
Provably Efficient Model-based Policy Adaptation

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Abstract

The high sample complexity of reinforcement learning challenges its use in practice. A promising approach is to quickly adapt pre-trained policies to new environments. Existing methods for this policy adaptation problem typically rely on domain randomization and meta-learning, by sampling from some distribution of target environments during pre-training, and thus face difficulty on out-of-distribution target environments. We propose new model-based mechanisms that are able to make online adaptation in unseen target environments, by combining ideas from no-regret online learning and adaptive control. We prove that the approach learns policies in the target environment that can recover trajectories from the source environment, and establish the rate of convergence in general settings. We demonstrate the benefits of our approach for policy adaptation in a diverse set of continuous control tasks, achieving the performance of state-of-the-art methods with much lower sample complexity. Our project website, including code, can be found at <https://yudasong.github.io/PADA>.

1. Introduction

Deep Reinforcement Learning (RL) methods typically require a very large number of interactions with environments, making them difficult to be used on practical systems (Tan et al., 2018). A promising direction is to adapt policies trained in one environment to similar but unseen environments, such as from simulation to real robots. Existing approaches for policy adaptation mostly focus on pre-training the policies to be robust to predefined distributions of disturbances in the environment, by increasing the sample diversity during training (Peng et al., 2018; Tobin et al., 2017;

Mordatch et al., 2015), or meta-learn policies or models that can be quickly adapted to in-distribution environments (Finn et al., 2017a; Nagabandi et al., 2019a,b; Yu et al., 2018a). A key assumption for these approaches is that the distribution of the target environments is known, and that it can be efficiently sampled during training. On out-of-distribution target environments, these methods typically do not deliver good performance, reflecting common challenges in generalization (Na et al., 2020). If we observe how humans and animals adapt to environment changes, clearly there is an online adaptation process in addition to memorization (Staddon, 2016). We can quickly learn to walk with a slightly injured leg even if we have not experienced the situation. We draw experiences from normal walking and adapt our actions online, based on how their effects differ from what we are familiar with in normal settings. Indeed, this intuition has recently led to practical approaches for policy adaptation. The work in (Christiano et al., 2016) uses a pre-trained policy and model of the training environment, and learns an inverse dynamics model from scratch in the new environment by imitating the behaviors of the pre-trained policy. However, it does not involve mechanisms for actively reducing the divergence between the state trajectories of the two environments, which leads to inefficiency and distribution drifting, and does not fully capture the intuition above. The work in (Zhu et al., 2018) uses Generative Adversarial Imitation Learning (GAIL) (Ho & Ermon, 2016) to imitate the source trajectories in the new environment, by adding the GAIL discriminator to the reward to reduce divergence, but relies on generic policy optimization methods with high sample complexity. In general, these recent approaches show the feasibility of policy adaptation, but are not designed for optimizing sample efficiency. There has been no theoretical analysis of whether policy adaptation methods can converge in general, or their benefits in terms of sample complexity.

In this paper, we propose a new model-based approach for the policy adaptation problem that focuses on efficiency with theoretical justification. In our approach, the agent attempts to predict the effects of its actions based on a model of the training environment, and then adapts the actions to minimize the divergence between the state trajectories in the new (target) environment and in the training (source) environment. This is achieved by iterating between two steps: a

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modeling step learns the divergence between the source environment and the target environment, and a planning step that uses the divergence model to plan actions to reduce the divergence over time. Under the assumption that the target environment is close to the source environment, the divergence modeling and policy adaptation can both be done locally and efficiently. We give the first theoretical analysis of policy adaptation by establishing the rate of convergence of our approaches under general settings. Our methods combine techniques from model-based RL (Wang et al., 2019) and no-regret online learning (Ross et al., 2011). We demonstrate that the approach is empirically efficient in comparison to the state-of-the-art approaches (Christiano et al., 2016; Zhu et al., 2018). The idea of recovering state trajectories from the source environment in the target environment suggests a strong connection between policy adaptation and imitation learning, such as Learning from Observation (LfO) (Torabi et al., 2018; 2019a; Sun et al., 2019b; Yang et al., 2019). A key difference is that in policy adaptation, the connection between the source and target environments and their difference provide both new challenges and opportunities for more efficient learning. By actively modeling the divergence between the source and target environments, the agent can achieve good performance in new environments by only making local changes to the source policies and models. On the other hand, because of the difference in the dynamics and the action spaces, it is not enough to merely imitate the experts (Bain & Sommut, 1999; Ross et al., 2011; Sun et al., 2017). Traditionally, adaptive control theory (Åström, 1983) studies how to adapt to disturbances by stabilizing the error dynamics. Existing work in adaptive control assumes closed-form dynamics and does not apply to the deep RL setting (Nagabandi et al., 2019a). In comparison to domain randomization and meta-learning approaches, our proposed approach does not require sampling of source environments during pre-training, and makes it possible to adapt in out-of-distribution environments. Note that the two approaches are complementary, and we demonstrate in experiments that our methods can be used in conjunction with domain randomization and meta-learning to achieve the best results.

The paper is organized as follows. We review related work in Section 2 and the preliminaries in Section 3. In Section 4, we propose the theoretical version of the adaptation algorithm and prove its rate of convergence. In section 5 we describe the practical implementation using deviation models and practical optimization methods. We show detailed comparison with competing approaches in Section 6.

2. Related Work

Our work connects to existing works on imitation learning, online adaptation, domain randomization and meta-learning, and model-based reinforcement learning.

