Bisection-Based Pricing for Repeated Contextual Auctions against Strategic Buyer: SUPPLEMENTARY MATERIALS

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Contents

A Missed proofs

A.1 Missed proofs from Subsection 6.1

A.1.1 Proof of Statement 1

Proof. The proof is similar to [10, Lemma 19]. Since the function $\phi_i : [\underline{p}_t, \overline{p}_t] \to \mathbb{R}, \phi_i(p) = V_i(S_t) - V_i(S_t^+(p, x_t))$ is continuous and monotone [10, Theorem 7] with $\phi_i(\underline{p}_t) = 0$. Thus, if $V_i(S_t) - V_i(S_t^+(\overline{p}_t, x_t)) > l^i_{k(i)+1}/(4i!)$ then $\phi(\overline{p}_t) > l^i_{k(i)+1}/(4i!)$. Using continuity of ϕ_i we get that the price p_i exists.

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To guarantee the existence of J we need to show only that there exists at least one j s.t. $L_{j-1} > w \geq L_{M(j)}$. Further, in the proof we use notation $[x, z)$ for a directional (oriented) interval/segment (1-dimensional euclidean vector): where x is the initial point and z is the terminal one of the interval, but it may be $x > z$ or $x < z$.

Let $0 = i_0 < i_1 < \ldots < i_a = d$ be the indices i such that $M(i) = i$. To remove subscripts, let $y_s := L_{i_s}$. Note that $y_0 = L_0 = \infty$ (see in Alg. 1) and $y_a = L_d = 0$ (the d-volume of $(d-1)$-dimensional plane). So, we have the set of points $y_s \in [0, \infty]$ and the sum of euclidean vectors $[y_a, y_{a-1}) + \ldots + [y_2, y_1) + [y_1, y_0) = [0, \infty)$. Since $w \in [0, \infty)$, then w be in at least one vector $[y_j, y_{j-1})$ that has the same direction as $[0, \infty)$. □

A.1.2 Proof of Statement 2

Proof. The proof is similar to [10, Lemma 20]. Using the fact that V_J is monotone and additive [10, Theorem 7] and that $S_t^-(p_J, x_t) = (S_t \setminus S_t^+(p_J, x_t)) \cup K_J$ we have

$$V_J(S_t^-(p_J, x_t)) = V_J(S_t) - V_J(S_t^+(p_J, x_t)) + V_J(K_J) \leq \frac{l^J_{k(J)+1}}{4J!} + V_J(K_J). \tag{A.1}$$

Here we used that $V_J(S_t) - V_J(S_t^+(p_J, x_t)) \leq l^J_{k(J)+1}/(4J!)$ due to the choice of the price p_J in our algorithm. So, it remains to show that $V_J(K_J) \leq l^J_{k(J)+1}/(2J!)$ to prove our statement. We will use the Cone Lemma [10, Lemma 13] to obtain the following inequalities:

$$\frac{1}{J+1} V_J(K_J) w \leq V_{J+1}(S_t) \leq \frac{1}{(J+1)!} l^{J+1}_{k(J+1)} \leq \frac{1}{(J+1)!} l^{J+1}_{(k(J))+1}. \tag{A.2}$$

The first inequality follows from the Cone Lemma and the fact that S_t contains a cone of base K_J and height at least w. The second inequality comes from the definition of $k(J)$ and the third comes from the fact that $J = M(J)$, so $k(J+1) \geq k(J) + 1$.

Finally, from our choice of J, $w \geq L_J = (V_J(K_J)/c_J)^{1/J}$. Substituting it in Inequality A.2 we obtain:

$$\frac{1}{J+1} V_J(K_J)^{(J+1)/J} c_J^{-1/J} \leq \frac{1}{(J+1)!} l^{J+1}_{k(J)+1}. \tag{A.3}$$

Substituting the definition of c_J and simplifying th last inequality, we get the desired bound of $V_J(K_J) \leq l^J_{k(J)+1}/(2J!)$. □

A.1.3 Proof of Statement 3

Proof. The proof is similar to [10, Lemmas 21, 22, 23].

