
CAUSE: Learning Granger Causality from Event Sequences using Attribution Methods

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Abstract

We study the problem of learning Granger causality between event types from asynchronous, interdependent, multi-type event sequences. Existing work suffers from either limited model flexibility or poor model explainability and thus fails to uncover Granger causality across a wide variety of event sequences with diverse event interdependency. To address these weaknesses, we propose CAUSE (Causality from Attribution on Sequence of Events), a novel framework for the studied task. The key idea of CAUSE is to first implicitly capture the underlying event interdependency by fitting a neural point process, and then extract from the process a Granger causality statistic using an axiomatic attribution method. Across multiple datasets riddled with diverse event interdependency, we demonstrate that CAUSE achieves superior performance on correctly inferring the inter-type Granger causality over a range of state-of-the-art methods.

1. Introduction

Many real-world processes generate a massive number of asynchronous and interdependent events in real time. Examples include the diagnosis and drug prescription histories of patients in electronic health records, the posting and responding behaviors of users on social media, and the purchase and selling orders executed by traders in stock markets, among others. Such data can be generally viewed as *multi-type event sequences*, in which each event consists of both a timestamp and a type label, indicating when and what the event is, respectively.

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In this paper, we focus on the fundamental problem of uncovering causal structure among event types from multi-type event sequence data. Since the question of “true causality” is deeply philosophical (Schaffer, 2003), and there are still massive debates on its definition (Pearl, 2009; Imbens & Rubin, 2015), we consider a weaker notion of causality based on predictability—Granger causality. While Granger causality was initially used for studying the dependence structure for multivariate time series (Granger, 1969; Dahlhaus & Eichler, 2003), it has also been extended to multi-type event sequences (Didelez, 2008). Intuitively, for event sequence data, an event type is said to be (strongly) *Granger causal* for another event type if the inclusion of historical events of the former type leads to better predictions of future events of the latter type.

Due to their asynchronous nature, in the literature, multi-type event sequences are often modeled by multivariate point process (MPP), a class of stochastic processes that characterize the random generation of points on the real line. Existing point process models for inferring inter-type Granger causality from multi-type event sequences appear to be limited to a particular case of MPPs—Hawkes process (Eichler et al., 2017; Xu et al., 2016; Hall & Willett, 2016; Yang et al., 2017; Achab et al., 2018), which assumes past events can only independently and additively *excite* the occurrence of future events according to a collection of pairwise kernel functions. While these Hawkes process-based models are very interpretable and many include favorable statistical properties, the strong parametric assumptions inherent in Hawkes processes render these models unsuitable for many real-world event sequences with potentially abundant *inhibitive* effects or event *interactions*. For example, maintenance events should reduce the chances of a system breaking down, and a patient who takes multiple medicines at the same time is more likely to experience unexpected adverse events.

Regarding event sequence modeling in general, a new class of MPPs, loosely referred to as neural point processes (NPPs), has recently emerged in the literature (Du et al., 2016; Xiao et al., 2017; Mei & Eisner, 2017; Xiao et al., 2019). NPPs use deep (mostly recurrent) neural networks to capture complex event dependencies, and thus excel at predicting future events due to their model flexibility. How-

ever, NPPs are uninterpretable and unable to provide insight into the Granger causality between event types.

To address this tension between model explainability and model flexibility in existing point process models, we propose CAUSE (Causality from Attribution on Sequence of Events), a framework for obtaining Granger causality from multi-type event sequences using information captured by a highly predictive NPP model. At the core of CAUSE are two steps: first, it trains a flexible NPP model to capture the complex event interdependency, then it computes a novel Granger causality statistic by inspecting the trained NPP with an axiomatic attribution method. In this way, CAUSE is the first technique that brings model-agnostic explainability to NPPs and endows NPPs with the ability to discover Granger causality from multi-type event sequences exhibiting various types of event interdependencies.

Contributions. Our contributions are:

- We bring model-agnostic explainability to NPPs and propose CAUSE, a novel framework for learning Granger causality from multi-type event sequences exhibiting various types of event interdependency.
- We design a two-level batching algorithm that enables efficient computation of Granger causality scalable to millions of events.
- We evaluate CAUSE on both synthetic and real-world datasets riddled with diverse event interdependency. Our experiments demonstrate that CAUSE outperforms other state-of-the-art methods.

Reproducibility. We publish our data and our code at <https://github.com/razhangwei/CAUSE>.

2. Background

In this section, we first establish some notation and then briefly introduce the background for several highly relevant topics.

2.1. Notations

Suppose there are S subjects and each subject s is associated with a multi-type event sequence $\{(t_i^s, k_i^s)\}_{i=1}^{n_s}$, where $t_i^s \in \mathbb{R}_+$ is the timestamp of the i -th event of the s -th sequence, $k_i^s \in [K]$ is the corresponding event type, and n_s is the sequence length. We denote by $\mathbf{z}_i^s \in \{0, 1\}^K$ the one-hot representation of each event type k_i^s , and use $[n]$ to represent the set $\{1, \dots, n\}$ for any positive integer n . To avoid clutter, we omit the subscript/superscript of index s when we are discussing a single event sequence and no confusion arises.

2.2. Multivariate Point Process

Multivariate point processes (MPPs) (Daley & Vere-Jones, 2003) are a particular class of stochastic processes that

characterize the dynamics of discrete events of multiple types in continuous time. The most common way to define an MPP is through a set of *conditional intensity functions (CIFs)*, one for each event type. Specifically, let $N_k(t) \triangleq \sum_{i=1}^{\infty} \mathbb{1}(t_i \leq t \wedge k_i = k)$ be the number of events of type k that have occurred up to t , and let $\mathcal{H}(t) \triangleq \{(t_i, k_i) | t_i < t\}$ be the *history* of all types of events before t . The CIF for event type k is defined as the expected instantaneous event occurrence rate conditioned on history, i.e.,

$$\lambda_k^*(t) \triangleq \lim_{\Delta t \downarrow 0} \frac{\mathbb{E}[N_k(t + \Delta t) - N_k(t) | \mathcal{H}(t)]}{\Delta t},$$

where the use of the asterisk is a notational convention to emphasize that intensity is conditioned upon $\mathcal{H}(t)$.