Imitation Learning. In imitation learning, there is typically no separation between training environments and test environments. Existing imitation learning approaches aim to learn a policy that generates state distributions (Tobin et al., 2017; Torabi et al., 2019b; Sun et al., 2019b; Yang et al., 2019) or state-action distributions (Ho & Ermon, 2016; Fu et al., 2017; Ke et al., 2019; Ghasemipour et al., 2019) that are similar to the ones given by the expert policy. The difference in the policy adaptation setting is that the expert actions do not work in the first place in the new environment, and we need to both model the divergence and find a new policy for the target environment. In light of this difference, the work (Zhu et al., 2018) considers a setting that is the closest to ours (which we will compare with in the experiments). It uses a state-action-imitation (GAIL (Ho & Ermon, 2016)) approach to learn a policy in the target environment, to generate trajectories that are similar to the trajectories of the expert from the source environments. It also relies on using the true reward signals in the target environment to train the policy besides state imitation. In recent work, (Liu et al., 2020) approaches a similar problem by using Wasserstein distance between the states as the reward. It uses adversarial training and model-free policy optimization methods. Our approach is model-based and relies on reduction to Data Aggregation (Ross et al., 2011) for efficiency. The reduction allows us to derive provable convergence guarantee with the rate of convergence. Experimentally we show that our approach is more sample efficient than model-free and minmax-based imitation approaches in general.

Online Adaptation. Online adaptation methods transfer policies by learning a mapping between the source and target domains (Daftry et al., 2016; Tzeng et al., 2015). Such methods have achieved success in vision-based robotics but require extra kernel functions or learning feature spaces. In contrast, we focus on control problems where the policy adaptation is completely autonomous. (Christiano et al., 2016) trains an inverse dynamics model (IDM) which can serve as the target policy by inquiring the source policy online. However, the approach does not focus on optimizing sample efficiency, which is crucial for the use of policy adaptation. In (Yu et al., 2018b), the agent selects from a range of pre-trained policies online, and does not perform further adaptation, and thus experiences problems similar to domain randomization approaches.

Domain Randomization and Meta-Learning. Domain randomization and meta-learning methods are popular ways of transferring pre-trained policies to new environments. These methods rely on the key assumption that the training environments and test environments are sampled from the same predefined distribution. Domain randomization methods train robust agents on diverse samples of target environments (Tobin et al., 2017; Mordatch et al., 2015; Antonova et al., 2017; Chebotar et al., 2019). When the configuration

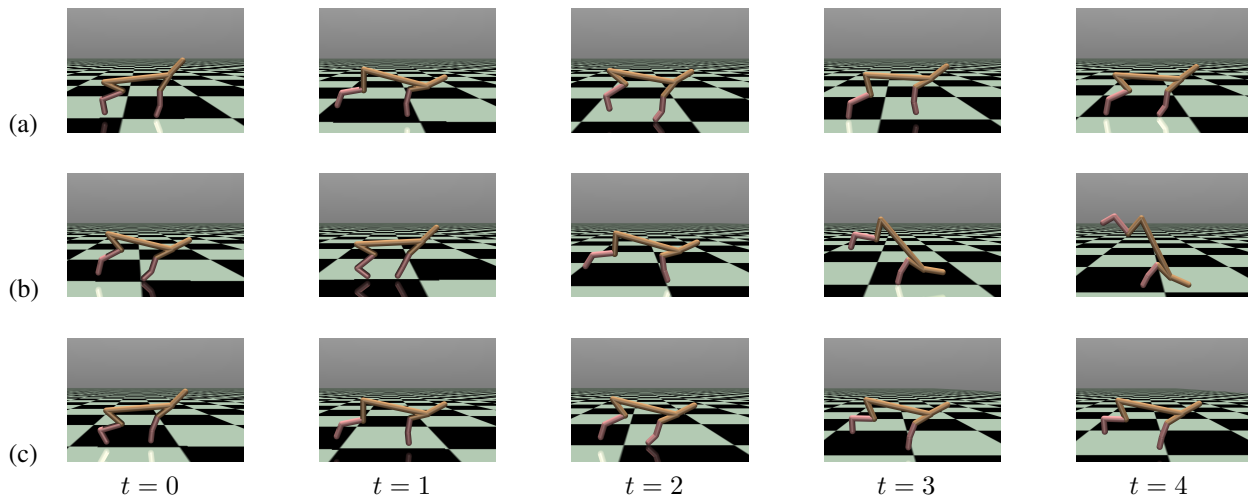


Figure 1. (a) Halfcheetah of mass m using the source policy $\pi^{(s)}$ in the source environment $\mathcal{M}^{(s)}$. (b) Halfcheetah of mass $1.5 \times m$ using the source policy $\pi^{(s)}$ in the target environment $\mathcal{M}^{(t)}$. (c) Halfcheetah of mass $1.5 \times m$ using the learned target policy $\pi^{(t)}$ in $\mathcal{M}^{(t)}$ with our method. Using the policy trained in the source environment without adapting it to the target environment yields suboptimal results. The adapted policy $\pi^{(t)}$ can recover gaits close to the source trajectory.

of the target environment lies outside the training distribution, there is no performance guarantee. In meta-learning, such as Model Agnostic Meta-Learning (MAML) (Finn et al., 2017a;b; Nagabandi et al., 2018), meta-learned dynamics policies and models can adapt to perturbed environments with notable success including in physical robots. However, similar to domain randomization based approaches, they experience difficulties on new environments that are not covered by the training distribution. Our proposed approach focuses on online adaptation in unseen environments. It is orthogonal to domain randomization and meta-learning approaches. We show in experiments that these different approaches can be easily combined.

Model-based Reinforcement Learning. Model-based reinforcement learning (MBRL) provides a paradigm that learns the environment dynamics and optimizes the control actions at the same time. Recent work has shown that MBRL has much better sample efficiency compared to model-free approaches both theoretically and empirically (Tu & Recht, 2018; Chua et al., 2018; Sun et al., 2019a). Our setting is different from the traditional MBRL setting. We consider test environments that are different from the training environment, and adapt the policy from the training environment to the test environment.

