Supplementary Material: Spectral Subsampling for Stationary Time Series

1. Asymptotic Analysis

Let $\boldsymbol{\theta}^\star$ be a mode of $\pi_W(\boldsymbol{\theta})$ and $\Delta(\boldsymbol{\theta}) := \partial^2 \log \pi_W(\boldsymbol{\theta})/\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top$. As in Quiroz et al. (2019), we need the following regularity conditions on the Whittle likelihood, which can be justified by the asymptotic normality of the maximum Whittle-likelihood estimator (Fox & Taqqu, 1986) and the Bernstein-von Mises theorem for the Whittle measure (Tamaki, 2008).

Assumption 1 *(A1) For each i,*

$$\ell_{W,i}(\boldsymbol{\theta}) := -\log f_{\boldsymbol{\theta}}(\omega_i) - \mathcal{I}(\omega_i)/f_{\boldsymbol{\theta}}(\omega_i)$$

is three times differentiable with $\max_{j,k,l\in\{1,\ldots,p\}} \sup_{\boldsymbol{\theta}\in\boldsymbol{\theta}} \left|\frac{\partial^3 \ell_{W,i}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_j\partial\boldsymbol{\theta}_k\partial\boldsymbol{\theta}_l}\right|$ bounded.

(A2) $\Delta(\boldsymbol{\theta}^\star)$ is negative definite.

(A3) $\|\Sigma\|_2 = O(n^{-1})$, where $\Sigma = \big(-\Delta(\boldsymbol{\theta}^\star)\big)^{-1}$.

(A4) For any $\epsilon > 0$, there exist a $\delta_\epsilon > 0$ and an integer $N_{1,\epsilon}$ such that for any $n > N_{1,\epsilon}$ and $\boldsymbol{\theta} \in H(\boldsymbol{\theta}^\star, \delta_\epsilon)$, $\Delta(\boldsymbol{\theta})$ exists and satisfies

$$-A(\epsilon) \leq \Delta(\boldsymbol{\theta})\big(\Delta(\boldsymbol{\theta}^\star)\big)^{-1} - I \leq A(\epsilon)$$

where $A(\epsilon)$ is a positive semidefinite matrix whose largest eigenvalue goes to 0 as $\epsilon \to 0$.

(A5) For any $\delta > 0$, there exists a positive integer $N_{2,\delta}$ and two positive numbers c and κ such that for $n > N_{2,\delta}$ and $\boldsymbol{\theta} \notin H(\boldsymbol{\theta}^\star, \delta)$

$$\frac{\pi_W(\boldsymbol{\theta})}{\pi_W(\boldsymbol{\theta}^\star)} < \exp\left(-c\big[(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)^\top \Sigma^{-1}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)\big]^\kappa\right).$$

2. Additional results

The accuracy of all marginal posteriors is illustrated for each example in Figures 1-4; see the captions for details. The Taylor control variates is very accurate or close to very accurate for all parameters. The coreset control variate is often accurate, however, for some of the parameters the bias is noticeable. This is because the variance of the coreset control variate in the examples considered is larger than that of the Taylor series control variate. We expect the coreset control variate to outperform the Taylor series control variate in examples where the grouped data density is multimodal on the logarithmic scale. This case is clearly outside the scope of the Taylor series control variate, since it assumes that the grouped data density is approximately quadratic on the logarithmic scale. We leave this for future research.

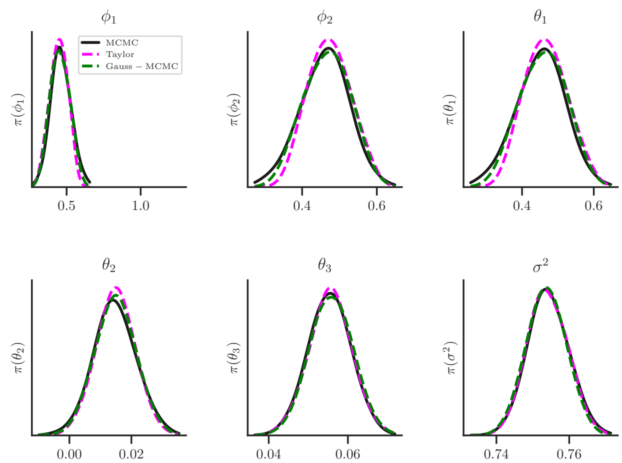

Figure 1. Kernel Density Estimates of all marginal distributions for the ARMA(2,3) example. MCMC is the full-data MCMC on the Whittle likelihood. Taylor is the Subsampling MCMC method using the Taylor series expanded control variates. Gauss-MCMC is the full-data MCMC on the time domain Gaussian likelihood.

