Maximum Entropy Gain Exploration for Long Horizon Multi-goal Reinforcement Learning

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Abstract

What goals should a multi-goal reinforcement learning agent pursue during training in long-horizon tasks? When the desired (test time) goal distribution is too distant to offer a useful learning signal, we argue that the agent should not pursue unobtainable goals. Instead, it should set its own intrinsic goals that maximize the entropy of the historical achieved goal distribution. We propose to optimize this objective by having the agent pursue past achieved goals in sparsely explored areas of the goal space, which focuses exploration on the frontier of the achievable goal set. We show that our strategy achieves an order of magnitude better sample efficiency than the prior state of the art on long-horizon multi-goal tasks including maze navigation and block stacking.[1]

1. Introduction

Multi-goal reinforcement learning (RL) agents (Plappert et al., 2018; Schaul et al., 2015b; Kaelbling, 1993) learn goal-conditioned behaviors that can achieve and generalize across a range of different goals. Multi-goal RL forms a core component of hierarchical agents (Sutton et al., 1999; Nachum et al., 2018), and has been shown to allow unsupervised agents to learn useful skills for downstream tasks (Warde-Farley et al., 2019; Hansen et al., 2020). Recent advances in goal relabeling (Andrychowicz et al., 2017) have made learning possible in complex, sparse-reward environments whose goal spaces are either dense in the initial state distribution (Plappert et al., 2018) or structured as a curriculum (Colas et al., 2018). But learning without demonstrations in long-horizon tasks remains a challenge (Nair et al., 2018a; Trott et al., 2019), as learning signal decreases exponentially with the horizon (Osband et al., 2014).

In this paper, we improve upon existing approaches to intrinsic goal setting and show how multi-goal agents can form an automatic behavioural goal curriculum that allows them to master long-horizon, sparse reward tasks. We begin with an algorithmic framework for goal-seeking agents that contextualizes prior work (Baranes & Oudeyer, 2013; Florensa et al., 2018; Warde-Farley et al., 2019; Nair et al., 2018b; Pong et al., 2019) and argue that past goal selection mechanisms are not well suited for long-horizon, sparse reward tasks (Section 2). By framing the long-horizon goal seeking task as optimizing an initially ill-conditioned distribution matching objective (Lee et al., 2019), we arrive at our unsupervised Maximum Entropy Goal Achievement (MEGA) objective, which maximizes the entropy of the past achieved goal set. This early unsupervised objective is annealed into the original supervised objective once the latter becomes tractable, resulting in our OMEGA objective (Section 3).

We propose a practical algorithmic approach to maximizing entropy, which pursues past achieved goals in sparsely explored areas of the achieved goal distribution, as measured by a learned density model. The agent revisits and explores around these areas, pushing the frontier of achieved goals forward (Ecoffet et al., 2019). This strategy, similar in spirit to Baranes & Oudeyer (2013) and Florensa et al. (2018), encourages the agent to explore at the edge of its abilities, which avoids spending environment steps in pursuit of already mastered or unobtainable goals. When used in combination with hindsight experience replay and an off-policy learning algorithm, our method achieves more than an order of magnitude better sample efficiency than the prior state of the art on difficult exploration tasks, including long-horizon mazes and block stacking (Section 4). Finally, we draw connections between our approach and the empowerment objective (Klyubin et al., 2005; Salge et al., 2014) and identify a key difference to prior work: rather than maximize empowerment on-policy by setting maximally diverse goals during training (Gregor et al., 2016; Warde-Farley et al., 2019; Nair et al., 2018b; Pong et al., 2019), our proposed approach maximizes it off-policy by setting goals on the frontier of the past achieved goal set. We conclude with discussion of related and future work (Sections 5-7).

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[1]Code available at `https://github.com/spitis/mrl`

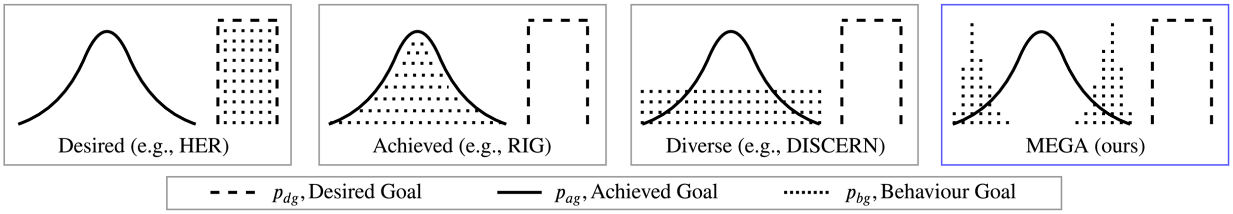

Figure 1. Illustration of density-based SELECT mechanisms at start of training, when achieved (p_{ag}) and desired (p_{dg}) goal distributions are disconnected. HER samples goals from the desired distribution p_{dg}. RIG samples from the achieved distribution p_{ag}. DISCERN and Skew-Fit skew p_{ag} to sample diverse achieved goals. Our approach (MEGA) focuses on low density regions of p_{ag}. See Subsection 2.3.