Different parameterizations of CIFs lead to different MPPs. One classic example of MPP is the multivariate Hawkes process (Hawkes, 1971a;b), which assumes each $\lambda_k^*(t)$ to be of the following form:

$$\lambda_k^*(t) = \mu_k + \sum_{i: t_i < t} \phi_{k, k_i}(t - t_i), \quad (1)$$

where $\mu_k \in \mathbb{R}_+$ is the baseline rate for event type k , and $\phi_{k, k'}(\cdot)$ for any $k, k' \in [K]$ is a non-negative-valued function (usually referred to as *kernel function*) that characterizes the excitation effect of event type k' on type k .

More recently, a class of MPPs loosely referred to as *neural point processes* have emerged in the literature (Du et al., 2016; Xiao et al., 2017; Mei & Eisner, 2017; Xiao et al., 2019). These models parameterize CIFs with deep neural networks and generally follow an encoder-decoder design: an *encoder* successively embeds the event history $\{(t_j, k_j)\}_{j=1}^i$ into a vector $\mathbf{h}_i \in \mathbb{R}^{N_h}$ for each i , and a *decoder* then predicts with \mathbf{h}_i the future CIFs $\lambda_k^*(t)$ after time t_i (until the next event is generated).

Most MPPs are trained by minimizing the negative log-likelihood (NLL):

$$\sum_{s=1}^S \sum_{i=1}^{n_s} \left[-\log \lambda_{k_i^s}^*(t_i^s) + \sum_{k=1}^K \int_{t_{i-1}^s}^{t_i^s} \lambda_k^*(t') dt' \right], \quad (2)$$

where $\lambda_k^{*s}(t) \triangleq \lambda_k^*(t | \mathcal{H}_s(t))$ is the CIF of the s -th sequence for the type k . In (2), the first term corresponds to the NLL of an event of type k_i^s being observed at t_i^s for the s -th sequence, whereas the second term is the NLL of the observation that no events of any type occurred during the window (t_{i-1}^s, t_i^s) . When there are no closed-form expressions for the integrals $\int_{t_{i-1}^s}^{t_i^s} \lambda_k^*(t') dt'$, Monte-Carlo approximation needs to be used to approximate either the integrals themselves or their gradients with respect to the parameters. However, these approximation techniques are inefficient and generally suffer from large variances, resulting in low convergence rate.

2.3. Granger Causality for Multi-Type Event Sequences

The Granger causality definition for multi-type event sequences is established based on point process theory (Daley & Vere-Jones, 2003). To proceed formally, for any $\mathcal{K} \subseteq [K]$, we denote by $\mathcal{H}_{\mathcal{K}}(t)$ the natural filtration expanded by the sub-process $\{N_k(t)\}_{k \in \mathcal{K}}$; that is, the sequence of smallest σ -algebra expanded by the past event history of any type $k \in \mathcal{K}$ and $t \in \mathbb{R}_+$, i.e., $\mathcal{H}_{\mathcal{K}}(t) = \sigma(\{N_k(s) | k \in \mathcal{K}, s < t\})$.¹ We further write $\mathcal{H}_{-k}(t) = \mathcal{H}_{[K] \setminus \{k\}}(t)$ for any $k \in [K]$.

Definition 1. (Eichler et al., 2017) For a K -dimensional MPP, event type k is *Granger non-causal* for event type k' if $\lambda_{k'}^*(t)$ is $\mathcal{H}_{-k}(t)$ -measurable for all t .

The above definition amounts to saying that a type k is Granger non-causal for another type k' if, given the history of events other than type k , historical events of type k do not further contribute to future $\lambda_{k'}^*(t)$ at any time. Otherwise, type k is said to be *Granger causal* for type k' .

Uncovering Granger causality from event sequences generally is a very challenging task, as the underlying MPP may have rather complex CIFs with abundant event interaction and non-excitative effect. As a result, existing work tends to restrict consideration to certain classes of parametric MPPs, such as Hawkes processes (Eichler et al., 2017; Xu et al., 2016; Hall & Willett, 2016; Yang et al., 2017; Achab et al., 2018). Specifically, for multivariate Hawkes process, it is straightforward from (1) that a type k is Granger non-causal for another type k' if and only if the corresponding kernel function $\phi_{k',k}(\cdot) = 0$.

2.4. Attribution Methods

We view an *attribution method* for black-box functions as another “black box”, which takes in a function, an input, and a baseline, and outputs a set of meaningful attribution scores, one per feature. The following is a formal definition of attribution method.

Definition 2 (Attribution Method). Suppose $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^d$ is a d -dimensional input and $\underline{\mathbf{x}} \in \mathcal{X}$ a suitable baseline. Let \mathcal{F}_d be a class of functions from \mathcal{X} to \mathbb{R} . A functional $A : \mathcal{F}_d \times \mathcal{X} \times \mathcal{X} \mapsto \mathbb{R}^d$ is called an *attribution method* for \mathcal{F}_d if $A_i(f, \mathbf{x}, \underline{\mathbf{x}})$ measures the contribution of x_i to the prediction $f(\mathbf{x})$ relative to $\underline{\mathbf{x}}$ for any $f \in \mathcal{F}_d$, $\mathbf{x}, \underline{\mathbf{x}} \in \mathcal{X}$, and $i \in [d]$.

Since it is very challenging (and often subjective) to compare different attribution methods, Sundararajan et al. (2017) argue that attribution methods should ideally satisfy a num-

¹Here, we abuse our previous notation $\mathcal{H}(t)$ that denotes the set of events that occurred prior to t . Appendix D includes a primer on measure and probability theory for readers who are less familiar with some concepts in this subsection.

ber of axioms (i.e., desirable properties), which we re-state in Definition 3.