3. Preliminaries

We consider finite-horizon Markov Decision Processes (MDP) $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, f, H, R \rangle$ with the following components. \mathcal{S} denotes the state space, and \mathcal{A} the action space. The transition function $f: \mathcal{S} \times \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$ deter-

mines the probability $f(s'|s, a)$ of transitioning into state s' from state s after taking action a . The reward function $R: \mathcal{S} \rightarrow \mathbb{R}$ is defined *only* on states. We write π_θ to denote a stochastic policy $\pi_\theta: \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$ parameterized by θ . Each policy π_θ determines a distribution over trajectories $\{(s_i, a_i, r_i)\}_{i=1}^H$ under a fixed dynamics f . The goal of the agent is to maximize the expected cumulative reward $J(\theta) = \mathbb{E}_{\pi_\theta, f} \left[\sum_{h=1}^H R(s_h) \right]$ over all possible trajectories that can be generated by π_θ . Without loss of generality, in the theoretical analysis we always assume the normalized total reward is in the $[0, 1]$ range, i.e., $\max_{s_1, \dots, s_H} \sum_{h=1}^H R(s_h) \in [0, 1]$.

We write the set $\{1, 2, \dots, N\}$ as $[N]$, and the uniform distribution over set A as $U(A)$ throughout the paper. For any two distributions d_1 and d_2 , we use $\|d_1 - d_2\|$ to denote the total variation distance between the two distributions.

4. Policy Adaptation with Data Aggregation

4.1. Basic Definitions

In policy adaptation, we consider a pair of MDPs and call them a source MDP and a target MDP. We define the source MDP as $\mathcal{M}^{(s)} := \{\mathcal{S}, \mathcal{A}^{(s)}, f^{(s)}, H, R\}$ and the target MDP as $\mathcal{M}^{(t)} := \{\mathcal{S}, \mathcal{A}^{(t)}, f^{(t)}, H, R\}$. Note that the two MDPs share the same state space and reward functions, but can have different action spaces and transition dynamics. Fig. 1 demonstrates the problem of adapting a policy from a source environment to a target environment. Because of the difference in the action space and dynamics, directly using good policies from the source environment (Fig.1(a))

does not work in the target environment (Fig.1(b)). The objective is to adapt the policy from the source to the target environment to achieve good performance (Fig.1(c)).

We focus on minimizing the samples needed for adaptation in the target MDP, by leveraging $\mathcal{M}^{(s)}$ to quickly learn a policy in $\mathcal{M}^{(t)}$. To achieve this, we assume that a pre-trained policy $\pi^{(s)}$ from $\mathcal{M}^{(s)}$ achieves high rewards in $\mathcal{M}^{(s)}$. We wish to adapt $\pi^{(s)}$ to a policy $\pi^{(t)}$ that works well in $\mathcal{M}^{(t)}$. For ease of presentation, we consider $\pi^{(s)}$ and $\pi^{(t)}$ as deterministic throughout the theoretical analysis.

Given a policy π , we write $\mathbb{E}_\pi^{(s)}(\cdot)$ for the expectation over random outcomes induced by π and $\mathcal{M}^{(s)}$. We write $d_{\pi;h}^{(s)}$ to denote the state distribution induced by π at time step h under $\mathcal{M}^{(s)}$, and $d_\pi^{(s)} = \sum_{h=1}^H d_{\pi;h}^{(s)}/H$ as the average state distribution of π under $\mathcal{M}^{(s)}$. We write $\rho_\pi^{(s)}$ to represent the distribution of the state trajectories from π : for $\tau = \{s_h\}_{h=0}^H$, $\rho_\pi^{(s)}(\tau) = \prod_{h=1}^H f^{(s)}(s_h|s_{h-1}, \pi(s_{h-1}))$. For the target MDP $\mathcal{M}^{(t)}$, we make the same definitions but drop the superscript (t) for ease of presentation. Namely, $\mathbb{E}_\pi(\cdot)$ denotes the expectation over the randomness from π and $\mathcal{M}^{(t)}$, d_π denotes the induced state distribution of π under $\mathcal{M}^{(t)}$, and ρ_π denotes the state trajectory distribution.

4.2. Algorithm

We now introduce the main algorithm Policy Adaptation with Data Aggregation (PADA). Note that this is the theoretical version of the algorithm, and the practical implementation will be described in detail in Section 5. To adapt a policy from a source environment to a target environment, PADA learns a model \hat{f} to approximate the target environment dynamics $f^{(t)}$. Based on the learned model, the algorithm generates actions that attempt to minimize the divergence between the trajectories in the target environment and those in the source environment generated by $\pi^{(s)}$ at $\mathcal{M}^{(s)}$. Namely, the algorithm learns a policy $\pi^{(t)}$ that reproduces the behavior of $\pi^{(s)}$ on $\mathcal{M}^{(s)}$ in the target MDP $\mathcal{M}^{(t)}$. Since the state space \mathcal{S} is often large, learning a model \hat{f} that can accurately approximate $f^{(t)}$ globally is very costly. Instead, we only aim to iteratively learn a locally accurate model, i.e., a model that is accurate near the states that are generated by $\pi^{(t)}$. This is the key to efficient adaptation.