2. The Long-Horizon Problem

2.1. Preliminaries

We consider the multi-goal reinforcement learning (RL) setting, described by a generalized Markov Decision Process (MDP) $\mathcal{M} = \langle S, A, T, G, [p_{dg}] \rangle$, where S, A, T, and G are the state space, action space, transition function and goal space, respectively (Sutton & Barto, 2018; Schaul et al., 2015a) and p_{dg} is an optional desired goal distribution. In the most general version of this problem each goal is a tuple $g = \langle R_g, \gamma_g \rangle$, where $R_g : S \to \mathbb{R}$ is a reward function and $\gamma_g \in [0, 1]$ is a discount factor (Sutton et al., 2011), so that "solving" goal $g \in G$ amounts to finding an optimal policy in the classical MDP $\mathcal{M}_g = \langle S, A, T, R_g, \gamma_g \rangle$. Although goal-oriented methods are general and could be applied to dense reward MDPs (including the standard RL problem, as done by Warde-Farley et al. (2019), among others), we restrict our present attention to the sparse reward case, where each goal g corresponds to a set of "success" states, S_g, with $R_g : S \to \{-1, 0\}$ (Plappert et al., 2018) defined as $R_g(s) = \mathbb{I}\{s \in S_g\} + c$. Following Plappert et al., we use base reward $c = -1$, which typically leads to more stable training than the more natural $c = 0$ (see Van Seijen et al. (2019) for a possible explanation). We also adopt the usual form $S_g = \{s \,|\, d(\text{AG}(s), g) < \epsilon\}$, where $\text{AG} : S \to G$ maps state s to an "achieved goal" $\text{AG}(s)$ and d is a metric on G. An agent's "achieved goal distribution" p_{ag} is the distribution of goals achieved by states s (i.e., $\text{AG}(s)$) the agent visits (not necessarily the final state in a trajectory). Note that this may be on-policy (induced by the current policy) or historical, as we will specify below. The agent must learn to achieve success and, if the environment is not episodic, maintain it. In the episodic case, we can think of each goal $g \in G$ as specifying a skill or option $o \in \Omega$ (Sutton et al., 1999; Eysenbach et al., 2018), so that multi-goal reinforcement learning is closely related to hierarchical reinforcement learning (Nachum et al., 2018).

A common approach to multi-goal RL, which we adopt, trains a goal-conditioned state-action value function, $Q : S \times A \times G \to \mathbb{R}$, using an off-policy learning algorithm that can leverage data from other policies (past and exploratory) to optimize the current policy (Schaul et al., 2015b). A goal-conditioned policy, $\pi : S \times G \to A$, is either induced via greedy action selection (Mnih et al., 2013) or learned using policy gradients. Noise is added to π during exploration to form exploratory policy π_{explore}. Our continuous control experiments all use the DDPG algorithm (Lillicrap et al., 2015), which parameterizes actor and critic separately, and trains both concurrently using Q-learning for the critic (Watkins & Dayan, 1992), and deterministic policy gradients (Silver et al., 2014) for the actor. DDPG uses a replay buffer to store past experiences, which is then sampled from to train the actor and critic networks.

2.2. Sparse rewards and the long horizon problem

Despite the success of vanilla off-policy algorithms in dense-reward tasks, standard agents learn very slowly—or not at all—in sparse-reward, goal-conditioned tasks (Andrychowicz et al., 2017). In order for a vanilla agent to obtain a positive reward signal and learn about goal g, the agent must stumble upon g through random exploration *while it is trying to achieve* g. Since the chance of this happening when exploring randomly decreases exponentially with the horizon ("the long horizon problem") (Osband et al., 2014), successes are infrequent even for goals that are relatively close to the initial state, making learning difficult.

One way to ameliorate the long horizon problem is based on the observation that, regardless of the goal being pursued, $\langle$state, action, next state$\rangle$ transitions are unbiased samples from the environment dynamics. An agent is therefore free to pair transitions with any goal and corresponding reward, which allows it to use experiences gained in pursuit of one goal to learn about other goals ("goal relabeling") (Kaelbling, 1993). Hindsight Experience Replay (HER) (Andrychowicz et al., 2017) is a form of goal relabeling that relabels experiences with goals that are achieved later in the same trajectory. For every real experience, Andrychowicz et al. (2017)'s `future` strategy produces k relabeled experiences, where the k goals are sampled uniformly from goals achieved by future states in the same trajectory. This forms an implicit optimization curriculum, and allows an agent to learn about any goal g it encounters during exploration.

