \vec{\nabla}\sigma\cdot\vec{\nabla}\sigma + \Pi_\Psi^2 + \sum_{i=1}^{3}\vec{\nabla}\Psi_i\cdot\vec{\nabla}\Psi_i) + \frac{K}{4}\int d^3x\,(\sigma^2+\Psi^2-\sigma_0^2)^2$$
$$+2\xi\int\frac{d^3p d^3x}{(2\pi)^3}\sqrt{p^2+g^2(\sigma^2+\Psi^2)}\,\Theta(\Lambda^2-p^2) + \frac{\lambda-1}{2}\int d^3x\,(\Pi_\sigma^2+\Pi_\Psi^2)\,, \qquad (2.1)$$

where $\vec{\alpha}$, β and γ_5 are the usual Dirac matrices, $\vec{\tau}$ correspond to the matrices of the fundamental flavor representation SU(2), Π_σ and $\vec{\Pi}_\Psi$ are the conjugate momenta associated with the classical fields σ and $\vec{\Psi}$ respectively. The coupling constant g and the constants K and σ_0 will be fixed in the calculations in order to attribute physically reasonable masses for quarks and mesons. The factor ξ stands for the degeneracy of the system and will be taken equal to six (we shall be considering three colours). The last two terms in eq.(2.1) are regularization terms, which depend on a cutoff parameter Λ. The first one assures the stability of the ground state and the second term which contains the parameter λ allows for the definition of the scalar meson mass in the vacuum. Note that our hamiltonian is adjusted to the configuration space spanned by $|\vec{p}| < \Lambda$ and is invariant under a chiral rotation in the $\gamma_5 - isospin$ space. More precisely, the replacements

$$\beta \to \beta + i\vec{\epsilon}.[\gamma_5\vec{\tau},\beta]\,,$$
$$i\beta\gamma_5\vec{\tau} \to i\beta\gamma_5\vec{\tau} + i\epsilon_j[\gamma_5\tau_j, i\beta\gamma_5\vec{\tau}]\,,$$
$$\sigma \to \sigma - 2\vec{\epsilon}.\vec{\Psi}\,,$$
$$\vec{\Psi} \to \vec{\Psi} + 2\vec{\epsilon}\sigma\,,$$

where $\vec{\epsilon}$ is an infinetesimal constant vector, leave the hamiltonian, including counterterms, invariant.

In what follows we shall be considering an extended system of fermions, which are treated quantum mechanically in the mean field approximation, and interact with classical fields, as described above.

The ground state of the model is determined variationally. For the fermions we choose the family of Slater determinants $| \phi_0 >$, or equivalently, of density matrices ρ_0 obtained by occupying single-particle positive energy states with momentum lower than P_F and single-particle negative energy states with momentum lower than Λ ($\Lambda > P_F$). In homogeneous quark matter we can write

$$\rho_0 = \frac{1}{2}\left(I + \frac{\vec{p}.\vec{\alpha} + \beta M^*}{\sqrt{p^2 + M^{*2}}}\right)\Theta(P_F^2 - p^2) + \frac{1}{2}\left(I - \frac{\vec{p}.\vec{\alpha} + \beta M^*}{\sqrt{p^2 + M^{*2}}}\right)\Theta(\Lambda^2 - p^2) . \quad (2.2)$$

Here M^* is a variational parameter, representing the quark mass, to be fixed by the usual energy minimization procedure. The ground state energy can be immediately calculated and is given by

$$E = 2\xi \sum_p{}' \left(\sqrt{p^2 + M^{*2}} + \frac{M^*(g\sigma - M^*)}{\sqrt{p^2 + M^{*2}}}\right) + \frac{\lambda\Omega}{2}(\Pi_\sigma^2 + \Pi_\Psi^2)$$
$$+ \frac{K\Omega}{4}(\sigma^2 + \Psi^2 - \sigma_0^2)^2 + 2\xi \sum_{p<\Lambda} \sqrt{p^2 + g^2(\sigma^2 + \Psi^2)} , \quad (2.3)$$

where $\sum_p' = \sum_{p<P_F} - \sum_{p<\Lambda}$ and Ω is the normalization volume. Variations with respect to M^*, σ and $\vec{\Psi}$ in the static case give the following relations

$$M^* = g\sigma , \quad (2.4)$$

$$\frac{2\xi}{\Omega}\sum_p{}' \frac{g^2\sigma}{\sqrt{p^2 + g^2\sigma^2}} + K(\sigma^2 + \Psi^2 - \sigma_0^2)\sigma + \frac{2\xi}{\Omega}\sum_{p<\Lambda} \frac{g^2\sigma}{\sqrt{p^2 + g^2(\sigma^2 + \Psi^2)}} = 0 , \quad (2.5)$$

$$K(\sigma^2 + \Psi^2 - \sigma_0^2)\Psi_i + \frac{2\xi}{\Omega}\sum_{p<\Lambda} \frac{g^2\Psi_i}{\sqrt{p^2 + g^2(\sigma^2 + \Psi^2)}} = 0 . \quad (2.6)$$

The only solution of eq. (2.6) is $\Psi_i = 0$, if $\sigma \neq 0$. The corresponding energy density is

$$\frac{E}{\Omega} = \frac{2\xi}{\Omega}\sum_{p<P_F} \epsilon + \frac{K}{4g^4}(M^{*2} - g^2\sigma_0^2)^2 , \quad (2.7)$$

where $g\sigma$ has been replaced by M^*. We also get the self-consistency condition

$$\frac{K}{g^4}(M^{*2} - g^2\sigma_0^2)M^* = -\frac{2\xi M^*}{\Omega}\sum_{p<P_F} \frac{1}{\epsilon} , \quad (2.8)$$

where $\epsilon = \sqrt{p^2 + M^{*2}}$.

From eq. (2.8) we see that the vacuum state ($P_F = 0$) is characterized by $M^* = M = g\sigma_0$ which is the constituent quark mass. If we are dealing with extended quark matter ($P_F \neq 0$) this effective mass will change according to the self consistency condition eq.(2.8). This implies that the energy per volume will be a function of P_F (or the effective quark mass) for a fixed set of parameters K, σ_0

and g. In Fig. 1 we show the energy per unit volume, eq.(2.7), as a function of M^* for some values of P_F. For small values of P_F, there are only two solutions for eq. (2.8), one corresponding to $M^* = 0$ (which remains as a solution for any values of P_F), and one correspondig to $M^* \neq 0$ which is the minimal energy solution. However, as P_F grows the situation changes and we come into a region where eq. (2.8) provides three solutions. One of them corresponds to a maximum of the energy and must be discarded, the other two solutions correspond to a local and an absolute minimum of the energy. If we keep increasing P_F we will obtain only the solution $M^* = 0$. The symmetry is restored at $P_F = P_{FC}$ for which the two minima are degenerate.