The detailed algorithm is summarized in Alg. 1. Given a model \hat{f}_e at the e -th iteration, we define the ideal policy $\pi_e^{(t)}$

$$\pi_e^{(t)}(s) \triangleq \operatorname{argmin}_{a \in \mathcal{A}^{(t)}} \|\hat{f}_e(\cdot|s, a) - f^{(s)}(\cdot|s, \pi^{(s)}(s))\|. \quad (1)$$

The intuition is that, assuming \hat{f}_e is accurate in terms of modelling $f^{(t)}$ at state s , $\pi_e^{(t)}(s)$ aims to pick an action such that the resulting next state distribution under \hat{f}_e is similar to the next state distribution resulting from $\pi^{(s)}$ under the

Algorithm 1 Policy Adaptation with Data Aggregation

Require: Source domain policy $\pi^{(s)}$, source dynamics $f^{(s)}$, model class \mathcal{F}

- 1: Initialize dataset $\mathcal{D} = \emptyset$
- 2: Initialize \hat{f}_1
- 3: **for** $e = 1, \dots, T$ **do**
- 4: Define policy $\pi_e^{(t)}$ as in Eq. 1
- 5: **for** $n = 1, \dots, N$ **do**
- 6: Reset $\mathcal{M}^{(t)}$ to a random initial state
- 7: Uniformly sample a time step $h \in [H]$
- 8: Execute $\pi_e^{(t)}$ in $\mathcal{M}^{(t)}$ for h steps to get state s
- 9: Sample exploration action $a \sim U(\mathcal{A}^{(t)})$
- 10: Take a in $\mathcal{M}^{(t)}$ and get next state s'
- 11: Add (s, a, s') into \mathcal{D} (Data Aggregation)
- 12: **end for**
- 13: Update to \hat{f}_{e+1} via MLE by Eq. 2
- 14: **end for**
- 15: **Output:** $\{\pi_e^{(t)}\}_{e=1}^T$

source dynamics $f^{(s)}$. We then execute $\pi_e^{(t)}$ in the target environment $\mathcal{M}^{(t)}$ to generate a batch of data (s, a, s') . We further aggregate the newly generated data to the dataset \mathcal{D} (i.e., data aggregation). We update model to \hat{f}_{e+1} via Maximum Likelihood Estimation (MLE) on \mathcal{D} :

$$\hat{f}_{e+1} = \operatorname{argmax}_{\hat{f} \in \mathcal{F}} \sum_{s, a, s' \in \mathcal{D}} \log f(s'|s, a). \quad (2)$$

Note that Algorithm 1 relies on two black-box offline computation oracles: (1) a *one-step* minimization oracle (Eq. 1) and (2) a Maximum Likelihood Estimator (Eq. 2). In Section 5, we will introduce practical methods to implement these two oracles. We emphasize here that these two oracles are offline computation oracles and the computation itself does not require any fresh samples from the target environment $\mathcal{M}^{(t)}$.

4.3. Analysis

We now prove the performance guarantee of Alg.1 for policy adaptation and establish its rate of convergence. At a high level, our analysis of Alg. 1 is inspired from the analysis of DAgger (Ross et al., 2011; Ross & Bagnell, 2012) which leverages a reduction to no-regret online learning (Shalev-Shwartz et al., 2012). We will first make the connection with the Follow-the-Leader (FTL) algorithm, a classic no-regret online learning algorithm, on a sequence of loss functions. We then show that we can transfer the no-regret property of FTL to performance guarantee on the learned policy $\pi^{(t)}$. Our analysis uses the FTL regret bound $\tilde{O}(1/T)$ where T is the number of iterations (Shalev-Shwartz et al., 2012). Since our analysis is a reduction to general no-regret online learning, in theory we can also replace FTL by other no-regret

online learning algorithms as well (e.g., Online Gradient Descent (Zinkevich, 2003) and AdaGrad (Duchi et al., 2011)). Intuitively, for fast policy adaptation to succeed, one should expect that there is similarity between the source environment and the target environment. We formally introduce the following assumption to quantify this.

Assumption 4.1 (Adaptability). *For any state action pair (s, a) with source action $a \in \mathcal{A}^{(s)}$, there exists a target action $a' \in \mathcal{A}^{(t)}$ in target environment, such that:*

$$\|f^{(s)}(\cdot|s, a) - f^{(t)}(\cdot|s, a')\| \leq \epsilon_{s,a},$$

for some small $\epsilon_{s,a} \in \mathbb{R}^+$.

Remark 4.2. *When $\epsilon_{s,a} \rightarrow 0$ in the above assumption, the target environment can perfectly recover the dynamics of the source domain at (s, a) . However, $\epsilon_{s,a} = 0$ does not mean the two transitions are the same, i.e., $f^{(t)}(s, a) = f^{(s)}(s, a)$. First the two action spaces can be widely different. Secondly, there may exist states s , where one may need to take completely different target actions from $\mathcal{A}^{(t)}$ in order to match the source transition $f^{(s)}(\cdot|s, a)$, i.e., $\exists a' \in \mathcal{A}^{(t)}$ such that $f^{(t)}(\cdot|s, a') = f^{(s)}(\cdot|s, a)$, but $a \neq a'$.*

Assumption 4.3 (Realizability). *Let the model class \mathcal{F} be a subset of $\{f : \mathcal{S} \times \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]\}$. We assume $f^{(t)} \in \mathcal{F}$.*

Here we assume that our model class \mathcal{F} is rich enough to include $f^{(t)}$. Note that the assumption on realizability is just for analysis simplicity. Agnostic results can be achieved with more refined analysis similar to (Ross et al., 2011; Ross & Bagnell, 2012).