Note, however, that a HER agent must still encounter g (or

Algorithm 1 Unified Framework for Multi-goal Agents

function TRAIN($*args$):
  Alternate between collecting experience using ROLLOUT and optimizing the parameters using OPTIMIZE.

function ROLLOUT (policy π_{explore}, buffer $\mathcal{B}$, $*args$):
  $\mathtt{g} \leftarrow$ SELECT($*args$)
  $\mathtt{s}_0 \leftarrow$ initial state
  for t in $0 \ldots T-1$ **do**
    $\mathtt{a}_t, \mathtt{s}_{t+1} \leftarrow$ execute $\pi_{\text{explore}}(\mathtt{s}_t, \mathtt{g})$ in environment
    $\mathtt{r}_t \leftarrow$ REWARD($\mathtt{s}_t, \mathtt{a}_t, \mathtt{s}_{t+1}, \mathtt{g}$)
    Store $(\mathtt{s}_t, \mathtt{a}_t, \mathtt{s}_{t+1}, \mathtt{r}_t, \mathtt{g})$ in replay buffer $\mathcal{B}$

function OPTIMIZE (buffer $\mathcal{B}$, algorithm $\mathcal{A}$, parameters θ):
  Sample mini-batch $B = \{(\mathtt{s}, \mathtt{a}, \mathtt{s}', \mathtt{r}, \mathtt{g})_i\}_{i=1}^{N} \sim \mathcal{B}$
  $B' \leftarrow$ RELABEL($B, *args$)
  Optimize θ using $\mathcal{A}$ (e.g., DDPG) and relabeled B'

function SELECT ($*args$):
  Returns a behavioural goal for the agent. Examples include the environment goal g_{ext}, a sample from the buffer of achieved goals $\mathcal{B}_{ag}$ (Warde-Farley et al., 2019), or samples from a generative model such as a GAN (Florensa et al., 2018) or VAE (Nair et al., 2018b). Our approach (MEGA) selects previously achieved goals in sparsely explored areas of the goal space according to a learned density model.

function REWARD ($\mathtt{s}_t, \mathtt{a}_t, \mathtt{s}_{t+1}, \mathtt{g}$):
  Computes the environment reward or a learned reward function (Warde-Farley et al., 2019; Nair et al., 2018b).

function RELABEL ($B, *args$):
  Relabels goals and rewards in minibatch B according to some strategy; e.g., don't relabel, `future`, mix `future` and generated goals (Nair et al., 2018b), or `rfaab` (ours).

goals sufficiently similar to g) in order to learn about g, and the long horizon problem persists for goals that are too far removed from the agent's initial state distribution. This is illustrated in Figure 2, and is most easily understood by considering the tabular case, where no generalization occurs between a finite set of MDPs $\mathcal{M}_g$: since a learning signal is obtained only when transitioning into $s \in S_g$, the agent's achieved goal distribution must overlap with S_g for learning to occur. Empirically, this means that DDPG+HER agents that explore using only action noise or epsilon random actions fail to solve *long-horizon tasks*, whose desired goal distribution does not overlap with the initial state distribution. This includes the original version of `FetchPickAndPlace` (with all goals in the air) (Andrychowicz et al., 2017), block stacking (Nair et al., 2018a), and mazes (Trott et al., 2019).

2.3. Setting intrinsic goals

We propose to approach the long-horizon problem by ignoring long-horizon goals: rather than try to achieve unobtainable goals, an agent can set its own intrinsic goals and slowly expand its domain of expertise in an unsupervised fashion. This is inspired by a number of recent papers on

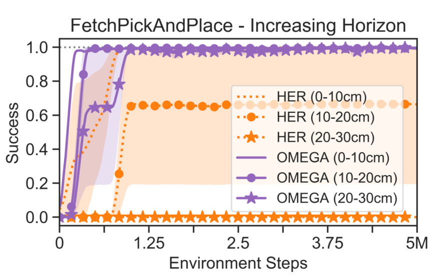

Figure 2. Performance of a DDPG+HER agent that must lift a box to reach goals at increasing heights (3 seeds). As the horizon (desired height) increases, the agent loses the ability to solve the task in reasonable time. Our approach, OMEGA (Section 3), is robust to the horizon length. Specific details in Appendix.

unsupervised multi-goal RL, to be described below. Our main contributions relative to past works are (1) a novel goal selection mechanism designed to address the long-horizon problem, and (2) a practical method to anneal initial unsupervised selection into training on the desired goals.

To capture the differences between various approaches, we present Algorithm 1, a unifying algorithmic framework for multi-goal agents. Variations occur in the subprocedures SELECT, REWARD, and RELABEL. The standard HER agent Andrychowicz et al. (2017) SELECTS the environment goal g_{ext}, uses the environment REWARD and uses the `future` RELABEL strategy. Functions used by other agents are detailed in Appendix A. We assume access to the environment REWARD and propose a novel SELECT strategy—MaxEnt Goal Achievement (MEGA)—that initially samples goals from low-density regions of the achieved goal distribution. Our approach also leads to a novel RELABEL strategy, `rfaab`, which samples from Real, Future, Actual, Achieved, and behavioural goals (detailed in Appendix C).