Definition 3. An attribution method A is said to satisfy the axiom of:

1. *Linearity*, if for any $f, g \in \mathcal{F}_d$, $\mathbf{x}, \underline{\mathbf{x}} \in \mathcal{X}$, and $c \in \mathbb{R}$,

$$\begin{aligned} A(f, \mathbf{x}, \underline{\mathbf{x}}) + A(g, \mathbf{x}, \underline{\mathbf{x}}) &= A(f + g, \mathbf{x}, \underline{\mathbf{x}}), \\ A(cf, \mathbf{x}, \underline{\mathbf{x}}) &= c \cdot A(f, \mathbf{x}, \underline{\mathbf{x}}). \end{aligned} \quad (\text{A1})$$

2. *Completeness/Efficiency*, if for any $f \in \mathcal{F}_d$ and $\mathbf{x}, \underline{\mathbf{x}} \in \mathcal{X}$,

$$f(\mathbf{x}) - f(\underline{\mathbf{x}}) = \sum_{i=1}^d A_i(f, \mathbf{x}, \underline{\mathbf{x}}). \quad (\text{A2})$$

3. *Null player*, if for any $f \in \mathcal{F}_d$ such that f does not mathematically depend on a dimension i ,

$$A_i(f, \mathbf{x}, \underline{\mathbf{x}}) = 0, \quad (\text{A3})$$

for all $\mathbf{x}, \underline{\mathbf{x}} \in \mathcal{X}$.

4. *Implementation invariance*, if for any $\mathbf{x}, \underline{\mathbf{x}} \in \mathcal{X}$, and any $f, g \in \mathcal{F}_d$ such that $f(\mathbf{x}') = g(\mathbf{x}')$ for all $\mathbf{x}' \in \mathcal{X}$,

$$A(f, \mathbf{x}, \underline{\mathbf{x}}) = A(g, \mathbf{x}, \underline{\mathbf{x}}). \quad (\text{A4})$$

Besides these four axioms, we also identify two other useful properties of attribution methods, which are less explicitly mentioned in the literature. We state these two properties in Definition 4.

Definition 4. An attribution method A is said to satisfy

1. *Fidelity-to-control*, if for any $f \in \mathcal{F}_d$, $\mathbf{x}, \underline{\mathbf{x}} \in \mathcal{X}$, and $i \in [d]$,

$$x_i = \underline{x}_i \Rightarrow A_i(f, \mathbf{x}, \underline{\mathbf{x}}) = 0. \quad (\text{P1})$$

2. *Batchability*, if for any $f \in \mathcal{F}_d$ and any n input/baseline pairs $\{(\mathbf{x}_i, \underline{\mathbf{x}}_i)\}_{i \in [n]}$, there exists a function $F : \mathcal{X}^n \mapsto \mathbb{R}$ such that

$$A(F, \mathbf{X}, \underline{\mathbf{X}}) = [A(f, \mathbf{x}_1, \underline{\mathbf{x}}_1), \dots, A(f, \mathbf{x}_n, \underline{\mathbf{x}}_n)], \quad (\text{P2})$$

where $\mathbf{X} \triangleq [\mathbf{x}_1, \dots, \mathbf{x}_n]$ and $\underline{\mathbf{X}} \triangleq [\underline{\mathbf{x}}_1, \dots, \underline{\mathbf{x}}_n]$.

Many popular attribution methods satisfy most of these six properties, as we show in the Proposition 1 and 2.

Proposition 1. *Integrated Gradients (Sundararajan et al., 2017) satisfies all four axioms (A1–A4) and two properties (P1–P2), and DeepLIFT (Shrikumar et al., 2017) satisfies all but the implementation invariance (A4). In particular, a choice of F for both methods satisfying batchability (P2) is $F(\mathbf{X}) \triangleq \sum_{i=1}^n f(\mathbf{x}_i)$.*

Proposition 2. *For any $U \subseteq [d]$, let $\bar{U} \triangleq [d] \setminus U$ and define $\mathbf{x}_U \sqcup \underline{\mathbf{x}}_{\bar{U}}$ to be the spliced data point between \mathbf{x} and $\underline{\mathbf{x}}$ such that for any $i \in [d]$*

$$[\mathbf{x}_U \sqcup \underline{\mathbf{x}}_{\bar{U}}]_i \triangleq \begin{cases} x_i & i \in U, \\ \underline{x}_i & i \in \bar{U}. \end{cases} \quad (3)$$

Then Shapley values (Shapley, 1953) with a value function $v_{f, \mathbf{x}, \underline{\mathbf{x}}}(U) \triangleq f(\mathbf{x}_U \sqcup \underline{\mathbf{x}}_{\bar{U}})$ satisfies all four axioms (A1–A4) and the fidelity-to-control (P1).

We include in Appendix B the proofs for both propositions, as well as a description of Shapley values.

3. Proposed: CAUSE

In this section, we formally present CAUSE, a novel framework for learning Granger causality from multi-type event sequences. Our framework consists of two steps: first, it trains a neural point process (NPP) to fit the underlying event sequence data; then it inspects the predictions of the trained NPP to compute a Granger causality statistic with some “well-behaved” attribution method $A(\cdot)$, which we assume satisfies the following properties: linearity (A1), completeness (A2), null player (A3), fidelity-to-control (P1), and batchability (P2).

In what follows, we first describe the architecture of the used NPP in Section 3.1. Then we elaborate the intuition and the definition of our Granger causality statistic in Section 3.2. Section 3.3 explains the computational challenges and presents a highly efficient algorithm for computing such statistic. We conclude this section by discussing the choice of attribution methods for CAUSE in Section 3.4.

3.1. A Semi-Parametric Neural Point Process

The design of our NPP follows the general encoder-decoder architecture of existing NPPs (Section 2.2), but we innovate the decoder part to enable both modeling flexibility and computational feasibility.

Encoder. First, we convert each event i into an embedding vector \mathbf{v}_i that summarizes both the temporal and the type information for that event, as follows:

$$\mathbf{v}_i = [\vartheta(t_i - t_{i-1}); \mathbf{V}^T \mathbf{z}_i], \quad (4)$$

where $\vartheta(\cdot)$ is a pre-specified function that transforms the elapsed time into one or more temporal features (simply chosen to be identity function in our experiments), \mathbf{V} is the embedding matrix for event types, and recall that \mathbf{z}_i is the one-hot encoding of the event type k_i .

We then obtain the embedding of a history from event embedding sequences by

$$\mathbf{h}_i = \text{Enc}(\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_i), \quad (5)$$

where $\text{Enc}(\cdot)$ is a sequence encoder and chosen to be a Gated Recurrent Unit (GRU) (Cho et al., 2014) in our experiments.