(a) First we prove that for chosen J follows that $V_J(S_t) - V_J(S_t^+(p_J, x_t)) = l^J_{k(J)+1}/(4J!)$. Note, that this inequality is guaranteed by the algorithm's choice of p_J, except when $V_J(S_t) - V_J(S_t^+(\overline{p}_t, x_t)) < l^J_{k(J)+1}/(4J!)$ and $p_J = \overline{p}_t$. So, we need to show that this case is impossible.

Indeed, in this case, $S_t^-(\overline{p}_t, x_t) = S_t$ by the definition of $\overline{p}_t$. Then from Statement 2 we get that $[J! V_J(S_t^-(p_J, x_t))]^{1/J} \leq l_{k(J)+1}$, but this contradicts to the definition of $k(J)$:

$$\varphi_J = [J! V_J(S_t = S_t^-(p_J, x_t))]^{1/J} \in (l_{k(J)+1}, l_{k(J)}]. \tag{A.4}$$

(b) Now, we show that in each round t, the width of the knowledge set $w \leq 2 l_{k(J)}$. To prove it we provide an upper and lower bound on $V_J(S_t)$. To get a lower bound we apply the Cone Lemma [10, Lemma 13]:

$$V_J(S_t) \geq \frac{1}{J} V_{J-1}(K_{J-1}) w \geq \frac{1}{J} (c_{J-1} w^{J-1}) w. \tag{A.5}$$

If $J > 1$, then the first inequality holds since S_t contains a cone of base K_{J-1} and height w, and the second inequality follows from the fact that $w \le L_{J-1}$. If $J = 1$, than we observe that S_t contains a segment of length w, so $V_1(S_t) \ge w$.

To get an upper bound on $V_J(S_t)$ simply note that $V_J(S_t) \le l^J_{k(J)}/J!$, where the inequality follows from the definition of $k(J)$. Together the upper and lower bounds of $V_J(S_t)$ imply that $c_{J-1}w^J/J \le l^J_{k(J)}/J!$. Substituting the value of c_{J-1} and simplifying the last inequality we obtain that $w \le 2l_{k(J)}$.

(c) Finally, we prove that $k(J) \le 4(d \log_2 dT + 1)$ in the rounds where $w \ge 1/T$. It straightforwardly follows from the previous point. In the learning phase we know that $w \le 2l_{k(J)}$. Combining lower and upper bounds of w and using definition of l_k we get:

$$1/T \le w \le 2l_{k(J)} = 2d^2(1 + 1/d)^{-k(J)}. \tag{A.6}$$

Simplifying Inequality A.6 we get the desired bound of $k(J)$. □

A.2 Missed proofs from Subsection 7.2

A.2.1 Proof of Corollary 1

Proof. Let us to use the notations of Lemma 3. In order to prove the corollary we will show that $P_{3,h} \cap S_t \ne \varnothing$, where $h = \gamma^{m_t} \mathrm{diam}(S_t)/(1-\gamma)$ (thus, we will get that $p_J - \gamma^{m_t}\mathrm{diam}(S_t)/(1-\gamma) \ge \underline{p}_t$).

Applying Lemma 3 it is enough to show that

$$h = \frac{\gamma^{m_t}}{1-\gamma}\mathrm{diam}(S_t) \le 2w \left[\left(\frac{4(1+1/d)^J}{4(1+1/d)^J - 1}\right)^{1/J} - 1\right]\left[1 - (3/4)^{1/J}\right]. \tag{A.7}$$

in order to prove that $P_{3,h} \cap S_t \ne \varnothing$. Inequality A.7 is equivalent to

$$\begin{aligned} m_t \ge \log_\gamma(1-\gamma) + \log_\gamma(2w) - \log_\gamma(\mathrm{diam}(S_t)) + \\ + \log_\gamma\left(1 - (3/4)^{1/J}\right) + \log_\gamma\left(\left(\frac{4(1+1/d)^J}{4(1+1/d)^J - 1}\right)^{1/J} - 1\right). \end{aligned} \tag{A.8}$$

Note that from the definition of m_t, we have

$$\begin{aligned} m_t = \lceil \log_\gamma(1-\gamma) + \log_\gamma w - \log_\gamma \mathrm{diam}(S_t) + \\ + \log_\gamma\left(\left(\frac{8(1+1/d)^d}{8(1+1/d)^d - 1}\right)^{1/d} - 1\right) + \log_\gamma\left(1 - (3/4)^{1/d}\right)\rceil \end{aligned} \tag{A.9}$$