Prior work also considers intrinsic SELECT functions. The approaches used by DISCERN (Warde-Farley et al., 2019), RIG (Nair et al., 2018b) and Skew-Fit (Pong et al., 2019) select goals using a model of the past achieved goal distribution. DISCERN samples from a replay buffer (a non-parametric model), whereas RIG and Skew-Fit learn and sample from a variational autoencoder (VAE) (Kingma & Welling, 2013). These approaches are illustrated in Figure 1, alongside HER and MEGA. Prior density-based approaches were not tailored to the long-horizon problem; e.g., DISCERN was primarily focused on learning an intrinsic REWARD function, and left "the incorporation of more explicitly instantiated [SELECT] curricula to future work." By contrast, MEGA focuses on the low density, or sparsely explored, areas of the achieved goal distribution, forming a curriculum that crosses the gap between the initial state distribution and the desired goal distribution in record time. Although Diverse sampling (e.g., Skew-Fit) is less biased towards already mastered areas of the goal space than Achieved sampling (e.g., RIG), we show in our experiments that it still under-explores relative to MEGA.

MEGA's focus on the frontier of the achieved goal set makes it similar to SAGG-RIAC (Baranes & Oudeyer, 2013), which seeks goals that maximize learning progress, and Goal GAN (Florensa et al., 2018), which seeks goals of intermediate difficulty.

3. Maximum Entropy Goal Achievement

3.1. The MEGA and OMEGA objectives

To motivate the MEGA objective, we frame exploration in episodic, multi-goal RL with goal relabeling as a distribution matching problem (Lee et al., 2019). We note that the original distribution matching objective is ill-conditioned in long-horizon problems, which suggests maximizing the entropy of the achieved goal distribution (the MEGA objective). We then show how this can be annealed into the original objective (the OMEGA objective).

We start by noting that, for a truly off-policy agent, the actual goals used to produce the agent's experience do not matter, as the agent is free to relabel any experience with any goal. This implies that only the distribution of experience in the agent's replay buffer, along with the size of the buffer, matters for effective off-policy learning. How should an agent influence this distribution to accumulate useful data for achieving goals from the desired distribution p_{dg}?

Though we lack a precise characterization of which data is useful, we know that all successful policies for goal g pass through g, which suggests that useful data for achieving g monotonically increases with the number of times g is achieved during exploration. Past empirical results, such as the success of Andrychowicz et al. (2017)'s `future` strategy and the effectiveness of adding expert demonstrations to the replay buffer (Nair et al., 2018a), support this intuition. Assuming a relatively fixed initial state distribution and uniformly distributed p_{dg}[2], it follows that the intrinsic goal g^t at episode t should be chosen to bring the agent's historical achieved goal distribution p^t_{ag} closer to the desired distribution p_{dg}. We can formalize this as seeking g^t to minimize the following distribution matching objective:

$$J_{\text{original}}(p^t_{ag}) = \mathrm{D}_{\mathrm{KL}}(p_{dg} \parallel p^t_{ag}), \tag{1}$$

where p^t_{ag} represents the *historical* achieved goal distribution in the agent's replay buffer after executing its exploratory policy in pursuit of goal g^t. It is worth highlighting that objective (1) is a forward KL: we seek p_{ag} that "covers" p_{dg} (Bishop, 2006). If reversed, it would always be infinite when p_{dg} and the initial state distribution s_0 do not overlap, since p_{dg} cannot cover s_0.

So long as (1) is finite and non-increasing over time, the support of p_{ag} covers p_{dg} and the agent is accumulating data that can be used to learn about all goals in the desired distribution. In those multi-goal environments where HER has been successful (e.g., `FetchPush`), this is easily achieved by setting the behavioural goal distribution p_{bg} to equal p_{dg} and using action space exploration (Plappert et al., 2018). In long-horizon tasks, however, the objective (1) is usually ill-conditioned (even undefined) at the beginning of training when the supports of p_{dg} and p_{ag} do not overlap. While this explains why HER with action space exploration fails in these tasks, it isn't very helpful, as the ill-conditioned objective is difficult to optimize.

When p_{ag} does not cover p_{dg}, a natural objective is to expand the support of p_{ag}, in order to make the objective (1) finite as fast as possible. We often lack a useful inductive bias about which direction to expand the support in; e.g., a naive heuristic like Euclidean distance in feature space can be misleading due to dead-ends or teleportation (Trott et al., 2019), and should not be relied on for exploration. In absence of a useful inductive bias, it is sensible to expand the support as fast as possible, in any and all directions as in breadth-first search, which can be done by maximizing the entropy of the achieved goal distribution $H[p_{ag}]$. We call this the Maximum Entropy Goal Achievement (MEGA) objective:

$$J_{\text{MEGA}}(p^t_{ag}) = -H[p^t_{ag}], \tag{2}$$

The hope is that by maximizing $H[p_{ag}]$ (minimizing J_{MEGA}), the agent will follow a natural curriculum, expanding the size of its achievable goal set until it covers the support of the desired distribution p_{dg} and objective (1) becomes tractable.