Decoder. Our design of decoder aims to fulfill the following *two desiderata*: (a) it should be *flexible enough*

to produce from \mathbf{h}_i a wide variety of $\lambda_k^*(t)$ with complex time-varying patterns; and (b) it should also be *computationally manageable*, particularly in terms of computing the cumulative intensity $\int_{t_i}^{t_{i+1}} \lambda_k^*(t) dt'$, a key term in the log-likelihood-based training given in (2) and the definition of our Granger causality statistic in the subsequent subsections.

We propose a novel semi-parametric decoder that enjoys both the flexible modeling of CIF and computational feasibility. Specifically, for each $i \in [n]$, we define the CIF $\lambda_k^*(t)$ on $(t_i, t_{i+1}]$ to be a weighted sum of a set of basis functions, as follows:

$$\lambda_k^*(t) = \sum_{r=1}^R \alpha_{k,r}(\mathbf{h}_i) \psi_r(t - t_i), \quad (6)$$

where $\{\psi_r(\cdot)\}_{r=1}^R$ is a set of pre-specified positive-valued basis functions, and $\alpha : \mathbb{R}^{N_h} \mapsto \mathbb{R}_+^{K \times R}$ is a feedforward neural network that computes R positive weights for each of the K event types. In this way, by choosing $\{\psi_r(\cdot)\}_{r=1}^R$ to be a rich-enough function family, the CIFs defined in (6) are able to express a wide variety of time-varying patterns. More importantly, since the parts relevant to neural networks— $\alpha(\cdot)$ and $\text{Enc}(\cdot)$ —are separated from the basis functions, we can evaluate the integral $\int_{t_i}^{t_{i+1}} \lambda_k^*(t) dt'$ analytically, as follows:

$$\int_{t_i}^{t_{i+1}} \lambda_k^*(t') dt' = \sum_{r=1}^R \alpha_{k,r}(\mathbf{h}_i) \Psi_r(t_{i+1} - t_i), \quad (7)$$

where $\Psi_r(\Delta t) \triangleq \int_0^{\Delta t} \psi_r(t) dt$ is generally available for many parametric basis functions.

Inspired by the dyadic interval bases used by Bao et al. (2017), we choose the basis functions $\{\psi_r(\cdot)\}_{r=1}^R$ to be the densities for a Gaussian family $\{\mathcal{N}(\mu_r, \sigma_r^2)\}_{r=1}^R$, whose means are given by

$$\mu_r = \begin{cases} 0, & r = 1, \\ L/2^{R-r}, & r = 2, \dots, R, \end{cases} \quad (8)$$

and the standard deviations by $\sigma_r = \max(\mu_r/3, \mu_2/3)$ for $r \in [R]$. This design of basis functions reflects a reasonable inductive bias that the CIFs should vary more smoothly as the time increases. As shown in Figure 1 for an example of $L = 100$ and $R = 5$, the first a few bases, due to their small means and variances, capture the short-term effects, whereas the last several characterize the mid/long-term effects.

3.2. From Event Contributions to a Granger Causality Statistic

Now that we have trained a flexible NPP that can successively update the history embedding after each event i occurrence and then predict the future CIFs $\lambda_k^*(t)$ after t_i until

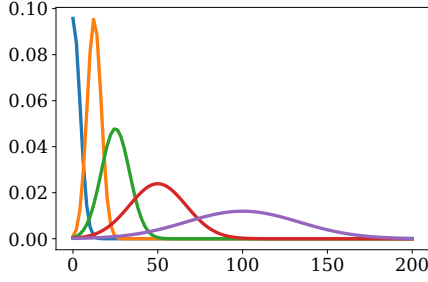


Figure 1: An example of dyadic Gaussian bases for $L = 100$ and $R = 5$. The first a few bases, due to their small means and variances, can capture the short-term effects, whereas the last several characterize the mid/long-term effects.

t_{i+1} ; we would like to ask: *can we quantify the contribution of each past event to each prediction?* Since in our case $\lambda_k^*(t)$'s are instantiated by two potentially highly nonlinear neural networks (i.e., $\text{Enc}(\cdot)$ and $\alpha(\cdot)$), it is not as straightforward to obtain the past event's contribution to current event occurrence as in the case of some parametric MPPs (e.g., Hawkes processes).

A natural idea for the aforementioned question would be applying some attribution method to $\lambda_k^*(t)$'s. To do so, however, there are two challenges. First, the predictions in our case are time-varying functions rather than static quantities (e.g., the probability of a class, as commonly seen in existing applications of attribution methods); thus it is unclear which target should be attributed. Second, as the input to $\lambda_k^*(t)$'s are multi-type event sequences with asynchronous timestamps, it is also unclear which baseline should be used.

We tackle the first challenge by setting the *cumulative intensity* $\int_{t_i}^{t_{i+1}} \lambda_k^*(t') dt'$ to be the attribution target. This is not only because the cumulative intensity reflects the overall effect of $\lambda_k^*(t')$ on $(t_i, t_{i+1}]$, but also because it has a clear meaning in the context of point processes: it is the rate of the Poisson distribution that the number of events of type k on $(t_i, t_{i+1}]$ satisfies. More importantly, since the cumulative intensity has a closed form as in (7), its gradients with respect to its input can be computed both precisely and efficiently. Note that by adopting this target, the input now includes not only $\{(t_i, \mathbf{z}_j)\}_{j \leq i}$ but also t_{i+1} ; thus we define $\mathbf{x}_i \triangleq [t_1, \mathbf{z}_1, \dots, t_i, \mathbf{z}_i, t_{i+1}]$ and write the target $\int_{t_i}^{t_{i+1}} \lambda_k^*(t') dt'$ as $f_k(\mathbf{x}_i)$.