The energy per particle represented in Fig. 2, refers to the absolute minimum solution. The curve is continuous, in spite of the discontinuous behaviour of M^* which is represented in Fig. 3 as a function of P_F. The reason for this becomes clear from Fig. 1, observing that at the point where the discontinuity in M^* occurs, namely P_{FC}, the two minima are degenerate and from that point onwards we have always $M^* = 0$. Note that the energy per particle exhibits a pronounced minimum at the Fermi momentum $P_F = 1.55 fm^{-1}$ (for the values of the parameters given in the figure). This value is compatible with the nucleon radius.

It is also important to emphasize that the inclusion of the Dirac sea plays an essential role in insuring the stability of the vacuum against fluctuations of M^* [20].

3. SCALAR AND PSEUDOSCALAR MESON SPECTRA FOR BOUND STATES

In this section we present the time evolution of small homogeneous excitations (carrying zero momentum) around equilibrium. In considering this, the last counterterm in eq. (2.1) is very important to guarantee that the scalar meson mass in the vacuum corresponds to its physical mass (or the mass we choose as physically reasonable).

The time evolution of a density matrix $\rho(t)$ corresponding to a Slater determinant slightly displaced from equilibrium can be written as

$$\rho(t) = e^{iS(t)}\rho_0 e^{-iS(t)} , \quad (3.1)$$

where $S(t)$ is a hermitian, one-body, time-dependent operator. For small amplitude motion, it is sufficient to consider the effect of $S(t)$ up to second order. Obviously, due to the coupling terms in the hamiltonian (eq. (2.1)) there will also be fluctuations in the scalar and pseudoscalar classical fields, respectively $\delta\sigma$ and $\delta\Psi_i$, which are non vanishing and time dependent. Consistently we shall treat these terms up to second order only. In order to obtain the equations of motion and corresponding eigenfrequencies and eigenvectors we follow the method presented in ref. [9].

We are considering a mean field reduction of the linear σ model. This means that the time dependence of the system we are considering is provided by the time-dependent Hartree-Fock (TDHF) equations. It is well known that these equations may be derived from a lagrangian formalism. The TDHF lagrangian describing small amplitude oscillations around the equilibrium state is

$$L = \frac{i}{2}tr(\rho_0[S,\dot{S}]) - \frac{1}{2}tr(\rho_0[S,[h_0,S]]) - itr(\rho_0[S,\delta h]) + \frac{\Omega}{2}\left(\frac{(\delta\dot{\sigma})^2}{\lambda} + \frac{(\delta\dot{\Psi}_i)^2}{\lambda}\right)$$
$$- \frac{K\Omega}{2g^2}[(3M^{*2} - M^2)(\delta\sigma)^2 + (M^{*2} - M^2)(\delta\Psi_i)^2] - \xi g^2 \sum_{p<\Lambda}\left(\frac{p^2}{\epsilon^3}(\delta\sigma)^2 + \frac{1}{\epsilon}(\delta\Psi_i)^2\right), \quad (3.2)$$

where $h_0, \delta h$ are given by

$$h_0 = \vec{p}.\vec{\alpha} + \beta M^*, \quad (3.3)$$

$$\delta h = \beta g\delta\sigma + ig\beta\gamma_5\vec{\tau}.\delta\vec{\Psi}. \quad (3.4)$$

The use of TDHF lagrangian eq.(3.2) is meaningful because the stability of the ground state ρ_0 is insured by the inclusion of the Dirac sea, by appropriate counterterms and, as we will see, by a cutoff Λ.

The generators of the scalar and pseudoscalar homogeneous excitations for zero momentum transfer are respectively given by

$$S_\sigma = \vec{p}.\vec{\alpha}\Phi_1(p^2,t) + i\beta\vec{p}.\vec{\alpha}\Phi_2(p^2,t), \quad (3.5)$$

$$S_\Psi = i\beta\gamma_5\vec{\tau}.\vec{S}_1(p^2,t) + \gamma_5\vec{\tau}.\vec{S}_2(p^2,t). \quad (3.6)$$

Inserting eq. (3.5) in eq. (3.2) leads to the following equations of motion

$$\dot{\Phi}_2 + 2M^*\Phi_1 = 0, \quad (3.7a)$$

$$M^*\dot{\Phi}_1 - 2\epsilon^2\Phi_2 + g\delta\sigma = 0, \quad (3.7b)$$

$$\frac{\delta\ddot{\sigma}}{\lambda} + \frac{K}{g^2}(3M^{*2} - M^2)\delta\sigma + \frac{2\xi g^2\delta\sigma}{\Omega}\sum_{p<\Lambda}\frac{p^2}{\epsilon^3} + \frac{4\xi g\Phi_2}{\Omega}\sum_p{}' \frac{p^2}{\epsilon^3} = 0. \quad (3.7c)$$

The eigenfrequencies and eigenmodes are now easily obtained from the above equations. For the eigenfrequencies we get the following dispersion relation, corresponding to the scalar mode

$$\frac{\omega_\sigma^2}{\lambda} = \frac{K}{g^2}(3M^{*2} - M^2) + \frac{2\xi g^2}{\Omega}\sum_{p<\Lambda}\frac{p^2}{\epsilon^3} + \frac{8\xi g^2}{\Omega}\sum_p{}' \frac{p^2}{\epsilon(4\epsilon^2 - \omega_\sigma^2)}. \quad (3.8)$$

When describing the vacuum state we should have $\omega_\sigma = m_\sigma$, the scalar meson mass. We choose $m_\sigma = 2M$ (the justification for this choice relies on the comparison with the results obtained for the same spectrum in the NJL model ref. [9–12]). This requirement fixes the relation between K and g^2 to be

$$\frac{K}{g^2} = 2, \quad (3.9)$$

and also determines the parameter λ :

$$\frac{1}{\lambda} = 1 - \frac{\xi g^2}{2\Omega}\sum_{p<\Lambda}\frac{1}{\epsilon_0^3}, \quad (3.10)$$

where $\epsilon_0 = \sqrt{p^2 + M^2}$. From this equation we see that $\frac{1}{\lambda}$ may assume any value from 1 to $-\infty$ depending on the chosen value for Λ (from 0 to ∞ respectivelly). Finally, the dispersion relation for the scalar meson mass reads

$$\omega_\sigma^2 = 6M^{*2} - 2M^2 + \frac{\xi g^2}{2\Omega}\sum_{p<\Lambda}\left(\frac{4p^2}{\epsilon^3} + \frac{\omega_\sigma^2}{\epsilon_0^3}\right) + \frac{8\xi g^2}{\Omega}\sum_p{}' \frac{p^2}{\epsilon(4\epsilon^2 - \omega_\sigma^2)}. \quad (3.11)$$

There are two types of solution of eq. (3.11) [21]. If $\omega_\sigma = \omega_{\sigma z}$, with $0 < \omega_{\sigma z} < 2\epsilon_F$, or $\omega_{\sigma z} > 2\epsilon_\Lambda$, there are two collective discrete modes (bound states). On the other hand, if $2\epsilon_F < \omega_\sigma < 2\epsilon_\Lambda$, there is a continuum of solutions analogous to those which will be discussed in detail in section 4, in connection with the pseudoscalar excitation.