In the unsupervised case, where p_{dg} is not specified, the agent can stop at the MEGA objective. In the supervised case we would like the agent to somehow anneal objective (2) into objective (1). We can do this by approximating (2) using a distribution matching objective, where the desired distribution is uniform over the current support:

$$\tilde{J}_{\text{MEGA}}(p^t_{ag}) = \mathrm{D}_{\mathrm{KL}}(\mathcal{U}(\text{supp}(p^t_{ag})) \parallel p^t_{ag}). \tag{3}$$

This is a sensible approximation, as it shares a maximum with (2) when the uniform distribution over G is obtainable, and encourages the agent to "cover" the current support of the achieved goal distribution as broadly as possible, so that the diffusion caused by action space exploration will increase entropy. We may now form the mixture distribution $p^t_\alpha = \alpha p_{dg} + (1-\alpha)\mathcal{U}(\text{supp}(p^t_{ag}))$ and state our

[2] For diverse initial state distributions, we would need to condition both p_{dg} and p_{ag} on the initial state. For non-uniform p_{dg}, we would likely want to soften the desired distribution as the marginal benefit of additional data is usually decreasing.

final "OMEGA" objective, which anneals the approximated MEGA into the original objective:

$$J_{\text{OMEGA}}(p^t_{ag}) = \mathrm{D}_{\mathrm{KL}}(p_\alpha \parallel p^t_{ag}). \tag{4}$$

The last remaining question is, how do we choose α? We would like $\alpha = 0$ when p_{ag} and p_{dg} are disconnected, and α close to 1 when p_{ag} well approximates p_{dg}. One way to achieve this, which we adopt in our experiments, is to set

$$\alpha = 1/\max(b + \mathrm{D}_{\mathrm{KL}}(p_{dg} \parallel p_{ag}), 1),$$

where $b \leq 1$. The divergence is infinite ($\alpha = 0$) when p_{ag} does not cover p_{dg} and approaches 0 ($\alpha = 1$) as p_{ag} approaches p_{dg}. Our experiments use $b = -3$, which we found sufficient to ensure $\alpha = 1$ at convergence (with $b = 1$, we may never have $\alpha = 1$, since p_{ag} is historical and biased towards the initial state distribution s_0).

3.2. Optimizing the MEGA objective

We now consider choosing behavioural goal $\hat{g} \sim p_{bg}$ in order to optimize the MEGA objective (2), as it is the critical component of (4) for early exploration in long-horizon tasks and general unsupervised goal-seeking. In supervised tasks, the OMEGA objective (4) can be approximately optimized by instead using the environment goal with probability α.

We first consider what behavioural goals we would pick if we had an oracle that could predict the conditional distribution $q(g' \mid \hat{g})$ of goals g' that would be achieved by conditioning the policy on $\hat{g}$. Then, noting that this may be too difficult to approximate in practice, we propose a minimum density heuristic that performs well empirically. The resulting SELECT functions are shown in Algorithm 2.

Oracle strategy If we knew the conditional distribution $q(g' \mid \hat{g})$ of goals g' that would be achieved by conditioning behaviour on $\hat{g}$, we could compute the expected next step MEGA objective as the expected entropy of the new empirical $p_{ag \mid g'}$ after sampling g' and adding it to our buffer:

$$\begin{aligned} J_{\text{MEGA}}(p_{ag \mid g'}) &= -\mathbb{E}_{g' \sim q(g' \mid \hat{g})} H[p_{ag \mid g'}] \\ &= \sum_{g'} q(g'|\hat{g}) \sum_{g} p_{ag \mid g'}(g) \log p_{ag \mid g'}(g), \end{aligned}$$

To explicitly compute this objective one must compute both the new distribution and its entropy for each possible new achieved goal g'. The following result simplifies matters in the tabular case. Proofs may be found in Appendix B.

Proposition 1 (Discrete Entropy Gain)**.** *Given buffer $\mathcal{B}$ with $\eta = \frac{1}{|\mathcal{B}|}$, maximizing expected next step entropy is equivalent to maximizing expected point-wise entropy gain $\Delta H(g')$:*

$$\begin{aligned} \hat{g}^* &= \arg\max_{\hat{g} \in \mathcal{B}} \mathbb{E}_{g' \sim q(g' \mid \hat{g})} H[p_{ag \mid g'}] \\ &= \arg\max_{\hat{g} \in \mathcal{B}} \mathbb{E}_{g' \sim q(g' \mid \hat{g})} \Delta H(g'), \end{aligned} \tag{5}$$

where $\Delta H(g') = p_{ag}(g') \log p_{ag}(g') - (p_{ag}(g') + \eta) \log(p_{ag}(g') + \eta)$.

For most agents η will quickly approach 0 as they accumulate experience, so that choosing $\hat{g}$ according to (9) becomes equal (in the limit) to choosing $\hat{g}$ to maximize the directional derivative $\langle \nabla_{p_{ag}} H[p_{ag}], q(g' \mid \hat{g}) - p_{ag} \rangle$.