As for the second challenge, we choose the baseline of an input \mathbf{x}_i to be $\underline{\mathbf{x}}_i \triangleq [t_1, \mathbf{0}, \dots, t_{i-1}, \mathbf{0}, t_{i+1}]$; that is, the one-hot encodings of all observed event types are replaced with zero vectors. Since \mathbf{x}_i and $\underline{\mathbf{x}}_i$ only differ in the dimensions corresponding to the event types, i.e., $\{z_{j,k_j}\}_{j \leq i}$, by the fidelity-to-control (P1), then only these dimensions will have non-zero attributions. With completeness (A2), it

further implies that for every type k

$$f_k(\mathbf{x}_i) - f_k(\underline{\mathbf{x}}_i) = \sum_{j=1}^i A_j(f_k, \mathbf{x}_i, \underline{\mathbf{x}}_i), \quad (9)$$

where $A_j(f_k, \mathbf{x}_i, \underline{\mathbf{x}}_i)$ is the attribution to z_{j,k_j} . Thus, the term $A_j(f_k, \mathbf{x}_i, \underline{\mathbf{x}}_i)$ can be viewed as the *event contribution* of the j -th event to the cumulative intensity prediction $f_k(\mathbf{x}_i)$ relative to the baseline $f_k(\underline{\mathbf{x}}_i)$. Besides, event *timestamps* are identical in \mathbf{x}_i and $\underline{\mathbf{x}}_i$, thus this contribution comes only from the event *type* and denotes how type k_j contributes to the prediction of type k for a specific event history \mathbf{x}_i .

A Granger Causality Statistic. We have established $A_j(f_k, \mathbf{x}_i, \underline{\mathbf{x}}_i)$'s as the past events' contribution to the cumulative intensity $f_k(\mathbf{x}_i)$ on interval $(t_i, t_{i+1}]$. A further question is: *can we infer from these event contributions for individual predictions the population-level Granger causality among event types?*

To answer this question, we define a novel statistic indicating the Granger causality for type k' to type k as follows:

$$Y_{k,k'} \triangleq \frac{\sum_{s=1}^S \sum_{i=1}^{n_s} \sum_{j=1}^i \mathbb{I}(k_j^s = k') A_j(f_k, \mathbf{x}_i^s, \underline{\mathbf{x}}_i^s)}{\sum_{s=1}^S \sum_{j=1}^{n_s} \mathbb{I}(k_j^s = k')}. \quad (10)$$

Here the numerator aggregates the event contributions for all event occurrences over the whole dataset, and denominator accounts for the fact that some event types may occur far more frequently than other types, which can lead to unreasonably large scores if used without such normalization. Note that an event contribution $A_j(f_k, \mathbf{x}_i^s, \underline{\mathbf{x}}_i^s)$ may be negative when the event j exerts an inhibitive effect; thus $Y_{k,k'}$ can also be negative and characterize the Granger causality from type k' to type k even when the influence is inhibitive.

Attribution Regularization. One caveat in (9) and (10) is that our chosen baselines $\underline{\mathbf{x}}_i$ have never appeared in the training procedure, thus the value of $f(\underline{\mathbf{x}}_i)$ may be meaningless or even misleading. Ideally, we would like $f_k(\underline{\mathbf{x}}_i)$ to be the cumulative intensity of type k given that history prior to t_i consists of “null” events at t_1, t_2, \dots, t_i . Thus a natural prior reflecting this idea is to make $f_k(\underline{\mathbf{x}}_i)$ nearly zero for any handcrafted baseline $\underline{\mathbf{x}}_i$. Such an “invariance” property on f can be achieved by adding an auxiliary l_1 regularization for each \mathbf{x}_i in the NLL given in (2), leading to a training objective

$$\sum_{s=1}^S \sum_{i=1}^{n_s} \left\{ \underbrace{-\log \lambda_{k_i^s}^*(t_i^s)}_{\text{negative log-likelihood}} + \underbrace{\sum_{k=1}^K f_k(\mathbf{x}_{i-1}^s)}_{\text{regularization}} + \underbrace{\sum_{k=1}^K \eta f_k(\underline{\mathbf{x}}_{i-1}^s)}_{\text{regularization}} \right\}, \quad (11)$$

where η is a hyperparameter.

Algorithm 1: Computation of the Granger causality statistic.

Input: Event sequences $\{\{(t_i^s, k_i^s)\}_{i \in [n_s]}\}_{s \in [S]}$, an attribution method $A(\cdot)$, and a trained NPP

Output: Granger causality statistic \mathbf{Y} .