This dispersion relation is independent of the cutoff parameter Λ in the limit $\Lambda \to \infty$, but if Λ is too large one imaginary root appears [21] related to Perry's instability. However for values of the cutoff below a given critical value, of the order of $2M$, $\frac{1}{\lambda}$ is positive (cf. (3.10)) and all roots are real implying dynamical stability of the system. This is the main reason why we work with a cutoff. Fig. 3 illustrates diagramatically the solutions for the dispersion relation (3.11) for $P_F = 0$ and for a suitable value of Λ.

The discrete scalar mass spectrum is shown in Fig. 4 and exhibits a qualitative behaviour very similar to the one found in ref. [9] for the NJL model. In particular, a reduction of the scalar meson effective mass in a hadronic medium is predicted. This effect increases with the density. The main difference is connected with the way in which chiral symmetry is restored in the two models: in the NJL model (see ref. [9], Fig. 5) chiral symmetry is restored in a continuous fashion as the quark density matter increases, whereas in the present model a discontinuous behaviour may occur (see Fig. 4). There remains however a small quantitative difference: whereas in the NJL model the scalar meson mass is always given by $\omega_\sigma = 2M^*$, this is not the case any more in the present model, although the difference is not too large as shown in Fig. 4.

The scalar RPA eigenmode for the bound states is given by

$$\Phi_1^{(\pm)} = -\frac{\pm ig\omega_{\sigma z}}{M^*}\,\frac{\sigma^{(\pm)}}{4\epsilon^2 - {\omega_{\sigma z}}^2}\,, \tag{3.12a}$$

$$\Phi_2^{(\pm)} = \frac{2g}{4\epsilon^2 - {\omega_{\sigma z}}^2}\,\sigma^{(\pm)}\,, \tag{3.12b}$$

$$\sigma^{(\pm)} = \frac{1}{\sqrt{2\Omega\mid\omega_{\sigma z}\mid}}\,\frac{1}{\sqrt{\frac{1}{\lambda} - \frac{8\xi g^2}{\Omega}\sum_p{}'\frac{p^2}{\epsilon(4\epsilon^2 - {\omega_{\sigma z}}^2)^2}}}\,, \tag{3.12c}$$

$$\Pi_\sigma^{(\pm)} = \pm\frac{i\omega_{\sigma z}}{\lambda}\,\sigma^{(\pm)}\,. \tag{3.12d}$$

The modes are normalized according to

$$i\Omega\left(\sigma^{(\pm)}\Pi_\sigma^{(\pm)^*} - \sigma^{(\pm)^*}\Pi_\sigma^{(\pm)} - \frac{4\xi M^*}{\Omega}\sum_p{}'\frac{p^2}{\epsilon}(\Phi_1^{(\pm)}\Phi_2^{(\pm)^*} - \Phi_1^{(\pm)^*}\Phi_2^{(\pm)})\right) = \pm\frac{\omega_{\sigma z}}{\mid\omega_{\sigma z}\mid}\,. \tag{3.13}$$

We turn now to the description of the pseudoscalar excitation. Its generator is given by eq. (3.6) and a dispersion relation can be obtained in a similar way as in the case of the scalar mode (note, however that the parameter λ has already been fixed). We get

$$\omega_\Psi^2 = 2(M^{*2} - M^2) + \frac{\xi g^2\omega_\Psi^2}{2\Omega}\sum_{p<\Lambda}\left(\frac{1}{\epsilon_0^3} - \frac{1}{\epsilon(\epsilon^2 - \omega_\Psi^2/4)}\right) + \frac{2\xi g^2}{\Omega}\sum_{p<P_F}\frac{\epsilon}{\epsilon^2 - \omega_\Psi^2/4}\,. \tag{3.14}$$

For values of the Fermi momentum P_F such that $P_F < P_{FC}$ the above equation has one collective low-energy solution $\omega_\Psi = 0$. This solution corresponds to the pseudoscalar Goldstone boson. For $P_F \geq P_{FC}$, the chiral symetry is restored and the scalar meson and pion masses are degenerate as expected. This result is also displayed in Fig. 4.

In order to generate normalizable RPA like states of the pionic excitation it is convenient to eliminate zero-valued frequencies. This is achieved if we introduce a perturbative term in the hamiltonian eq.(2.1) which explicitly breaks the chiral symmetry. This term is simply

$$W = \Omega\sigma c\,. \tag{3.15}$$

This term will not have any influence on the equilibrium value of the $\vec{\Psi}$ field, but now the expectation value of the scalar field in the vacuum will be given by

$$M^* = M = g\sigma_0 + \frac{c}{4g\sigma_0^2}\,, \tag{3.16}$$

as a consequence an extra c/σ_0 term appears in the r.h.s. of eq. (3.14). We can adjust c/σ_0 in such a way as to yield $\omega_\psi = 138MeV$ the pion mass in the vacuum ($P_F = 0$). We get $\sqrt{c/\sigma_0} = 140MeV$, for the values of M and g used in the figures. The presence of this new term (eq.(3.15)) in the hamiltonian has also the consequence of removing the degeneracy in the scalar and pseudoscalar spectrum.

Again we have a discrete solution of eq. (3.14), $\omega_\psi = \omega_\pi$, and the pseudoscalar RPA eigenmodes are given by

$$\vec{\Psi}^{(\pm)} = \frac{\vec{n}}{\sqrt{2\Omega\mid\omega_\pi\mid}}\,\frac{1}{\sqrt{\frac{1}{\lambda} - \frac{8\xi g^2}{\Omega}\sum_p{}'\frac{\epsilon}{(4\epsilon^2 - \omega_\pi^2)^2}}}\,, \tag{3.17a}$$

$$\vec{\Pi}_\Psi^{(\pm)} = \pm\frac{i\omega_\pi}{\lambda}\,\vec{\Psi}^{(\pm)}\,, \tag{3.17b}$$

$$\vec{S}_1^{(\pm)} = \pm\frac{ig\omega_\pi}{4\epsilon^2 - \omega_\pi^2}\,\vec{\Psi}^{(\pm)}\,, \tag{3.17c}$$

$$\vec{S}_2^{(\pm)} = \frac{2g\epsilon^2}{M^*}\,\frac{\vec{\Psi}^{(\pm)}}{4\epsilon^2 - \omega_\pi^2}\,, \tag{3.17d}$$