We define the following loss function:

$$\ell_e(f) \triangleq \mathbb{E}_{s \sim d_{\pi_e^{(t)}}, a \sim U(\mathcal{A}^{(t)})} \left[D_{KL} \left(f^{(t)}(\cdot|s, a), f(\cdot|s, a) \right) \right],$$

for all $e \in [T]$. The loss function $\ell_e(f)$ measures the difference between $f^{(t)}$ and f under the state distribution induced by $\pi_e^{(t)}$ under $\mathcal{M}^{(t)}$ and the uniform distribution over the action space. This definition matches the way we collect data inside each episode. We generate (s, a, s') triples via sampling s from $d_{\pi_e^{(t)}}$, a from $U(\mathcal{A}^{(t)})$, and then $s' \sim f^{(t)}(\cdot|s, a)$. At the end of the iteration e , the learner uses FTL to compute \hat{f}_{e+1} as:

$$\hat{f}_{e+1} = \operatorname{argmin}_{f \in \mathcal{F}} \sum_{i=1}^e \ell_i(f).$$

Using the definition of KL-divergence, it is straightforward to show that the above optimization is equivalent to the following Maximum Likelihood Estimation:

$$\operatorname{argmax}_{f \in \mathcal{F}} \sum_{i=1}^e \mathbb{E}_{s \sim d_{\pi_e^{(t)}}, a \sim U(\mathcal{A}^{(t)}), s' \sim f^{(t)}(\cdot|s, a)} [\log f(s'|s, a)].$$

At the end of the episode e , the aggregated dataset \mathcal{D} contains triples that are sampled based on the above procedure from the first to the e -th episode.

With no-regret learning on \hat{f}_e , assumptions 4.1, and 4.3, we can obtain the following main results. We first assume that the target environment $\mathcal{M}^{(t)}$ has a discrete action space, i.e., $\mathcal{A}^{(t)}$ is discrete, and then show that the result can be easily extended to continuous action spaces.

Theorem 4.4 (Main Theorem). *Assume $\mathcal{M}^{(t)}$ has a discrete action space $\mathcal{A}^{(t)}$ and denote $A \triangleq |\mathcal{A}^{(t)}|$. Among the sequence of policies computed in Alg. 1, there exists a policy $\hat{\pi}$ such that:*

$$\begin{aligned} \mathbb{E}_{s \sim d_{\hat{\pi}}} \|f^{(t)}(\cdot|s, \hat{\pi}(s)) - f^{(s)}(\cdot|s, \pi^{(s)}(s))\| \\ \leq O \left(AT^{-1/2} + \mathbb{E}_{s \sim d_{\hat{\pi}}} [\epsilon_{s, \pi^{(s)}(s)}] \right), \end{aligned}$$

which implies that:

$$\left\| \rho_{\hat{\pi}} - \rho_{\pi^{(s)}} \right\| \leq O \left(HAT^{-1/2} + H \mathbb{E}_{s \sim d_{\hat{\pi}}} [\epsilon_{s, \pi^{(s)}(s)}] \right),$$

where we recall that ρ_{π} stands for the state-trajectory distribution of policy π under $\mathcal{M}^{(t)}$ and $\rho_{\pi^{(s)}}$ stands for the state-trajectory distribution of $\pi^{(s)}$ under $\mathcal{M}^{(s)}$.

The full proof is in the Appendix A. The theorem shows that our algorithm can provide a policy in the target environment that induces trajectories close to those induced by the experts in the source environment. For instance, if the target and source MDPs are completely adaptable (i.e., $\epsilon_{s,a} = 0$ in Assumption 4.1 for all (s, a)) and the number of iterations approach to infinity, then we can learn a policy $\hat{\pi}$ that generates state trajectories in $\mathcal{M}^{(t)}$ that match the state trajectories generated via the source policy $\pi^{(s)}$ at the source MDP $\mathcal{M}^{(s)}$.

Remark 4.5. *The error $\mathbb{E}_{s \sim d_{\hat{\pi}}} [\epsilon_{s, \pi^{(s)}(s)}]$ is averaged over the state distribution induced by the learned policy rather than in an ℓ_{∞} form, i.e., $\max_{s,a} \epsilon_{s,a}$.*

Although the analysis is done on discrete action space, the algorithm can be naturally applied to compact continuous action space as follows. The proof of the following corollary and its extension to the d -dimensional continuous action spaces are in the Appendix.

Corollary 4.6 (Continuous Action Space). *Assume $\mathcal{A}^{(t)} = [0, 1]$, $f^{(t)}$ and functions $f \in \mathcal{F}$ are Lipschitz continuous with (and only with) actions in $\mathcal{A}^{(t)}$. Among policies returned from Alg. 1, there exists a policy $\hat{\pi}$ such that:*

$$\left\| \rho_{\hat{\pi}} - \rho_{\pi^{(s)}} \right\| \leq O \left(HT^{-1/4} + H \mathbb{E}_{s \sim d_{\hat{\pi}}} [\epsilon_{s, \pi^{(s)}(s)}] \right).$$

Remark 4.7. *As we assume the reward function only depends on states, $\|\rho_{\hat{\pi}} - \rho_{\pi^{(s)}}\| \leq \delta$ implies $|J^{(t)}(\hat{\pi}) - J^{(s)}(\pi^{(s)})| \leq \|\rho_{\hat{\pi}} - \rho_{\pi^{(s)}}\| \left(\max_{s_1 \dots s_H} \sum_h R(s_h) \right) \leq \delta$*

due to the normalization assumption on the rewards. Thus, though our algorithm runs without rewards, when $\pi^{(s)}$ achieves high reward in the source MDP $\mathcal{M}^{(s)}$, the algorithm is guaranteed to learn a policy $\hat{\pi}$ that achieves high rewards in the target environment $\mathcal{M}^{(t)}$.

5. Practical Implementation

In Algorithm 1 we showed the theoretical version of our approach, which takes an abstract model class \mathcal{F} as input and relies on two offline computation oracles (Eq. 1 and Eq. 2). We now design the practical implementation by specifying the parameterization of the model class \mathcal{F} and the optimization oracles. Algorithm 2 shows the practical algorithm, and we explain the details in this section.