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1 Initialize  $\tilde{\mathbf{Y}} = \mathbf{0}, \mathcal{I} = [S]$ ;
2 while  $|\mathcal{I}| > 0$  do
3     Sample a batch of sequence indices  $\mathcal{B} \subset \mathcal{I}$ ;
4     for  $k = 1, \dots, K$  do
5         Compute  $\mathbf{C} = A(\sum_{s \in \mathcal{B}} \sum_{i=1}^{n_s} F_{k,i}^s, \mathbf{X}, \underline{\mathbf{X}})$ ;
6         for  $k' = 1, \dots, K$  do
7              $\tilde{Y}_{k,k'} + = \sum_{s \in \mathcal{B}} \sum_{j=1}^{n_s} \mathbb{I}(k_j^s = k') C_j^s$ 
8          $\mathcal{I} \leftarrow \mathcal{I} \setminus \mathcal{B}$ ;
9 Compute  $Y_{k,k'} = \tilde{Y}_{k,k'} / \sum_{s=1}^S \sum_{j=1}^{n_s} \mathbb{I}(k_j^s = k')$ ,
     $\forall k, k' \in [K]$ .
```

3.3. Computing the Granger Causality Statistic

While (10) defines $Y_{k,k'}$'s analytically, it is rather challenging to compute them. This is because a naive implementation would require applying $A(\cdot)$ at each event occurrence, which is computationally prohibitive for a dataset of millions of events. Note that the normalization in (10) can be easily calculated; so if we write $\tilde{Y}_{k,k'}^s \triangleq \sum_{i=1}^{n_s} \sum_{j=1}^i \mathbb{I}(k_j^s = k') A_j(f_k, \mathbf{x}_i^s, \underline{\mathbf{x}}_i^s)$, the problem is reduced to how to efficiently compute $\sum_{s=1}^S \tilde{Y}_{k,k'}^s$.

We propose an efficient algorithm to compute $\sum_{s=1}^S \tilde{Y}_{k,k'}^s$, which is summarized in Algorithm 1. At the core of our algorithm are two levels of batching: (a) *intra-sequence batching*, which allow the computation of $\tilde{Y}_{k,k'}^s$ with only one call of $A(\cdot)$; and (b) *inter-sequence batching*, which enables batch computation of $\{Y_{k,k'}^s\}_{s \in \mathcal{B}}$ for a mini-batch of event sequences indexed by \mathcal{B} . We explain the details of these two levels of batching as follows.

Intra-Sequence Batching. As this part only deals with a particular event sequence, to simplify the notation, we omit the sequence index s for now. Note that $\mathbf{x}_1 \prec \mathbf{x}_2 \prec \dots \prec \mathbf{x}_n$ and due to the recurrent nature of f , all $f(\mathbf{x}_i)$ for $i \in [n]$ can be computed in a single forward pass with the shared input \mathbf{x}_n . Denote by $F = \{F_{k,i}(\cdot)\}_{k \in [K], i \in [n]}$ a matrix-valued function such that $F_{k,i}(\mathbf{x}_n) = f_k(\mathbf{x}_i)$ for any $k \in [K], i \in [n]$.

The equivalence between f and F means that,

$$A_j(f_k, \mathbf{x}_i, \underline{\mathbf{x}}_i) = A_j(F_{k,i}, \mathbf{x}_n, \underline{\mathbf{x}}_n),$$

which further implies that we can rewrite $\tilde{Y}_{k,k'}^s$ as a weighted sum of attribution scores for the same input \mathbf{x}_n and baseline $\underline{\mathbf{x}}_n$. Since we are not interested in computing the individual attribution scores but their sum, we can leverage the linearity

property (A1) to compute the attribution scores directly for the sum, as shown in the following proposition.

Proposition 3. For an attribution method $A(\cdot)$ with the linearity (A1) and the null player (A3), it holds that

$$\tilde{Y}_{k,k'}^s = \sum_{j=1}^{n_s} \mathbb{I}(k_j^s = k') A_j \left(\sum_{i=1}^{n_s} F_{i,k_i}, \mathbf{x}_n^s, \underline{\mathbf{x}}_n^s \right). \quad (12)$$

Proof. The proof is in Appendix B.3. \square

Inter-Sequence Batching. We now discuss how to efficiently compute $\{Y_{k,k'}^s\}_{s \in \mathcal{B}}$ for a mini-batch of event sequences indexed by \mathcal{B} . The key idea for a significant computational speed-up here is that if $A(\cdot)$ satisfies batchability (P2), we can then batch the computation of different sequences with a single call of $A(\cdot)$.

To simplify the discussion, we assume without loss of generality that $\mathcal{B} = \{1, \dots, |\mathcal{B}|\}$ and $n_s \equiv n$ for all $s \in \mathcal{B}$. Let $\mathbf{X} = [\mathbf{x}_s]_{s \in \mathcal{B}}$ and analogously the corresponding baselines $\underline{\mathbf{X}}$. We further override our previous notation and denote by $F = \{F_{k,i}^s(\cdot)\}_{s \in [S], k \in [K], i \in [n]}$ a new tensor-valued function such as that $F_{k,i}^s(\mathbf{X}) = f_k(\mathbf{x}_i^s)$. Then with Proposition 1, we have that

$$A \left(\sum_{s \in \mathcal{B}} \sum_{i=1}^{n_s} F_{k,i}^s, \mathbf{X}, \underline{\mathbf{X}} \right) = \left[A_j \left(\sum_{i=1}^{n_s} F_{i,k_i}, \mathbf{x}_n^s, \underline{\mathbf{x}}_n^s \right) \right]_{\substack{s \in \mathcal{B} \\ j \in [n]}}.$$

Time Complexity Analysis. With our two-level batching scheme, Algorithm 1 only requires $O(SK/B)$ invocations of $A(\cdot)$, a significant reduction from the $O(SNK)$ invocations required by a naive implementation that directly calculates $Y_{k,k'}$'s, where N is the average sequence length. Since modern computation hardware (such as GPUs) enables calling $A(\cdot)$ with a batch of inputs being almost as fast as calling it with a single input, our algorithm can achieve up to *three orders-of-magnitude speedup* over a naive implementation on datasets with relatively large N and B . (See Section 4.2.3 for empirical evaluations.)

3.4. Choice of Attribution Methods

In our experiments, we choose the attribution method $A(\cdot)$ to be the Integrated Gradients, which is defined as follows:

$$\text{IG}(f, \mathbf{x}, \underline{\mathbf{x}}) \triangleq (\mathbf{x} - \underline{\mathbf{x}}) \odot \int_0^1 \frac{\partial f(\tilde{\mathbf{x}})}{\partial \tilde{\mathbf{x}}} \Big|_{\tilde{\mathbf{x}} = \underline{\mathbf{x}} + \alpha(\mathbf{x} - \underline{\mathbf{x}})} d\alpha, \quad (13)$$

where \odot is the Hadamard product. Nevertheless, CAUSE does not depend on a specific attribution method but a set of properties that we have stated upfront; this means that any other attribution methods that satisfy these properties (e.g., DeepLIFT) should be applicable to CAUSE. Also