$\vec{n}$ being an arbitrary unit isovector. These modes are normalized according to

$$i\Omega(\vec{\Psi}^{(\pm)}\cdot\vec{\Pi}_\Psi^{(\pm)*} - \vec{\Psi}^{(\pm)*}\cdot\vec{\Pi}_\Psi^{(\pm)} + \frac{4\xi M^*}{\Omega}\sum_p{}'\frac{1}{\epsilon}(\vec{S}_2^{(\pm)*}\cdot\vec{S}_1^{(\pm)} - \vec{S}_2^{(\pm)}\cdot\vec{S}_1^{(\pm)*})) = \pm\frac{\omega_\pi}{\mid\omega_\pi\mid}\,. \tag{3.18}$$

We are now in a position to calculate the pion decay constant by using the above presented eigenmodes. This is done following a very simple and commonly used practice in nuclear structure calculations, which is well suited for the calculation of such a quantity. The pion decay constant is defined by the pion to vacuum transition amplitude induced by axial charges [22]

$$\langle 0\mid Q_5^j\mid\pi^k\rangle = i\sqrt{\frac{\Omega\omega_\pi}{2}}f_\pi\delta_{kj}\,, \tag{3.19}$$

where Q_5^j are the charge operators related to the time component of the axial current :

$$\vec{Q}_5 = \sum_{j=1}^N\gamma_5(j)\frac{\vec{\tau}(j)}{2} + \int d^3x\,\frac{M^*}{g}\,\vec{\Pi}_\Psi\,, \tag{3.20}$$

for a static scalar field.

In our description the fields $\vec{\Pi}_\Psi, \vec{\Psi}$ are classical fields. The $\vec{\Pi}_\Psi$ field being canonically conjugate to $\vec{\Psi}$ is to be understood as the generator of a fluctuation of the $\vec{\Psi}$ field. The axial charges are therefore associated with a special form of the generator in eq. (3.6) given by

$$S_{2i}^{(j)} = \frac{1}{2}\delta_{ij} , \tag{3.21a}$$

$$\vec{S}_1 = 0 , \tag{3.21b}$$

and $\vec{\Psi}, \vec{\Pi}_\Psi$ fields given by

$$\Psi_i^{(j)} = \frac{M^*}{g}\delta_{ij} , \tag{3.21c}$$

$$\vec{\Pi}_\Psi = 0 , \tag{3.21d}$$

where j is associated with the component j of the axial charges (eq.(3.20)).

Such a state may be expanded in our normal RPA modes according to

$$\begin{pmatrix} M^*/g \\ 0 \\ 0 \\ 1/2 \end{pmatrix} = c_+ \begin{pmatrix} \Psi_j^{(+)} \\ \Pi_j^{(+)} \\ S_{1j}^{(+)} \\ S_{2j}^{(+)} \end{pmatrix} + c_- \begin{pmatrix} \Psi_j^{(-)} \\ \Pi_j^{(-)} \\ S_{1j}^{(-)} \\ S_{2j}^{(-)} \end{pmatrix} , \tag{3.22}$$

where the signs + and − are related with positive and negative frequencies respectively, and $c_- = c_+^*$. It should be stressed that in eq.(3.22) the amplitudes of the components over continuum states vanish when the chiral symmetry is restored. Using eq. (3.18) we can write the coefficient c_+ as

$$c_+ = \sqrt{\frac{\Omega \mid \omega_\pi \mid}{2}} \frac{M^*/g}{\sqrt{\frac{1}{\lambda} - \frac{8\xi g^2}{\Omega}\sum_p{}' \frac{\epsilon}{(4\epsilon^2 - \omega_\pi^2)^2}}} \left(\frac{1}{\lambda} - \frac{2\xi g^2}{\Omega} \sum_p{}' \frac{1}{\epsilon(4\epsilon^2 - \omega_\pi^2)} \right) . \tag{3.23}$$

This coefficient can be interpreted as follows

$$c_+^* = -i < 0 \mid Q_5^j \mid \pi^j > = \sqrt{\frac{\Omega\omega_\pi}{2}} f_\pi , \tag{3.24}$$

and it yields the following expression for the pion decay constant (in the bound state we have $\omega_\pi \ll 2\epsilon$)

$$f_\pi = \frac{M^*}{g}\sqrt{\frac{1}{\lambda} - \frac{\xi g^2}{2\Omega}\sum_p{}' \frac{1}{\epsilon^3}} . \tag{3.25}$$

In the vacuum, and using eq. (3.10)

$$f_\pi = \frac{M}{g} , \tag{3.26}$$

holds exactly. Equation (3.26) agrees with the Goldberger-Treiman relation. In order to reproduce the experimental pion decay constant, for instance $f_\pi = 93MeV$, for a constituent quark mass

$M_{u,d} = 320MeV$ we get for the coupling constant $g = 3.44$. These were the values used in all numerical calculations.

4. PSEUDOSCALAR MESON SPECTRA IN THE CONTINUUM

In order to study the RPA normal modes including the continuum it is convenient to write the Lagrangian eq.(3.2) in a dimensionless form. Inserting eq.(3.6) in eq.(3.2), and using the dimensionless quantities

$$\vec{Q} = \delta\vec{\Psi}/M , \tag{4.1a}$$

$$\vec{P} = \vec{\Pi}_\Psi/M^2 = \delta\dot{\vec{\Psi}}/\lambda M^2 , \tag{4.1b}$$

$$x = \epsilon^2/M^2 \quad , \quad \epsilon = \sqrt{p^2 + M^{*2}} \tag{4.1c}$$

$$m = M^*/M , \tag{4.1d}$$

$$f(x) = \frac{\xi}{2\pi^2}\sqrt{\frac{x - m^2}{x}} \tag{4.1e}$$

$$4\alpha^2 = 2\left(m^2 - 1 + \frac{c}{2\sigma_0 M^2}\right) + g^2\int_{m^2}^{x_\Lambda} dx f(x) . \tag{4.1f}$$

we get for the pseudoscalar-isovector excitation the TDHF lagrangian

$$\frac{L_\psi}{M^4\Omega} = \frac{1}{2}(\vec{P}^* \cdot \dot{\vec{Q}} - \dot{\vec{P}} \cdot \vec{Q}^*) - m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{S}_2^* \cdot \dot{\vec{S}}_1 - \vec{S}_1^* \cdot \dot{\vec{S}}_2) - 2\int_{x_F}^{x_\Lambda} dx f(x)(m^2 \mid \vec{S}_2 \mid^2 + x \mid \vec{S}_1 \mid^2)$$
$$+ gm\vec{Q}^* \cdot \int_{x_F}^{x_\Lambda} dx f(x)\vec{S}_2 + gm\vec{Q} \cdot \int_{x_F}^{x_\Lambda} dx f(x)\vec{S}_2^* - \frac{1}{2}(\lambda \mid \vec{P} \mid^2 + 4\alpha^2 \mid \vec{Q} \mid^2) , \tag{4.2}$$

where all time derivatives are related to $\tau = M_0 t$.