Proposition 2 (Discrete Entropy Gradient)**.**

$$\begin{aligned} \lim_{\eta \to 0} \hat{g}^* &= \arg\max_{\hat{g} \in \mathcal{B}} \langle \nabla_{p_{ag}} H[p_{ag}], q(g' \mid \hat{g}) - p_{ag} \rangle \\ &= \arg\max_{\hat{g} \in \mathcal{B}} \mathrm{D}_{\mathrm{KL}}(q(g' \mid \hat{g}) \parallel p_{ag}) + H[q(g' \mid \hat{g})] \end{aligned} \tag{6}$$

This provides a nice intuition behind entropy gain exploration: we seek maximally diverse outcomes ($H[q(g' \mid \hat{g})]$) that are maximally different from historical experiences ($\mathrm{D}_{\mathrm{KL}}(q(g' \mid \hat{g}) \parallel p_{ag})$)—i.e., exploratory behavior should evenly explore under-explored regions of the state space. By choosing goals to maximize the entropy gain, an agent effectively performs constrained gradient ascent (Frank & Wolfe, 1956; Hazan et al., 2018) on the entropy objective.

Assuming the empirical p_{ag} is used to induce (abusing notation) a density p_{ag} with full support, Proposition 2 extends to the continuous case by taking the functional derivative of the differential entropy with respect to the variation $\eta(g) = q(g' \mid \hat{g})(g) - p_{ag}(g)$ (Appendix B).

Minimum density approximation Because we do not know $q(g' \mid \hat{g})$, we must approximate it with either a learned model or an effective heuristic. The former solution is difficult, because by the time there is enough data to make an accurate prediction conditional on $\hat{g}$, $q(g' \mid \hat{g})$ will no longer represent a sparsely explored area of the goal space. While it might be possible to make accurate few- or zero-shot predictions if an agent accumulates enough data in a long-lived, continual learning setting with sufficient diversity for meta-learning (Ren et al., 2018), in our experiments we find that a simple, minimum-density approximation, which selects goals that have minimum density according to a learned density model, is at least as effective (Appendix D). We can view this approximation as a special case where the conditional $q(g' \mid \hat{g}) = \mathbb{1}[g' = \hat{g}]$, i.e. that the agent achieves the behaviour goal.

Algorithm 2 O/MEGA SELECT functions

function OMEGA_SELECT (env goal g_{ext}, bias b, $*args$):
  $\alpha \leftarrow 1/\max(b + \mathrm{D}_{\mathrm{KL}}(p_{dg} \parallel p_{ag}), 1)$
  if $x \sim \mathcal{U}(0,1) < \alpha$ **then return** g_{ext}
  else return MEGA_SELECT($*args$)

function MEGA_SELECT (buffer $\mathcal{B}$, num_candidates N):
  Sample N candidates $\{g_i\}_{i=1}^N \sim \mathcal{B}$
  Eliminate unachievable candidates (see text)
  return $\hat{g} = \arg\min_{g_i} \hat{p}_{ag}(g_i)$ $(*)$

Proposition 3. *If $q(g'|\hat{g}) = \mathbb{1}[g' = \hat{g}]$, the discrete entropy gradient objective simplifies to a minimum density objective:*

$$\begin{aligned} \hat{g}^* &= \arg\max_{\hat{g}\in\mathcal{B}} -\log[p_{ag}(\hat{g})] \\ &= \arg\min_{\hat{g}\in\mathcal{B}} p_{ag}(\hat{g}). \end{aligned} \tag{7}$$

Our minimum density heuristic (Algorithm 2) fits a density model to the achieved goals in the buffer to form estimate $\hat{p}_{ag}$ of the historical achieved goal distribution p_{ag} and uses a generate and test strategy (Newell, 1969) that samples N candidate goals $\{g_i\}_{i=1}^N \sim \mathcal{B}$ from the achieved goal buffer (we use $N = 100$ in our experiments) and selects the minimum density candidate $\hat{g} = \arg\min_{g_i} \hat{p}_{ag}(g_i)$. We then adopt a Go Explore (Ecoffet et al., 2019) style strategy, where the agent increases its action space exploration once a goal is achieved. Intuitively, this heuristic seeks out past achieved goals in sparsely explored areas, and explores around them, pushing the frontier of achieved goals forward.

It is important for $\hat{g}$ to be achievable. If it is not, then $q(g' \mid \hat{g})$ may be disconnected from $\hat{g}$, as is the case when the agent pursues unobtainable g_{ext} (Figure 2), which undermines the purpose of the minimum density heuristic. To promote achievability, our experiments make use of two different mechanisms. First, we only sample candidate goals from the past achieved goal buffer $\mathcal{B}$. Second, we eliminate candidates whose estimated value (according to the agent's goal-conditioned Q-function) falls below a dynamic cutoff, which is set according to agent's goal achievement percentage during exploration. The specifics of this cutoff mechanism may be found in Appendix C. Neither of these heuristics are core to our algorithm, and they might be be replaced with, e.g., a generative model designed to generate achievable goals (Florensa et al., 2018), or a success predictor that eliminates unachievable candidates.