5.1. Model Parameterization

In the continuous control environments, we focus on stochastic transitions with Gaussian noise, where $f^{(t)}(s, a) = \bar{f}^{(t)}(s, a) + \epsilon$, $f^{(s)}(s, a) = \bar{f}^{(s)}(s, a) + \epsilon'$, with ϵ and ϵ' from $\mathcal{N}(0, \Sigma)$ and $\bar{f}^{(t)}$ and $\bar{f}^{(s)}$ being nonlinear deterministic functions. In this case, we consider the following model class with parameterization θ :

$$\mathcal{F} = \{\delta_\theta(s, a) + \hat{f}^{(s)}(s, \pi^{(s)}(s)), \forall s, a : \theta \in \Theta\}.$$

where $\hat{f}^{(s)}$ is a pre-trained model of the source dynamics $f^{(s)}$ and we assume $\hat{f}^{(s)}$ well approximates $f^{(s)}$ (and one has full control to the source environment such as the ability to reset). Then for each state s , $\hat{f}^{(s)}(s, \pi^{(s)}(s))$ is a fixed distribution of the next state in the source environment by following the source policy. Define $\Delta^{\pi^{(s)}}(s, a) \triangleq \hat{f}^{(s)}(s, \pi^{(s)}(s)) - f^{(t)}(s, a)$, which captures the deviation from taking action a in the target environment to following $\pi^{(s)}$ in the source environment. So $\delta_\theta(s, a)$ is trained to approximate the deviation $\Delta^{\pi^{(s)}}(s, a)$. Note that learning $\Delta^{\pi^{(s)}}$ is just an alternative way to capture the target dynamics since we know $\hat{f}^{(s)}(s, \pi^{(s)})$ foresight, thus it should be no harder than learning $f^{(t)}$ directly.

5.2. Model Predictive Control

For deterministic transition, Eq 1 reduces to one-step minimization $\operatorname{argmin}_{a \in \mathcal{A}^{(t)}} \|\hat{f}_e(s, a) - \hat{f}^{(s)}(s, \pi^{(s)}(s))\|_2$. Since $\hat{f}_e \in \mathcal{F}$, we have $\hat{f}_e(s, a) = \delta_{\theta_e}(s, a) + \hat{f}^{(s)}(s, \pi^{(s)}(s))$, and the optimization can be further simplified to: $\operatorname{argmin}_{a \in \mathcal{A}^{(t)}} \|\delta_{\theta_e}(s, a)\|_2$. We use the Cross Entropy Method (CEM) (Botev et al., 2013) which iteratively repeats: randomly draw N actions, evaluate them in terms of the objective value $\|\delta_{\theta_e}(s, a)\|_2$, pick the top K actions in the increasing order of the objective values, and then refit a new Gaussian distribution using the empirical mean and covariance of the top K actions.

Algorithm 2 Policy Adaptation with Data Aggregation via Deviation Model

Require: $\pi_s, \hat{f}^{(s)}$, deviation model class $\{\delta_\theta : \theta \in \Theta\}$, explore probability ϵ , replay buffer \mathcal{D} , learning rate η

- 1: Randomly initialize divergence model δ_θ
- 2: **for** T Iterations **do**
- 3: **for** n steps **do**
- 4: $s \leftarrow$ Reset $\mathcal{M}^{(t)}$
- 5: **while** current episode does not terminate **do**
- 6: With probability ϵ : $a \sim U(\mathcal{A}^{(t)})$
- 7: Otherwise: $a \leftarrow$ CEM($\mathcal{A}^{(t)}, s, \delta_\theta$)
- 8: Execute a in $\mathcal{M}^{(t)}$: $s' \leftarrow f^{(t)}(s, a)$
- 9: Update replay buffer: $\mathcal{D} \leftarrow \mathcal{D} \cup \{(s, a, s')\}$
- 10: $s \leftarrow s'$
- 11: **end while**
- 12: **end for**
- 13: Update θ with Eq. 3
- 14: **end for**

We emphasize here we only need to solve a *one-step* optimization problem without unrolling the system for multiple steps. We write the CEM oracle as $\text{CEM}(\mathcal{A}^{(t)}, s, \delta_\theta)$ which outputs an action a from $\mathcal{A}^{(t)}$ that approximately minimizes $\|\delta_\theta(s, a)\|_2$. Here, $\text{CEM}(\mathcal{A}^{(t)}, s, \delta_\theta) : \mathcal{S} \rightarrow \mathcal{A}^{(t)}$ can be considered as a policy that maps state s to a target action a .

5.3. Experience Replay for Model Update

Note that Alg. 1 requires to solve a batch optimization problem (MLE in Eq. 2) in every iteration, which could be computationally expensive in practice. We use Experience Replay (Adam et al., 2011; Mnih et al., 2013), which is more suitable to optimize rich non-linear function approximators (δ_θ is a deep neural network in our experiments). Given the current divergence model δ_θ and the aggregated dataset $\mathcal{D} = \{s, a, s'\}$ (aka, replay buffer) with $s' = f^{(t)}(s, a)$, we randomly sample a mini-batch $B \subset \mathcal{D}$ and perform a stochastic gradient descent step with learning rate η :

$$\theta \leftarrow \theta - \frac{\eta}{|B|} \nabla_\theta \left(\sum_{i=1}^{|B|} \|\hat{f}^{(s)}(s_i, \pi^{(s)}(s_i)) + \delta_\theta(s_i, a_i) - s'_i\|_2^2 \right). \quad (3)$$

5.4. Policy Adaptation with Data Aggregation

As show in Algorithm 2, we maintain a replay buffer that stores all experiences from the target model $\mathcal{M}^{(t)}$ (Line 2) and constantly update the model δ_θ using mini-batch SGD (Eq. 3). Alg 2 performs local exploration in an ϵ -greedy way. We refer our method as Policy Adaptation with Data Aggregation via Deviation Model (**PADA-DM**).