From the Euler-Lagrange equations, the following equations are obtained for the normal modes:

$$i\omega\vec{Q}_\omega = \lambda\vec{P}_\omega , \tag{4.3a}$$

$$i\omega\vec{P}_\omega = -4\alpha^2\vec{Q}_\omega + 2mg\int_{x_F}^{x_\Lambda} dx f(x)\vec{S}_{2\omega}(x) , \tag{4.3b}$$

$$i\omega\vec{S}_{1\omega}(x) = g\vec{Q}_\omega - 2m\vec{S}_{2\omega}(x) , \tag{4.3c}$$

$$i\omega\vec{S}_{2\omega}(x) = \frac{2x}{m}\vec{S}_{1\omega}(x) . \tag{4.3d}$$

As mentioned before, there are always two types of solutions of eqs. (4.3). Two discrete modes, $\omega = \omega_z$, if $\omega_z^2 < 4x_F$ or $\omega_z^2 > 4x_\Lambda$ and a continuum of solutions if $4x_F < \omega^2 < 4x_\Lambda$.

The dispersion relation for bound states is

$$\frac{\omega_z^2}{4\lambda} - \alpha^2 + \frac{g^2}{4}\int_{x_F}^{x_\Lambda} dx f(x)\frac{x}{x-\omega_z^2/4} = 0 \tag{4.4}$$

and the discrete modes are described by

$$\vec{Q}_\pm = \vec{n}\,, \tag{4.5a}$$

$$\vec{P}_\pm = \pm\frac{i\omega_z}{\lambda}\vec{n}\,, \tag{4.5b}$$

$$\vec{S}_{1\pm}(x) = \pm\frac{i\omega_z g}{4}\frac{\vec{n}}{x-\omega_z^2/4}\,, \tag{4.5c}$$

$$\vec{S}_{2\pm}(x) = \frac{gx}{2m}\frac{\vec{n}}{x-\omega_z^2/4}\,, \tag{4.5d}$$

$\vec{n}$ being an arbitrary unit vector and the only difference with eqs.(3.17) is the fact that now the eigenvectors are not normalized. In the continuum the normal modes are given by

$$\vec{Q}_\omega = -\frac{2m}{g}a(\omega^2/4)\vec{n}\,, \tag{4.6a}$$

$$\vec{P}_\omega = -\frac{i\omega}{\lambda}\frac{2m}{g}a(\omega^2/4)\vec{n}\,, \tag{4.6b}$$

$$\vec{S}_{1\omega}(x) = i\omega\frac{m}{2x}\vec{S}_{2\omega}(x)\,, \tag{4.6c}$$

$$\vec{S}_{2\omega}(x) = \left(\delta(\omega^2/4 - x) + \frac{x a(\omega^2/4)}{\omega^2/4 - x}\right)\vec{n}\,, \tag{4.6d}$$

where $a(\omega^2/4)$ satisfies the equation

$$a(\omega^2/4) = \frac{g^2 f(\omega^2/4)}{\omega^2/\lambda - 4\alpha^2 + g^2\int_{x_F}^{x_\Lambda} dx f(x)\frac{x}{x-\omega^2/4}}\,. \tag{4.7}$$

In what follows and in eq.(4.7), integrals involving the factor $1/(x-\omega^2/4)$ have to be interpreted as principal value integrals.

It can be seen from eqs.(4.3) that the normal modes are orthogonal, and using eqs.(4.5) and (4.6) we get the following orthogonality relations:

$$i\left(\vec{P}^*_{\omega'}\cdot\vec{Q}_\omega - \vec{Q}^*_{\omega'}\cdot\vec{P}_\omega - 2m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{S}^*{}_{2\omega'}\cdot\vec{S}_{1\omega} - \vec{S}^*{}_{1\omega'}\cdot\vec{S}_{2\omega})\right) = 2m^2\frac{f(\omega^2/4)}{\omega^2/4}\delta(\omega/2-\omega'/2)\,, \tag{4.8a}$$

$$i\left(\vec{Q}_\pm\cdot\vec{P}^*{}_\omega - \vec{P}_\pm\cdot\vec{Q}^*{}_\omega - 2m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{S}_{1\pm}\cdot\vec{S}^*{}_{2\omega} - \vec{S}_{2\pm}\cdot\vec{S}^*{}_{1\omega})\right) = 0\,, \tag{4.8b}$$

$$i\left(\vec{Q}_\pm\cdot\vec{P}^*{}_\pm - \vec{P}_\pm\cdot\vec{Q}^*{}_\pm - 2m\int_{x_f}^{x_\Lambda} dx f(x)(\vec{S}_{1\pm}\cdot\vec{S}^*{}_{2\pm} - \vec{S}_{2\pm}\cdot\vec{S}^*{}_{1\pm})\right) = \pm\eta_z\,, \tag{4.8c}$$

where

$$\eta_z = 2\omega_z\left(\frac{1}{\lambda} + \frac{g^2}{4}\int_{x_F}^{x_\Lambda} dx f(x)\frac{x}{(x-\omega_z^2/4)^2}\right)\,. \tag{4.9}$$

Following the arguments of van Kampem [23], for the electron plasma, and of ref. [24] for the nuclear case, it may be shown that this set of solutions is complete.

5. THE PION RESONANCES

It is well known that sum rules can be defined within the RPA approximation [25, 26]. They are very useful in providing a quantitative measure of the collectivity of physical states. We wish now to investigate the excitation of the vacuum due to some operador D, which represents an external field.

In our formalism the operator D is the generator of the initial fluctuation. The energy weighted sum rule (EWSR) m_1 states that [27]

$$m_1 = \sum_{r=0}^{\infty}\omega_r|\langle r|D|0\rangle|^2 = \frac{1}{2}\langle 0|[D,[H,D]]|0\rangle\,, \tag{5.1}$$

where $|r\rangle$ represent an exact eigenstate of the hamiltonian H, ω_r the excitation energies ($\omega_r = E_r - E_0$) and $|0\rangle$ is the vacuum state. The sum includes the discrete as well as continuum states.

We chose some initial condition

$$\Psi_0 = \begin{bmatrix}\vec{Q}(0)\\ \vec{P}(0)\\ \vec{S}_1(x,0)\\ \vec{S}_2(x,0)\end{bmatrix} = \begin{bmatrix}\vec{Q}_0\\ \vec{P}_0\\ \vec{\Pi}_1(x)\\ \vec{\Pi}_2(x)\end{bmatrix}\,, \tag{5.2}$$

and identify this generator with a transition operator $\hat{D}$.