note that batchability (P2) is only used in the inter-sequence batching for speeding up the computation; thus, if efficiency is less of a concern, or the computation of attributions for different inputs can be accelerated in alternative ways,² attribution methods that only violate batchability, such as Shapley values, should also be applicable.

4. Experiments

In this section, we present the experiments that are designed to evaluate CAUSE and answer the following three questions:

- **Goodness-of-Fit:** How good is CAUSE at fitting multi-type event sequences?
- **Causality Discovery:** How accurate is CAUSE at discovering Granger causality between event types?
- **Scalability:** How scalable is CAUSE?

The experimental results on five datasets show that CAUSE (a) outperforms state-of-the-art methods in both fitting and discovering Granger causality from event sequences of diverse event interdependency, (b) can identify Granger causality on real-world datasets that agrees with human intuition, and (c) can compute the Granger causality statistic three orders-of-magnitude faster due to our optimization.

4.1. Experimental Setup

Datasets. We designed three synthetic datasets to reflect various types of event interactions and temporal effects.

- **Excitation:** This dataset was generated by a multivariate Hawkes process, whose CIFs are defined in (1). The exponential decay kernels were used, and a weighted ground-truth causality matrix was constructed with the ℓ_1 norms of the kernel functions $\phi_{k,k'}(\cdot)$.
- **Inhibition:** This dataset was generated by a multivariate self-correcting process (Isham & Westcott, 1979), whose CIFs are of the form: $\lambda_k^*(t) = \exp(\alpha_k t + \sum_{i:t_i < t} w_{k,k_i})$, where $\alpha_k > 0$ and $w_{k,k'} \leq 0$. A weighted ground-truth causality matrix was formed with the pairwise weights $w_{k,k'}$.
- **Synergy:** Generated by a proximal graphical event model (PGEM) (Bhattacharjya et al., 2018), this dataset contains synergistic effects between a pair of event types to a third (outcome) event type; that is, the occurrence of that pair of event types together would have a greater effect on an outcome event type than the simple sum of their individual effects. A binary ground-truth causality matrix was constructed from the dependency graph of the PGEM.

²In fact, for almost all attribution methods, the attribution for different inputs is embarrassingly parallelizable.

We also included two real-world datasets used in existing literature.

- **IPTV (Luo et al., 2015):** Each sequence records the history of TV watching behavior of a user, and the event types are the TV program categories. This dataset, however, does not contain ground-truth causality between TV program categories.
- **MemeTracker (MT):**³ Each sequence represents how a phrase or quote appeared on various online websites over time during the period of August 2008 to April 2009, and the event types are the domains of the top websites. Like previous studies (Achab et al., 2018; Xiao et al., 2019), a weighted ground-truth causality matrix was *approximated* by whether one site contains any URLs linking to another site.

The parameter settings for the synthetic datasets and the preprocessing steps for the real-world datasets are detailed in Appendix C.1. The statistics of the five datasets are summarized in Table 1.

Table 1: Statistics for various datasets.

Dataset	S	K	# of events	Ground truth
Excitation	1,000	10	250,447	Weighted
Inhibition	1,000	10	250,442	Weighted
Synergy	1,000	10	178,338	Binary
IPTV	1,869	16	966,338	N/A
MT	382,014	100	3,419,399	Weighted

Methods for Comparison. We compared our method to the following baselines:

- **HExp:** Hawkes process with fixed exponential kernels.
- **HSG and NHPC:** Hawkes process with sum of Gaussian kernels (Xu et al., 2016) and nonparametric Hawkes process cumulant matching (Achab et al., 2018). These two methods represent the state-of-the-art parametric and nonparametric methods for learning Granger causality for Hawkes process, respectively.
- **RPPN:** Recurrent point process network (Xiao et al., 2019), to the best of our knowledge, the only NPP that can provide summary statistics for Granger causality, which is enabled by its use of an attention mechanism.

The implementation details and hyperparameter configurations for CAUSE and various baselines are provided in Appendix C.2

Evaluation Metrics. The hold-out negative log-likelihood (NLL) was used for evaluating the goodness-of-fit of each method on various datasets, and the Kendall’s τ coefficient and the area under the ROC curve (AUC) were

³<https://www.memetracker.org/data.html>

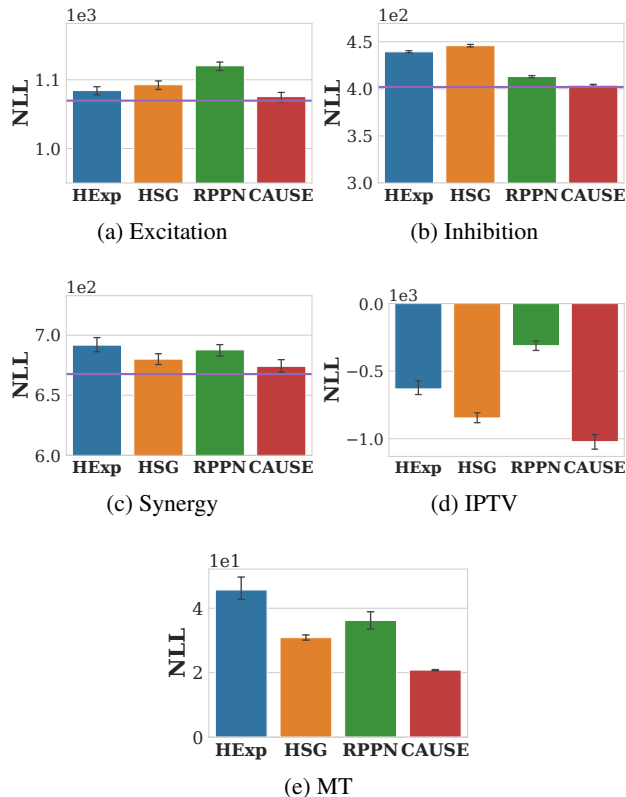


Figure 2: Hold-out NLLs of various methods, where horizontal lines denote the ground-truth NLLs. CAUSE attains the best NLLs on all datasets.

used for evaluating the estimated Granger causality matrix against the ground truth. Non-binary ground-truth causality matrices were binarized at zero in the evaluation of AUCs. We performed five-fold cross-validation and report the average results.

4.2. Detailed Results

4.2.1. GOODNESS-OF-FIT

We start by examining the goodness-of-fit of various methods on various datasets, since if a method fails to fit the data, it is unlikely to detect the true Granger causality between event types. As shown in Figure 2, CAUSE attains smaller NLLs than all baselines on all datasets, suggesting that CAUSE consistently has a better fit than all baselines. Notably, on all three synthetic datasets, the NLLs of CAUSE nearly match those computed by the ground-truth models. These results confirm the flexibility of CAUSE in learning the various types of event interactions and temporal effects.