Remark 5.1. Even being one-step, $\text{CEM}(\mathcal{A}^{(t)}, s, \delta_\theta)$ may be computationally expensive, we could obtain a MPC-free policy (target policy) by training an extra parameterized

policy to mimic $\text{CEM}(\mathcal{A}^{(t)}, s, \delta_\theta)$ via techniques of Behavior Cloning (Bain & Sommut, 1999). When we train this extra parameterized policy, we name the method as **PADA-DM with target policy** and we will show it does not affect the performance of the overall algorithm during training. However, during test time, such parameterized policy runs faster than CEM and thus is more suitable to be potentially deployed on real-world systems.

6. Experiments

In this section we compare our approach with the state-of-the-art methods for policy adaptation (Christiano et al., 2016; Zhu et al., 2018; Schulman et al., 2017; Finn et al., 2017a) and show that we achieve competitive results more efficiently. We also test the robustness of the approaches on multi-dimensional perturbations. We then compare to domain randomization and meta-learning approaches and show how they can be combined with our approach. We provide further experiments in Appendix D.

Following the same experiment setup as (Christiano et al., 2016), We focus on standard OpenAI Gym (Brockman et al., 2016) and Mujoco (Todorov et al., 2012) control environments such as HalfCheetah, Ant, and Reacher. We perturb the environments by changing their parameters such as mass, gravity, dimensions, motor noise, and friction. More details of task designs are in Appendix B.1.

6.1. Comparison with Existing Approaches

We compare our methods (PADA-DM, PADA-DM with target policy) with the following state-of-the-art methods for policy adaptation. The names correspond to the learning curves shown in Figure 2.

Christiano et al., 2016: (Christiano et al., 2016) uses a pre-trained policy $\pi^{(s)}$ and source dynamics $f^{(s)}$, to learn an inverse dynamics model $\phi: \mathcal{A} \times \mathcal{S} \times \mathcal{S} \rightarrow [0, 1]$, where $\phi(a|s, s')$ is the probability of taking action a that leads to s' from s .¹ That is, $\pi^{(t)}(s) = \phi(s, f^{(s)}(s, \pi^{(s)}(s)))$.

Zhu et al., 2018: (Zhu et al., 2018) proposed an approach for training policies in the target domain with a new reward $\lambda R(s_h) + (1 - \lambda)R_{gail}(s_h, a_h)$, $\lambda \in [0, 1]$. Here R_{gail} is from the discriminator from GAIL. Note that this baseline has access to true reward signals while ours do not.

For additional baselines, we also show the performance of directly running Proximal Policy Optimization (PPO) (Schulman et al., 2017) in the target environment, as well as directly using the **source policy** in the perturbed environ-

¹In general an inverse model is ill-defined without first specifying a policy, i.e., via Bayes rule, $\phi(a|s, s') \propto P(a, s, s') = f^{(t)}(s'|s, a)\pi(a|s)$. Hence one needs to first specify π in order to justify the existence of an inverse model $\phi(a|s, s')$.

ment without adaptation.

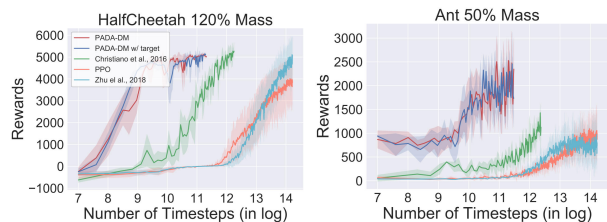


Figure 3. The long-term learning curves. The x-axis is the number of timesteps in natural logarithm scale.

Results. Figure 2 demonstrates the sample efficiency of our methods compared to the other methods and baselines. Both PADA-DM and PADA-DM with target policy converge within 10k to 50k training samples in the target environments. In contrast, (Christiano et al., 2016) requires 5 times more samples than our methods on average, and (Zhu et al., 2018) and PPO require about 30 times more. At convergence, our methods obtain the highest episodic rewards in 7 out of 8 tasks above among the policy adaptation methods. The baseline performance of PPO is better than the policy adaptation methods in HalfCheetah and Reacher (recall PPO uses true reward signals), but it takes significantly longer as shown in Fig. 2. Note that in the Ant environment, even at convergence our methods outperform PPO as well.

The only task where our methods failed to achieve top performance is Ant-v2 0.6 std motor noise. In this environment, the action noise causes high divergence between the target and source environments, making it hard to efficiently model the domain divergence. All the adaptation methods deliver bad performance in this case, indicating the difficulty of the task.

We observe that the learning curves of PADA-DM and PADA-DM with target policy are similar across all tasks without sacrificing efficiency or performance. The target policy can be directly used without any MPC step.

To further illustrate the sample efficiency of our method, we compare the long-term learning curves in Fig. 3. We plot the learning curves up to convergence of each method. We further include a long-term version of Fig 2 and the hyperparameters in the Appendix.

6.2. Performance on Multi-Dimensional Perturbations

We further evaluate the robustness of our methods by perturbing multiple dimensions of the target environment (Fig. 4). Note that online adaptation is particularly useful for multiple-dimension perturbations, because they generate an exponentially large space of source environments that are hard to sample offline. In Fig. 4(b), we show that even when perturbing 15 different degrees of freedom of the tar-