Since the set of the normal modes is complete we can expand Ψ_0 in that base. There is a function $c(\omega)$ and numbers C_+, C_- such that,

$$\begin{bmatrix}\vec{Q}_0\\ \vec{P}_0\\ \vec{\Pi}_1(x)\\ \vec{\Pi}_2(x)\end{bmatrix} = 2\int_{2\sqrt{x_F}}^{2\sqrt{x_\Lambda}} c(\omega)\begin{bmatrix}\vec{Q}_\omega\\ \vec{P}_\omega\\ \vec{S}_{1\omega}(x)\\ \vec{S}_{2\omega}(x)\end{bmatrix} d\omega + \sum_z\left[C_{+z}\begin{bmatrix}\vec{Q}_+\\ \vec{P}_+\\ \vec{S}_{1+}(x)\\ \vec{S}_{2+}(x)\end{bmatrix}_z + C_{-z}\begin{bmatrix}\vec{Q}_-\\ \vec{P}_-\\ \vec{S}_{1-}(x)\\ \vec{S}_{2-}(x)\end{bmatrix}_z\right]\,. \tag{5.3}$$

The factor 2 in the first term in the right hand side of eq. (5.3) appear because we have to consider the expansion in the ranges $2\sqrt{x_F} < \omega < 2\sqrt{x_\Lambda}$ and $-2\sqrt{x_\Lambda} < \omega < -2\sqrt{x_F}$. The sum $\sum_z$ means that we have to consider the two discrete normal modes each one corresponding to the discrete energies $\omega_z < 2\sqrt{x_F}$ and $\omega_z > 2\sqrt{x_\Lambda}$. The quantities $\sqrt{\eta(\omega)}\,c(\omega)$, $\sqrt{\eta_z}\,C_{+z}$, where $\eta(\omega)$, η_z

denotes the norms of the eigensolutions, represent matrix elements of D, $\langle r|D|0\rangle = C_r\sqrt{\eta_r}$. The EWSR (5.1) is preserved in the mean field approximation and may be written as

$$m_1 = \sum_{r=0}^{\infty} \omega_r\, \eta_r\, |C_r|^2 = \frac{1}{2}\,\langle 0|\,[D\,,[H,D]]\,|0\rangle\,. \tag{5.3}$$

For completeness sake, we observe that the solution of the initial value problem defined by eq.(5.2) is

$$\begin{bmatrix} \vec{Q}(\tau) \\ \vec{P}(\tau) \\ \vec{S}_1(x,\tau) \\ \vec{S}_2(x,\tau) \end{bmatrix} = 2\int_{2\sqrt{x_F}}^{2\sqrt{x_\Lambda}} c(\omega) \begin{bmatrix} \vec{Q}_\omega \\ \vec{P}_\omega \\ \vec{S}_{1\omega}(x) \\ \vec{S}_{2\omega}(x) \end{bmatrix} e^{-i\omega\tau} d\omega + \sum_z \left[C_{+z} \begin{bmatrix} \vec{Q}_+ \\ \vec{P}_+ \\ \vec{S}_{1+}(x) \\ \vec{S}_{2+}(x) \end{bmatrix}_z e^{-i\omega_z\tau} + C_{-z} \begin{bmatrix} \vec{Q}_- \\ \vec{P}_- \\ \vec{S}_{1-}(x) \\ \vec{S}_{2-}(x) \end{bmatrix}_z e^{i\omega_z\tau} \right].$$

Following van Kampen [23] and making use of appropriate auxiliary functions [28] it may be shown that the function $c(\omega)$

$$c(\omega) = \frac{\bar{c}(\omega)}{\left(1+\pi^2 a^2(\omega^2/4)\,\frac{\omega^4}{16}\right)}\,, \tag{5.4}$$

$$\bar{c}(\omega) = \frac{i\omega^2/4}{4m^2 f(\omega^2/4)}\left(\vec{Q}_0\cdot\vec{P}^*_\omega - \vec{P}_0\cdot\vec{Q}^*_\omega - 2m\int_{x_F}^{x_\Lambda} dx\, f(x)\left((\vec{\Pi}_1(x)\cdot\vec{S}^*_{2\omega}(x) - \vec{\Pi}_2(x)\cdot\vec{S}^*_{1\omega}(x)\right)\right) \tag{5.5}$$

and

$$C_{\pm z} = \pm\frac{i}{\eta_z}\left(\vec{P}^*_\pm\cdot\vec{Q}_0 - \vec{Q}^*_\pm\cdot\vec{P}_0 - 2m\int_{x_F}^{x_\Lambda} dx\, f(x)\left(\vec{\Pi}_1(x)\cdot\vec{S}^*_{2\pm}(x) - \vec{\Pi}_2(x)\cdot\vec{S}^*_{1\pm}(x)\right)\right). \tag{5.6}$$

The coefficients $\bar{c}(\omega)$, $C^*_{\pm z}$ satisfy, therefore, the EWSR [28]:

$$4m^2\int_{2\sqrt{x_F}}^{2\sqrt{x_\Lambda}} \frac{d\omega\, 4|\bar{c}(\omega)|^2 f(\omega^2/4)}{\omega\,(1+\pi^2(\omega^4/16)\,a^2(\omega^2/4))} + \sum_z \frac{\omega_z}{2}\,\eta_z\left(|C_{+z}|^2 + |C_{-z}|^2\right) =$$

$$= \frac{1}{2}\left(\lambda\vec{P}_0^2 + 4\alpha^2\vec{Q}_0^2\right) + 2\int_{x_F}^{x_\Lambda} dx\, f(x)(m^2\vec{\Pi}_2^2 + x\vec{\Pi}_1^2) - 2gm\vec{Q}_0\cdot\int_{x_F}^{x_\Lambda} dx\, f(x)\vec{\Pi}_2(x) \tag{5.7}$$

The strength function representing the pionic mode in the $q\bar{q}$ continuum is

$$s_\pi(\omega) = \frac{16m^2|\bar{c}(\omega)|^2 f(\omega^2/4)}{\omega\,(1+\pi^2(\omega^4/16)\,a^2(\omega^2/4))}\,. \tag{5.8}$$

We investigate now the meson mass spectra, which we identify with the excitations of the vacuum. We take, therefore, in our expressions $P_F = 0$ and $m = M^*/M = 1$.

We choose three initial conditions which as we said before, represent different transition operators D. For each operator D, we calculate the expansion coefficients given by (5.4) to (5.6), and obtain the EWSR and the strength exhausted by each mode (the discrete and continuum).

In table 1, we display our results for pseudoscalar-isovector excitations corresponding to three different transition operators.