4.2.2. CAUSALITY DISCOVERY

We now examined the performance of CAUSE on Granger causality discovery, both quantitatively and qualitatively.

Quantitative Analysis. Table 2 shows values of AUC and Kendall’s τ of various methods on the four datasets that have ground-truth causality. The results in the table support the following conclusions.

First, CAUSE performs the best overall and is most robust to various types of event interactions. It not only significantly outperforms all baselines on three of the four datasets (i.e., Inhibition, Synergy, and MT), but also achieves a close-second on Excitation, in which events were generated by a Hawkes process, and CAUSE is supposed to have a disadvantage relative to Hawkes process-based baselines.

Second, once the underlying data generation process violates the assumptions of Hawkes process and exhibits complex event interactions other than excitation, Hawkes process-based methods tend to perform poorly, as seen from Inhibition and Synergy.

Finally, despite both being NPP-based methods, RPPN performs significantly worse than CAUSE on all datasets. We suspect that this underperformance is caused by two issues in RPPN’s construction of the Granger causality statistics with the attention weights. First, RPPN restricts all attention weights to be positive, thus cannot distinguish between excitative and inhibitive effects. Second, attention mechanism may not correctly attribute the model’s prediction to its inputs, as shown in several recent studies (Jain & Wallace, 2019; Serrano & Smith, 2019).

Qualitative Analysis. Figure 3 shows the heat map for the Granger causality matrix of IPTV dataset estimated by CAUSE. Almost all diagonal entries have large positive values, indicating that users, on average, exhibit strong tendencies to watch the TV programs of the same category. Several positive associations between different TV program categories are also observed, such as from military, laws, finance, and education to news, and from kids and music to drama. These results agree with common sense and are consistent with the findings of an existing study with HSG (Xu et al., 2016). Our method also suggests several meaningful negative associations, including ads to drama and education to entertainment; such negative associations, however, can be completely ignored—or even mistakenly attributed as positive ones—by models that only consider the excitations between events, such as HSG.

Appendix C.4 provides a detailed analysis of the estimated Granger causality matrix for MT dataset. The analysis shows that CAUSE identifies major “information-consuming” domains, such as `news.google.com` and `googleheads.org`, an active forum for investment-related Q&A, and CAUSE detects credible excitative relationships between subdomains.

Table 2: Results for Granger causality discovery on the four datasets with ground-truth causality available. The best and the second best results on each dataset are **emboldened** and *italicized*, respectively.

Method	Excitation		Inhibition		Synergy		MT	
	AUC	Kendall's τ	AUC	Kendall's τ	AUC	Kendall's τ	AUC	Kendall's τ
HExp	0.858±0.004	0.453±0.005	<i>0.546±0.002</i>	<i>0.102±0.002</i>	0.872±0.058	0.251±0.039	0.404±0.009	-0.061±0.005
HSG	0.997±0.001	0.635±0.002	0.490±0.002	-0.013±0.002	0.876±0.007	0.254±0.039	<i>0.539±0.008</i>	<i>0.024±0.005</i>
NPHC	0.782±0.007	0.337±0.010	0.400±0.054	-0.138±0.067	0.741±0.129	0.163±0.087	N/A	N/A
RPPN	0.595±0.010	0.136±0.012	0.448±0.003	-0.066±0.002	<i>0.891±0.043</i>	<i>0.264±0.029</i>	0.492±0.004	-0.005±0.002
CAUSE	<i>0.920±0.012</i>	<i>0.533±0.013</i>	0.921±0.021	0.532±0.021	0.991±0.004	0.331±0.003	0.623±0.012	0.075±0.007

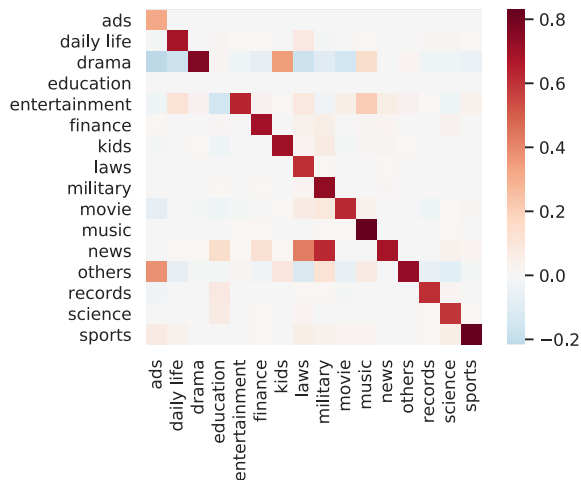


Figure 3: Visualization of the estimated Granger causality statistic matrices on IPTV. Better viewed on screen.

4.2.3. SCALABILITY

Finally, we investigate the scalability of CAUSE in computing the Granger causality statistic by Algorithm 1. Figure 4 shows how much speedup Algorithm 1 achieves over a naive implementation with different sequence lengths and batch sizes. The results demonstrate that with batch size and average sequence length both being relatively large (i.e., greater or equal to 16 and 100, respectively), our algorithm can achieve over *three orders-of-magnitude speedup* relative to a native implementation. Furthermore, the speedup scales almost linearly with sequence length and batch size when they do not exceed 150 and 16, respectively, which is consistent with our analysis in Section 3.3. Beyond this regime, only a sublinear relationship between the speedup and batch size or sequence length is observed, which is because the GPU we tested on was reaching its maximum utilization.

5. Conclusion

We have presented CAUSE, a novel framework for learning Granger causality between event types from multi-type

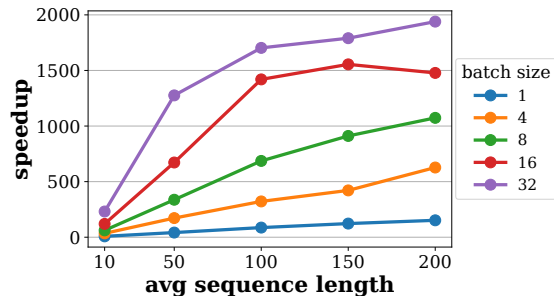


Figure 4: The speedup achieved by Algorithm 1 relative to a naive implementation with different average sequence lengths and batch sizes.

event sequences. At the core of CAUSE are two steps: first, it trains a flexible NPP model to capture the complex event interdependency, then it computes a novel Granger causality statistic by inspecting the trained model with an axiomatic attribution method. A two-level batching algorithm is derived to compute the statistic efficiently. We evaluate CAUSE on both synthetic and real-world datasets abundant with diverse event interactions and show the effectiveness of CAUSE on identifying the Granger causality between event types.

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