So, inequality A.8 holds, since $\log_\gamma$ is monotonically decreasing function. □

A.2.2 Proof of Corollary 2

Proof. Let us to use the notations of Lemma 3 and the abbreviation S_t^+ for $S_t^+(p_J, x_t)$. Let $h = 2\gamma^{m_t}\mathrm{diam}(S_t)/(1-\gamma)$. To prove this corollary we provide an upper bound of $S_t^+(p_J - h, x_t)$.

Since S_t is convex, we have $S_t^+(p_J - h, x_t) \subseteq S_t^+ \cup D_h \subseteq S_t^+ \cup (\mathrm{Hom}_h(\mathrm{Cone}) \setminus \mathrm{Cone})$. Since $\mathrm{Cone} \subseteq S_t^+$ and using monotone and homogeneity [10, Theorem 7, Theorem 9] of the intrinsic volumes, we get an upper bound of the J-th intrinsic volume $V_J(D_h)$ which equals to $V_J(\mathrm{Hom}_h(\mathrm{Cone})) - V_J(\mathrm{Cone})$:

$$V_J(\mathrm{Hom}_h(\mathrm{Cone})) - V_J(\mathrm{Cone}) = V_J(\mathrm{Cone})\left[\left(\frac{l+h}{l}\right)^J - 1\right] \le V_J(S_t^+)\left[\left(\frac{l+h}{l}\right)^J - 1\right]. \tag{A.10}$$

Thus, using Statement 3 to get that $V_J(S_t) - V_J(S_t^+) = l^J_{k(J)+1}/(4J!)$, we obtain

$$\begin{aligned} V_J(S_t) - V_J(S_t^+(p_J - h, x_t)) \geq V_J(S_t) - V_J(S_t^+) - V_J(S_t^+)[((l+h)/l)^J - 1] = \\ = l^J_{k(J)+1}/(4J!) - V_J(S_t^+)[((l+h)/l)^J - 1]. \end{aligned} \tag{A.11}$$

Therefore, we have to check that

$$V_J(S_t^+(p_J, x_t)) \left[\left(\frac{l+h}{l} \right)^J - 1 \right] \leq \frac{l^J_{k(J)+1}}{4J!} - \frac{l^J_{k(J)+1}}{8J!}. \tag{A.12}$$

From Statement 3 we can bound $V_J(S_t^+(p_J, x_t))$ as follows:

$$V_J(S_t^+(p_J, x_t)) = V_J(S_t) - l^J_{k(J)+1}/(4J!) \leq l^J_{k(J)}/J! - l^J_{k(J)+1}/(4J!), \tag{A.13}$$

here we also used that $\varphi_J \in (l_{k(J)+1}, l_{k(J)}]$ to get an upper bound of $V_J(S_t)$. Substituting Inequality A.13 and the definition of $l(k)$ in Inequality A.12 we have to prove that

$$h = 2\frac{\gamma^{m_t}}{1-\gamma}\mathrm{diam}(S_t) \leq l \left[\left(\frac{8(1+1/d)^J - 1}{8(1+1/d)^J - 2} \right)^{1/J} - 1 \right]. \tag{A.14}$$

Since $l \geq 2w[1 - (3/4)^{1/J}]$ from Lemma 2, it is enough to show that

$$\begin{aligned} m_t \geq \log_\gamma(1-\gamma) + \log_\gamma w - \log_\gamma \mathrm{diam}(S_t) + \\ + \log_\gamma \left(\left(\frac{8(1+1/d)^d}{8(1+1/d)^d - 1} \right)^{1/d} - 1 \right) + \log_\gamma \left(1 - (3/4)^{1/d} \right). \end{aligned} \tag{A.15}$$

So, inequality A.8 holds, since $\log_\gamma$ is monotonically decreasing function. □