4. Experiments

Having described our objectives and proposed approaches for optimizing them, we turn to evaluating our O/MEGA agents on four challenging, long-horizon environments that standard DDPG+HER agents fail to solve. We compare the performance of our algorithms with several goal selection baselines. To gain intuition on our method, we visualize qualitatively the behaviour goals selected and quantitatively the estimated entropy of the achieved goal distribution.

Environments We consider four environments. In `PointMaze` (Trott et al., 2019), a point must navigate a 2d maze, from the bottom left corner to the top right corner. In `AntMaze` (Nachum et al., 2018; Trott et al., 2019), an ant must navigate a U-shaped hallway to reach the target. In `FetchPickAndPlace` (hard version) (Plappert et al., 2018), a robotic arm must grasp a box and move it to the desired location that is at least 20cm in the air. In `FetchStack2` (Nair et al., 2018a), a robotic arm must move each of the two blocks into the desired position, where one of the block rests on top of the other. In `PointMaze` and `AntMaze` goals are 2-dimensional and the agent is successful if it reaches the goal once. In `FetchPickAndPlace` and `FetchStack2`, goals are 3- and 6-dimensional, respectively, and the agent must maintain success until the end of the episode for it to count.

Baselines We compare MEGA and OMEGA to the three density-based SELECT mechanisms shown in Figure 1 above: sampling from p_{dg} ("HER"), sampling from the historical p_{ag} as done approximately by RIG ("Achieved"), and sampling from a skewed historical p_{ag} that is approximately uniform on its support, as done by DISCERN and Skew-Fit ("Diverse"). We also compare against non density-based baselines as follows. `PointMaze` and `AntMaze` are the same environments used by the recent Sibling Rivalry paper (Trott et al., 2019). Thus, our results are directly comparable to Figure 3 of their paper, which tested four algorithms: HER, PPO (Schulman et al., 2017), PPO with intrinsic curiosity (Pathak et al., 2017), and PPO with Sibling Rivalry (PPO+SR). The `AntMaze` environment uses the same simulation as the `MazeAnt` environment tested in the Goal GAN paper (Florensa et al., 2018), but is four times larger. In Appendix D, we test MEGA on the smaller maze and obtain an almost 1000x increase in sample efficiency as compared to Goal GAN and the Goal GAN implementation of SAGG-RIAC (Baranes & Oudeyer, 2013). Results are not directly comparable as Goal GAN uses an on-policy TRPO base (Schulman et al., 2015), which is very sample inefficient relative to our off-policy DDPG base. Thus, we adapt the Goal GAN discriminator to our setting by training a success predictor to identify goals of intermediate difficulty (Appendix C) ("GoalDisc"). Finally, we compare against a minimum Q heuristic, which selects distant goals (Hartikainen et al., 2020) ("MinQ").

We note a few things before moving on. First, Sibling Rivalry (Trott et al., 2019) is the only prior work that directly addresses the long-horizon, sparse reward problem (without imitation learning). Other baselines were motivated by and tested on other problems. Second, the generative parts of Goal GAN and RIG are orthogonal to our work, and could be combined with MEGA-style generate-and-test selection, as we noted above in Section 3.2. We adopted the generative mechanism of DISCERN (sampling from a buffer) as it is simple and has a built-in bias toward achievable samples. For a fair comparison, all of our implemented baselines use the same buffer-based generative model and benefit from our base DDPG+HER implementation (Appendix C). The key difference between MEGA and our implemented baselines is the SELECT mechanism (line $(*)$ of Algorithm 2).

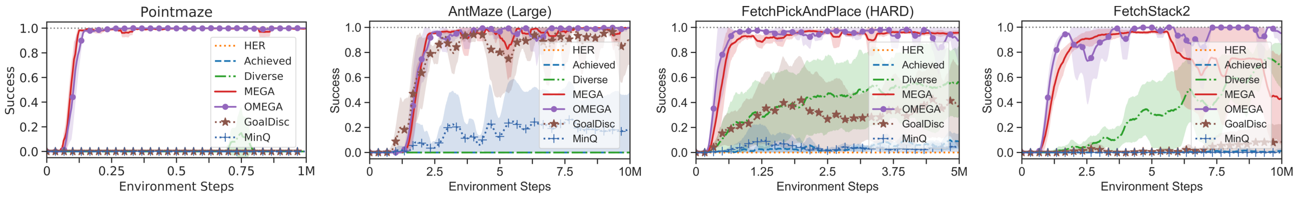

Figure 3. Test success on the desired goal distribution, evaluated throughout training, for several behaviour goal selection methods (3 seeds each). Our methods (MEGA and OMEGA) are the only the methods which are able to solve the tasks with highest sample efficiency. In `FetchStack2` we see that OMEGA's eventual focus on the desired goal distribution is necessary for long run stability.