As discussed before, we must work with a finite cutoff $\Lambda < 2.018M$ in order to avoid the imaginary eigenfrequency and to insure stability. In this situation we not only have the two low discrete eigenfrequencies ($\omega_\pi = \pm\, 138\, MeV$) but also two new discrete eigenfrequencies lying above the continuum, $\omega = \pm\omega_>$ with $\omega_>^2 > 4x_\Lambda$. The energy of these new modes are very sensitive to the cutoff. If we fix $\Lambda = 1.57M$ we get $\omega_> = 1771\, MeV$. As we can see in table 1 it is possible to identify the low discrete eigenfrequency with the experimental mass of the pion ($m_\pi = 138\, MeV$) and the high discret eigenfrequency with the mass of the pion's second resonance π (1770). In all these three initial conditions we have a distribution of strength in the continuum. We identify the maximum of the strength function with the first pion resonance π (1300). The percentage of the EWSR exhausted by the continuum and the position of the maximum depends on the transition operator. The operator which does excite the first pion resonance more strongly is $\vec{D} = i\beta\gamma_5\vec{\tau}$ and we get 14.2% of EWSR in the continuum states. Such results agree with the one obtained in the context of the NJL model [11,12]. Fig. 5 shows the strength distribution. It is very broad but most of the strength is concentrated in a energy range ($800 < \omega < 1200$) which covers the experimental value ($(1300 \pm 100)\, MeV$).

We may say that the pion particle is a very collective bound state because with the axial charge as the transition operator, 98.3% of EWSR lies in this low energy mode $\omega_\pi = 138\, MeV$. The reason for that is clear, because in our Hamiltonian, chiral symmetry is almost exact and so $\vec{Q}_5$ has a strong overlap with the corresponding RPA operator. We also remark that the EWSR for the operator $\vec{Q}_5$ is closely related to the so called GMOR relation [10, 11].

Moreover, we can identify the high energy discrete mode with the second pion resonance: the transition operator $\vec{D} = i\gamma_5\vec{\tau}$ exhausts almost all the EWSR for this state.

We have obtained a unified description of the pion and of its resonances.

6. FINAL DISCUSSION

Our results for the pseudoscalar-isovector excitation allow us to say that with a small cutoff ($\Lambda/M \simeq 1.57$ *to* 2) we get a good description for the pion and their resonances in the linear sigma model: a reasonable agreement with the experimental values was obtained. However, although the cutoff is small it corresponds to a cut in the complex plane between $2M$ and $2M\sqrt{3.56} \simeq 4M$.

Another interesting conclusion of our work is that by means of traditional many body techniques we were able to obtain a set of stationary small amplitude eigenmodes which include the continuum in a natural way and satisfy orthogonal conditions. We were able to define EWSR which proved to be a useful tool to study the collectivity of excited states and suggests the interpretation of these states as the pion and its resonances.

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FIGURE CAPTIONS

Fig.1 – The energy per volume in homogeneous quark matter in units of M^4 (the vacuum quark mass) for different values of the Fermi momentum for the parameter values: $g = 3.44$ and $K = 2g^2$. The curves are in different scales so that details can be observed.

Fig.2 – The energy per particle as a function of the Fermi momentum for the parameter values: $g = 3.44, K = 2g^2, M = 320MeV$.

Fig.3 – Diagrammatical illustration of the solutions for the dispersion relation corresponding to the scalar-isoscalar excitation for the vacuum ($P_F = 0$) and with a suitable value of cutoff Λ. The straight line represents the function $(\omega^2/\lambda) - 4M^2$ and the curves the function $-(\xi g^2/2\Omega)\omega^2 \sum_{p<\Lambda} p^2/(\epsilon^3(\epsilon^2 - (\omega^2/4)))$. The circles correspond to the two collective discrete solutions and the points to the continuum solutions.

Fig.4 – The effective quark mass (dotted line), the scalar dispersion relation (solid line) and the pseudoscalar dispersion relation (dashed line) as a function of the Fermi momentum for the parameter values : $g = 3.44$, $K = 2g^2$, $M = 320MeV$. The point P_{FC} indicates the value of P_F for which the chiral symmetry is restored.

Fig.5 – Strength function representing the first pion resonance as a function of ω for the transition operator $\vec{D} = i\beta\gamma_5\vec{\tau}$ with $\Lambda = 2M$. The arrow indicates the experimental mass.

TABLE CAPTION

Table I – Energy of the pseudoscalar-isovector excitations correspondin g to three different transition operators for the parameter values: $M = 320\,MeV$, $g = 3.44$, $K = 2g^2$, $\sqrt{c/\sigma_0} = 140\,MeV$. Λ is the cutoff.

Table 1

TRANSITION OPERATOR $\vec{D}$	$\frac{\Lambda}{M}$	π (138)		π (1300)		π (1770)	
		ENERGY (MeV)	EWSR %	ENERGY (MeV)	EWSR %	ENERGY (MeV)	EWSR %
$\vec{Q}_5$	1.57	138	98.3	767	0.3	1771	1.4
$i\beta\gamma_5\vec{\tau}$	1.57	138	64.4	786	3.3	1771	32.3
	2.00	138	84.5	952	14.2	10035	1.3
$\gamma_5\vec{\tau}$	1.57	138	1.5	1150	1.4	1771	97.1