A.2.3 Proof of Remark 1

Proof. Consider the modified number of penalization rounds:

$$\begin{aligned} m_t = \lceil \log_\gamma(1-\gamma) + \log_\gamma w - \log_\gamma \mathrm{diam}(S_t) + \\ + \log_\gamma \left(\left(\frac{8(1+1/d)^d}{8(1+1/d)^d - 1} \right)^{1/d} - 1 \right) + \log_\gamma \left(1 - (3/4)^{1/d} \right) \rceil \end{aligned}$$

At first, we note that penalization rounds occur in case $w \geq 1/T$, then $\log_\gamma(w) = O(\log_\gamma T)$. Since $S_t \subseteq \mathbb{X} = [0,1]^d$, the diameter of the knowledge set $\mathrm{diam}(S_t) \leq d+1$ for all rounds t and $-\log_\gamma(\mathrm{diam}(S_t)) \leq -\log_\gamma(d+1)$. The fifth and sixth terms of m_t have asymptotic $O(\log_\gamma d)$. Thus, the upper bound of m_t is $O(\log_\gamma(1-\gamma) + \log_\gamma(dT))$. □

B Discussion of Algorithm 1

B.1 Squaring trick

One can see that our algorithm requires knowledge of the horizon T (for example, the learning phase is defined by inequality $w \geq 1/T$). To be free of this assumption, we apply the standard technique "squaring trick" [11, 2]: we divide the range of possible values of the time horizon T into ranges $(b_{i-1}, b_i], i = 1, \dots, \infty$ such that $b_i - b_{i-1} = 2^{2^i}$ and $b_0 = 0$. Define the index M such that $b_{M-1} < T \leq b_M$ (this index is unknown to the seller). Then $2^{2^{M-1}} < T$, i.e. $M \leq O(\log \log T)$. After that we run independent instances of Algorithm 1: $\mathcal{A}_i$ during phase $(b_i, b_{i+1}]$ with the time horizon $T_i := b_i - b_{i-1}$. Note that decisions of the strategic buyer during current phase does not affect on other, since the algorithms run independently [3]. So, the seller's regret during the phase $(b_{i-1}, b_i]$ is $O(\log^2 T_i)$ (see Theorem 1). Since

$$\sum_{i=1}^{M} \log_\gamma^2 T_i = (\log_\gamma^2 2) \sum_{i=1}^{M} 4^i = \log_\gamma^2 2 \frac{4(4^M - 1)}{3} \leq O(4^{\log \log T}) = O(\log^2 T), \tag{B.1}$$

we get that the total seller's regret for such procedure is $O(\log^2 T)$.

B.2 Extension to nonlinear models

Our setup focuses on the linear valuation model, but it is easy to generalize our analysis to some of nonlinear models. Let us consider the valuation model $v_t := \phi(\langle \psi(x_t), \theta^* \rangle)$, where $\psi : \mathbb{X} \to \mathbb{X}$ is a mapping and $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing function s.t. $\phi(x) + h \leq \phi(x + h)$ for all $x \in \mathbb{R}_+$ and $h \geq 0$. In order to apply Algorithm 1 in this model, we perform the following procedure. After receiving a vector x_t, we consider $\tilde{x}_t := \psi(x_t)$ as a new contextual vector. Then, we find the price p_t from Algorithm 1 for the vector $\tilde{x}_t$ and propose the new price $\tilde{p}_t := (\phi(p_t + h) + \phi(p_t - h))/2$ to the buyer, where $h = \gamma^{m_t} \mathrm{diam}(S_t)/(1 - \gamma)$. From the properties of the function ϕ, the price $\tilde{p}_t$ satisfies the inequality

$$\phi(p_t - h) + h \leq \tilde{p}_t \leq \phi(p_t + h) - h. \tag{B.2}$$

We notice that after such modification the new algorithm has exactly the same regret bound as Algorithm 1 applied to the contextual vectors $\tilde{x}_t$ and proposing the prices p_t instead of $\tilde{p}_t$. It holds from the following considerations. From Propositions 1, 2 we know that after rejection and m_t penalization rounds we have

$$\phi(\langle \tilde{x}_t, \theta^* \rangle) \leq \tilde{p}_t + \frac{\gamma^{m_t}}{1 - \gamma} \mathrm{diam}(S_t). \tag{B.3}$$