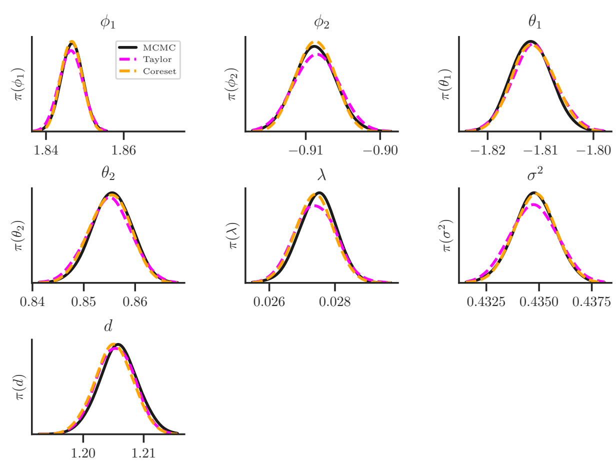

Figure 2. Kernel Density Estimates of all marginal distributions for the ARTFIMA(2,2) example.

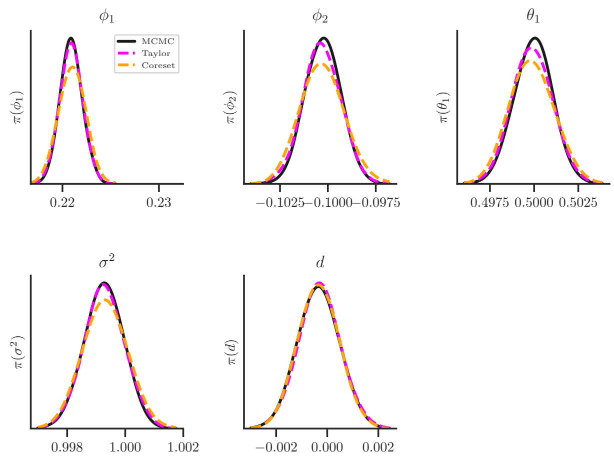

Figure 3. Kernel Density Estimates of all marginal distributions for the ARFIMA(2,1) example. MCMC is the full-data MCMC on the Whittle likelihood. Taylor and coreset are the Subsampling MCMC methods using the corresponding control variates.

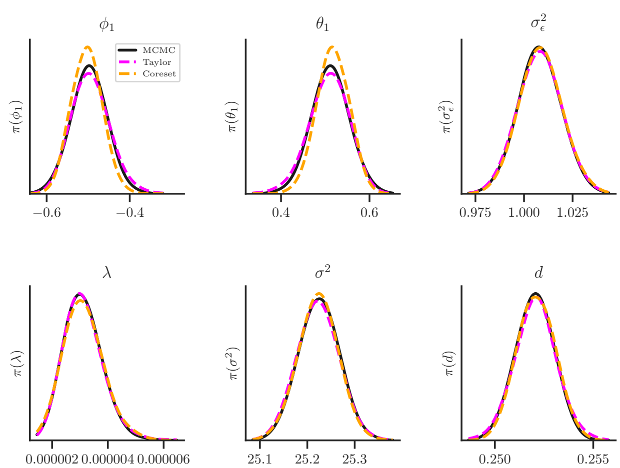

Figure 4. Kernel Density Estimates of all marginal distributions for the ARTFIMA-SV(1,1) example. MCMC is the full-data MCMC on the Whittle likelihood. Taylor and coreset are the Subsampling MCMC methods using the corresponding control variates.

References

Fox, R. and Taqqu, M. Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series. *The Annals of Statistics*, 14(2):517–532, 1986.

Quiroz, M., Villani, M., Kohn, R., Tran, M.-N., and Dang, K.-D. Subsampling MCMC - An introduction for the survey statistician. *Sankhya A*, 80(1):33–69, 2019.

Tamaki, K. The Bernstein-von Mises theorem for stationary processes. *Journal of the Japan Statistical Society*, 38(2): 311–323, 2008.