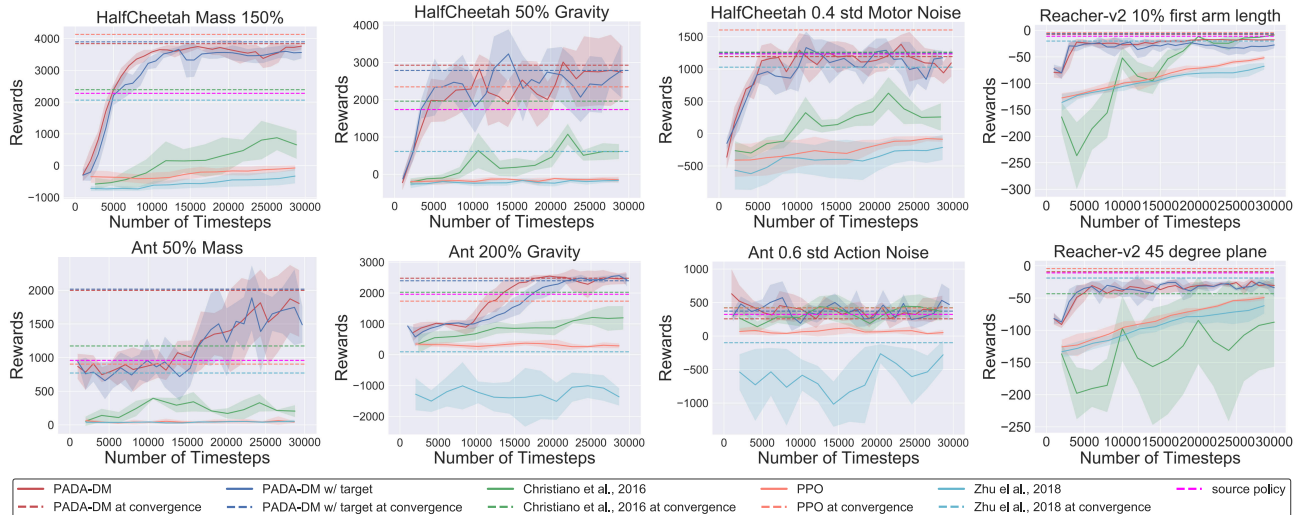


Figure 2. We plot the learning curves across 5 random seeds on a number of tasks. The title of each plot corresponds to the perturbation in the target domain, e.g., HalfCheetah Mass 150% means in mass of the agent in the target domain is 150% of that in the source domain.

get environment, our adaptation method can still achieve competitive performance at a much faster rate than all the other methods. We record the details of the configurations of the target environments in Appendix B.2.7.

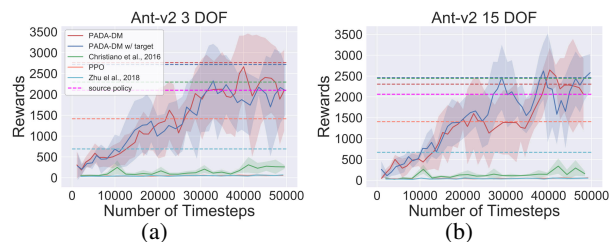


Figure 4. Learning curves for: changing 3 (a) and 15 (b) configurations in the target environment.

6.3. Comparison with Domain Randomization and Meta-Learning

We now compare with domain randomization and meta-learning approaches and show how they can be combined with our methods.

	single source domain	randomized source domains
no adaptation in target domain	source policy	DR
adaptation in target domain	PADA-DM	PADA-DM w/ DR; MAML

Table 1. Relationship of 5 methods in our ablation study.

Domain randomization (DR) (Peng et al., 2018): During

the training in the source domain, at the beginning of each episode, we randomly sample a new configuration for the source domain within a specified distribution. The total number of training samples here is the same as that for training the source policy. The policy outputted from DR is used in the target environment without further adaptation.

MAML: We adopt the RL version of MAML (Finn et al., 2017a) that meta-learns a model-free policy over a distribution of source environments and performs few-shot adaptation on the target environment.

We compare the following methods: (1) source policy trained in a fixed source environment, (2) domain randomization, (3) PADA-DM, (4) PADA-DM with DR (using domain randomization’s output as $\pi^{(s)}$ for PADA), and (5) MAML. Table 1 shows how these methods relate to each other.

For the first four methods, we train the source policy with 2m samples and perform adaptation with 80k samples. For MAML, we use 500 batches of meta-training (400m samples), and adapt 10 times with 80k samples in the target domain. We perform 100 trajectories across 5 random seeds in the target domain for each method and compare the episodic reward in Figure 5. We first observe that when the target domains lie in the distribution of domain randomization (70% – 130%), domain randomization outperforms source policy significantly, but does not help when the target lies far away from the distribution, which is the most notable shortcoming of domain randomization. Note that using domain randomization in conjunction with our adaptation method usually yields the best results. Domain randomization often

provides robustness within the environments' distribution, and online adaptation in real target environment using our approach further ensures robustness to out-of-distribution environments. We also observe that our method provides the most stable performance given the smallest test variances. We include additional experiments and detailed numbers of the performances of all methods (mean and standard deviations) in Appendix D.4.

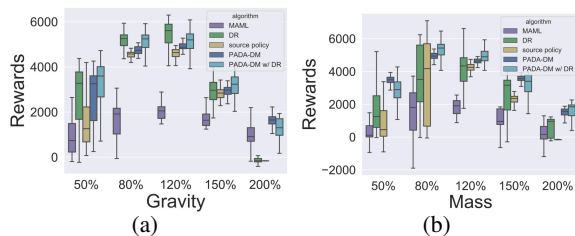


Figure 5. Ablation experiments using domain randomization and meta-learning. (a) Varying gravity. (b) Varying mass.

7. Conclusion

We proposed a novel policy adaptation algorithm that combines techniques from model-based RL and no-regret online learning. We theoretically proved that our methods generate trajectories in the target environment that converge to those in the source environment. We established the rate of convergence of the algorithms. We have shown that our algorithm achieves competitive performance across a diverse set of continuous control tasks with better sample efficiency. A natural extension is to use our approach on simulation-to-real problems in combination with domain randomization and meta-learning.

As our experiments indicated that the combination of domain randomization and our online adaptation approach together often yields good results, for future work, we plan to investigate general theoretical framework for combining domain randomization and online adaptive control techniques.

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