Using Inequality B.2 we get that the previous inequality is equivalent to

$$\langle \tilde{x}_t, \theta^* \rangle \leq p_t + \frac{\gamma^{m_t}}{1 - \gamma} \mathrm{diam}(S_t). \tag{B.4}$$

Similarly, after acceptance and m_t penalization rounds we have

$$\langle \tilde{x}_t, \theta^* \rangle \geq p_t - \frac{\gamma^{m_t}}{1 - \gamma} \mathrm{diam}(S_t). \tag{B.5}$$

Note that bounds from Inequalities B.4, B.5 match with bounds from Propositions 1, 2 applied to the contextual vector $\tilde{x}_t$ and the price p_t. Thus, our modification of Algorithm 1 satisfies all properties of Algorithm 1: after substituting $\tilde{x}_t$ instead of x_t, all statements and lemmas formulated in Section 6 of the main text remain true for the price p_t. So, we get the same upper bound of the seller's regret for this modification.

B.3 Independence of γ

Let us assume that we do not know the true value of the parameter γ, but its upper bound γ_0 (s.t. $\gamma \le \gamma_0 < 1$) is known. Then, we can obtain our regret upper bound as well. The technique is similar to the one used in [4, 7, 6]. Note that bounds from Propositions 1, 2 are monotonous in the parameter γ and blows up when $\gamma \to 1$. Therefore, the bounds from these statements are true for γ_0, i.e. after rejection and m_t penalization rounds we have

$$\langle x_t, \theta^* \rangle \le p_t + \frac{\gamma_0^{m_t}}{1-\gamma_0}\mathrm{diam}(S_t). \tag{B.6}$$

Also, after acceptance and m_t penalization rounds we get

$$\langle x_t, \theta^* \rangle \ge p_t - \frac{\gamma_0^{m_t}}{1-\gamma_0}\mathrm{diam}(S_t). \tag{B.7}$$

Note that all statements and lemmas formulated in Section 6 of the main text remain true after substituting the parameter γ_0 instead of γ. Thus, using Algorithm 1 for the parameter γ_0, we get that the asymptotic of the seller's regret still has the form of $O(\log^2 T)$. From this, we can conclude that our algorithm is also applicable in the situation when the true value of the parameter γ is unknown and we just have its upper bound.

C Discussion of setup of repeated contextual auctions

As we noted in Sections 2, 3 of the main text, we study the setting of *repeated contextual posted-price auctions* which is seemingly similar to the one described by [1, 8]. However, note that, in the setups considered by [1, 8], *expected* regret $\sup_{\theta^* \in [0,1]^d, D} \mathbb{E}_{x_{1:T}\sim D}\mathtt{SReg}(T, \mathcal{A}, \theta^*, \gamma, x_{1:T}, D)$ was minimized. In contrast to that studies, in our approach, we minimize *worst-case* regret $\sup_{x_{1:T}\in \mathbb{X}^T, \theta^* \in [0,1]^d, D} \mathtt{SReg}(T, \mathcal{A}, \theta^*, \gamma, x_{1:T}, D)$. Note that $\mathtt{SReg}$ is a function of both a distribution D (the buyer maximizes his *surplus* with the fixed distribution D) and context vectors $x_{1:T}$ (similarly to the scenario of [6]). Then, in the case of expected regret, fixing the distribution, we compute $\mathbb{E}_{x_{1:T}\sim D}\mathtt{SReg}(T, \mathcal{A}, \theta^*, \gamma, x_{1:T}, D)$ and lose an ability to variate vectors $x_{1:T}$. So, it is easy to see that

$$\sup_{x_{1:T}\in \mathbb{X}^T, \theta^* \in [0,1]^d, D} \mathtt{SReg}(T, \mathcal{A}, \theta^*, \gamma, x_{1:T}, D) \ge \sup_{\theta^* \in [0,1]^d, D} \mathbb{E}_{x_{1:T}\sim D}\mathtt{SReg}(T, \mathcal{A}, \theta^*, \gamma, x_{1:T}, D), \tag{C.1}$$

and algorithm $\mathcal{A}$ that gives an upper bound for our worst-case regret implies the same upper bound for expected regret.

Also we emphasize that an algorithm minimizing worst-case regret have to be deterministic and his regret does not depend on any randomness (it is not true for the setup of [1, 8]).

