Supplementary Material of PDO-eConvs: Partial Differential Operator Based Equivariant Convolutions

1. Numerical Schemes of Partial Differential Operators

1.1. Filters of Size 3×3

$$\widetilde{u}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_x = \frac{1}{h}\begin{bmatrix} 0 & 0 & 0 \\ -1/2 & 0 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_y = \frac{1}{h}\begin{bmatrix} 0 & 1/2 & 0 \\ 0 & 0 & 0 \\ 0 & -1/2 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xx} = \frac{1}{h^2}\begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xy} = \frac{1}{h^2}\begin{bmatrix} -1/4 & 0 & 1/4 \\ 0 & 0 & 0 \\ 1/4 & 0 & -1/4 \end{bmatrix}$$

$$\widetilde{u}_{yy} = \frac{1}{h^2}\begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xxy} = \frac{1}{h^3}\begin{bmatrix} 1/2 & -1 & 1/2 \\ 0 & 0 & 0 \\ -1/2 & 1 & -1/2 \end{bmatrix}$$

$$\widetilde{u}_{xyy} = \frac{1}{h^3}\begin{bmatrix} -1/2 & 0 & 1/2 \\ 1 & 0 & -1 \\ -1/2 & 0 & 1/2 \end{bmatrix}$$

$$\widetilde{u}_{xxyy} = \frac{1}{h^4}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

1.2. Filters of Size 5×5

$$\widetilde{u}_{xxx} = \frac{1}{h^3}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & -1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{yyy} = \frac{1}{h^3}\begin{bmatrix} 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1/2 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xxxx} = \frac{1}{h^4}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xxxy} = \frac{1}{h^4}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1/4 & 1/2 & 0 & -1/2 & 1/4 \\ 0 & 0 & 0 & 0 & 0 \\ 1/4 & -1/2 & 0 & 1/2 & -1/4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xyyy} = \frac{1}{h^4}\begin{bmatrix} 0 & -1/4 & 0 & 1/4 & 0 \\ 0 & 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 1/4 & 0 & -1/4 & 0 \end{bmatrix}$$

$$\widetilde{u}_{yyyy} = \frac{1}{h^4}\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$