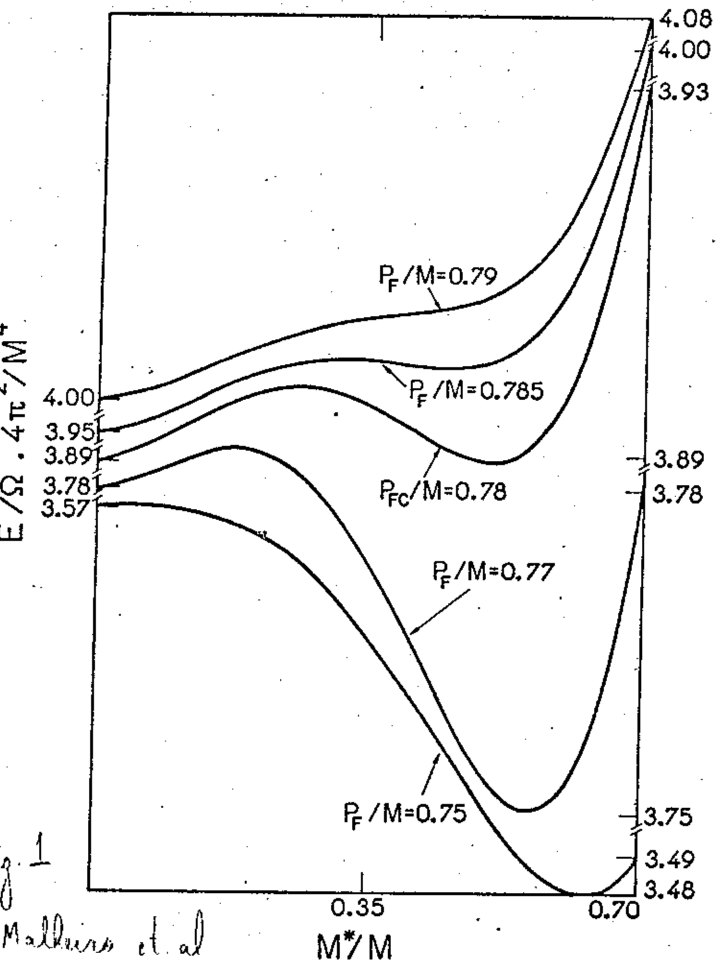

Fig. 1
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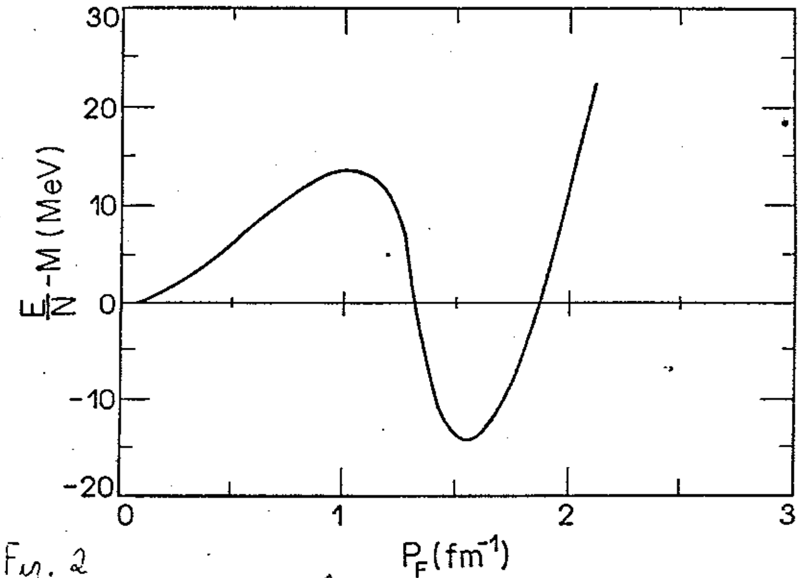

Fig. 2
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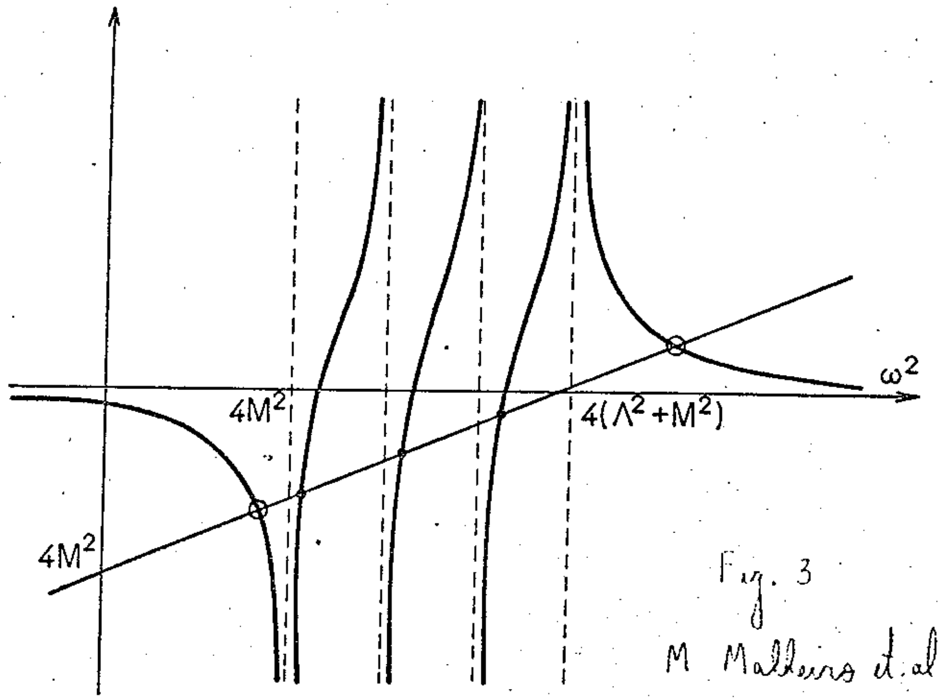

Fig. 3
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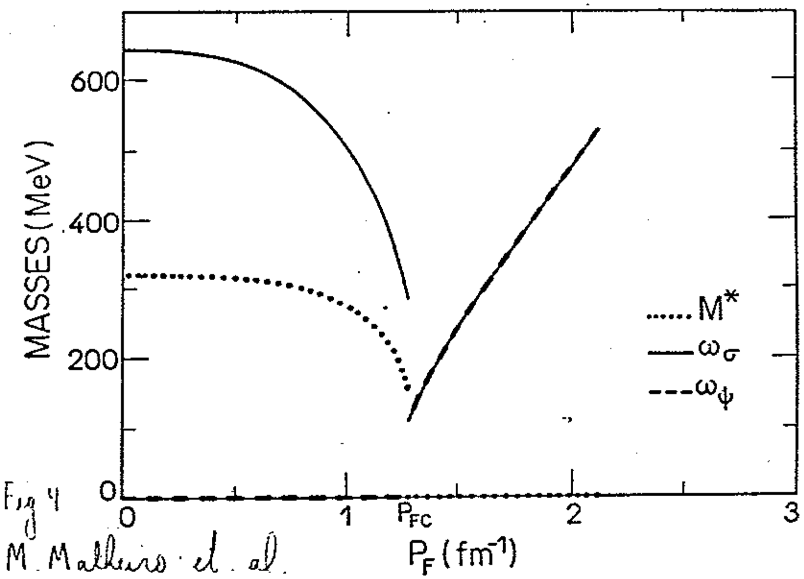

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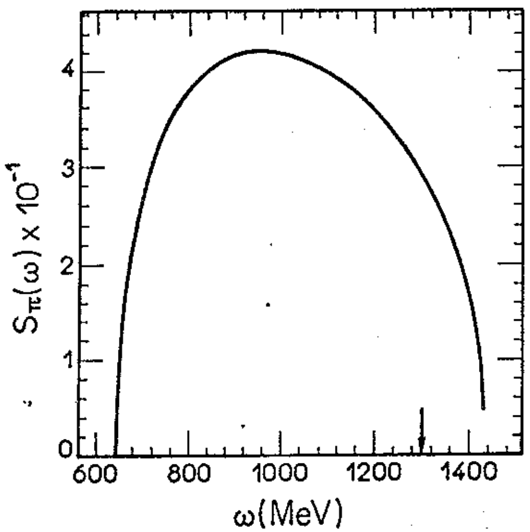

Fig 5
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