D Introduction to intrinsic volumes

We now present a formal definition of intrinsic volumes and summarize their most important properties. Our overview is similar to [10, Section 5]. We refer to the book [9] for a comprehensive introduction to integral geometry.

Let K be the convex set in $\mathbb{R}^d$ and B be an unit ball. We define by $K + \varepsilon B$ the (Minkowski) sum of sets K and B. Steiner shows that $Vol(K+\varepsilon B)$ is a polynomial in ε and the intrinsic volumes $V_j(K), j \in \{0, \dots, d\}$ can be defined as the normalized coefficients of this polynomial:

$$Vol(K+\varepsilon B) = \sum_{j=0}^{d} k_{d-j} V_j(K) \varepsilon^{d-j},$$

where k_{d-j} is the volume of the $(d-j)$-dimensional unit ball.

Definition D.1 (Valuations)**.** *Let* Conv_d *be the class of compact convex bodies in* $\mathbb{R}^d$. *A* valuation *is a map* $\nu\colon \mathrm{Conv}_d \to \mathbb{R}$ *such that* $\nu(\varnothing) = 0$ *and for every* $S_1, S_2 \in \mathrm{Conv}_d$ *satisfying* $S_1 \cup S_2 \in \mathrm{Conv}_d$ *it holds that* $\nu(S_1 \cup S_2) + \nu(S_1 \cap S_2) = \nu(S_1) + \nu(S_2)$. *A valuation is said to be* monotone *if* $\nu(S_1) \leq \nu(S_2)$ *whenever* $S_1 \subseteq S_2$. *A valuation is said to be* non-negative *is* $\nu(S) \geq 0$ *for any* $S \in \mathrm{Conv}_d$. *Finally, a valuation is* rigid *if* $\nu(S) = \nu(T(S))$ *for every rigid motion* T *of* $\mathbb{R}^d$.

To define what it means for a valuation to be continuous, we need a notion of distance between two convex sets. We define the Hausdorff distance $\delta(K, L)$ between two sets $K, L \in \mathrm{Conv}_d$ to be the minimum ε such that $K + \varepsilon B \subseteq L$ and $L + \varepsilon B \subseteq K$ where B is a unit ball. Finally, a sequence $K_t \in \mathrm{Conv}_d$ *converges* to $K \in \mathrm{Conv}_d$ ($K_t \to K$) if $\delta(K_t, K) \to 0$.

Definition D.2 (Continuity)**.** *A valuation function* ν *is* continuous *if whenever* $K_t \to K$ *then* $\nu(K_t) \to K$.

The following theorem states that the intrinsic volumes satisfy natural properties.

Theorem 1. *The intrinsic volumes are non-negative monotone continuous rigid valuation.*

Otherwise, the intrinsic volumes are quite special. They form the basis for the set of all valuations that are continuous and rigid.

Theorem 2 (Hadwiger)**.** *If* ν *is a continuous rigid valuation of* Conv_d, *then there are constants* $c_0, \ldots, c_d$ *such that* $\nu = \sum_{i=0}^{d} c_i V_i$, *where* V_i *are intrinsic volumes.*

Next we describe a few important properties of the intrinsic volumes that will be useful in the analysis of Algorithm 1.

Theorem 3 (Homogeneity)**.** *The map* V_j *is* j*-homogenous, i.e.,* $\forall \alpha \in \mathbb{R}_{\geq 0} : V_j(\alpha K) = \alpha^j V_j(K)$.

Following [10] we introduce inequlities

Lemma 1 (Isoperimetric inequality)**.** *For any* $S \in \mathrm{Conv}_d$ *and any* $i \geq 1$ *it holds that*

$$(i! V_i(S))^{1/i} \geq ((i+1)! V_{i+1}(S))^{1/(1+i)}.$$

Lemma 2 (Cone Lemma)**.** *Let* K *be a convex set in* $\mathbb{R}^d$, *and let* S *be a cone in* $\mathbb{R}^{d+1}$ *with base* K *and height* h. *Then, for all* $0 \leq j \leq d$,

$$V_{j+1}(S) \geq \frac{1}{j+1} h V_j(K).$$

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