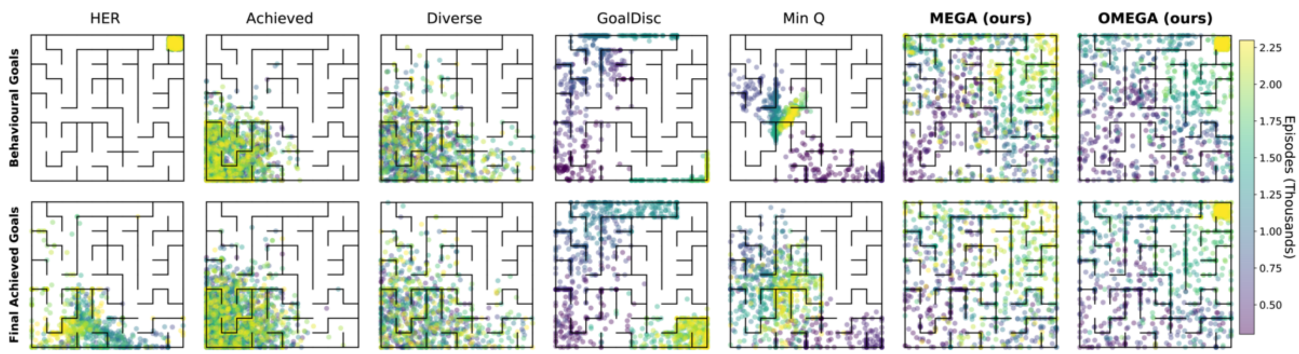

Figure 4. Visualization of behavioural (**top**) and terminal achieved (**bottom**) goals in `PointMaze`, colour-coded for over the course of training for several behavioural goal sampling methods. Only our methods reach the desired goal area in top right hand corner in approximately 2000 episodes, beating the previous state of the art (Trott et al., 2019) by almost 2 orders of magnitude (100 times).

Main results Our main results, shown in Figure 3 clearly demonstrate the advantage of minimum density sampling. We confirm that desired goal sampling (HER) is unable to solve the tasks, and observe that Achieved and Diverse goal sampling fail to place enough focus on the frontier of the achieved goal distribution to bridge the gap between the initial state and desired goal distributions. On `PointMaze`, none of the baselines were able to solve the environment within 1 million steps. The best performing algorithm from Trott et al. (2019) is PPO+SR, which solves `PointMaze` to 90% success in approximately 7.5 million time steps (O/MEGA is almost 100 times faster). On `AntMaze`, only MEGA, OMEGA and the GoalDisc are able to solve the environment. The best performing algorithm from Trott et al. (2019) is hierarchical PPO+SR, which solves `AntMaze` to 90% success in approximately 25 million time steps (O/MEGA is roughly 10 times faster). On a maze that is four times smaller, Florensa et al. (2018) tested four algorithms, including SAGG-RIAC (Baranes & Oudeyer, 2013), which was implemented, along with Goal GAN, using a TRPO base. Their best performing result achieves 71% coverage of the maze in about 175 million time steps (O/MEGA is roughly 100 times faster on a larger maze). O/MEGA also demonstrates that `FetchStack2` can be solved from scratch, without expert demonstrations (Duan et al., 2017; Nair et al., 2018a) or a task curriculum (Colas et al., 2018).

Maximizing entropy In Figure 5 (top), we observe that our approach increases the empirical entropy of the achieved goal buffer (the MEGA objective) much faster than other goal sampling methods. MEGA and OMEGA rapidly increase the entropy and begin to succeed with respect to the desired goals as the maximum entropy is reached. As OMEGA begins to shift towards sampling mainly from the desired goal distribution (Figure 5 (bottom)), the entropy declines as desired goal trajectories become over represented. We observe that the intermediate difficulty heuristic (GoalDisc) is a good optimizer of the MEGA objective on `AntMaze`, likely due to the environment's linear structure. This explains its comparable performance to MEGA.

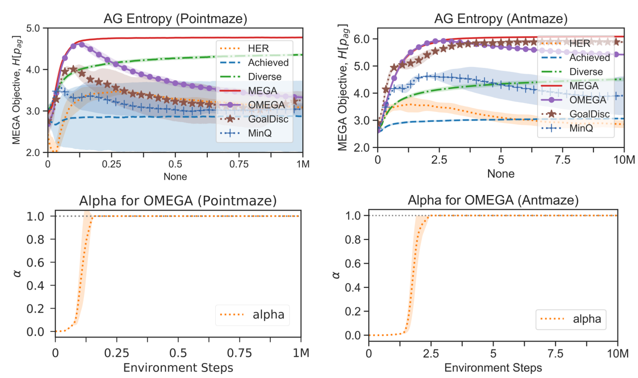

Figure 5. **Top**: Entropy of the achieved goal buffer for `Pointmaze` (left) and `Antmaze` (right) over course of training, estimated using a Kernel Density Estimator. O/MEGA expand the entropy much faster than the baselines. **Bottom**: α computed by OMEGA, which transitions from intrinsic to extrinsic goals.

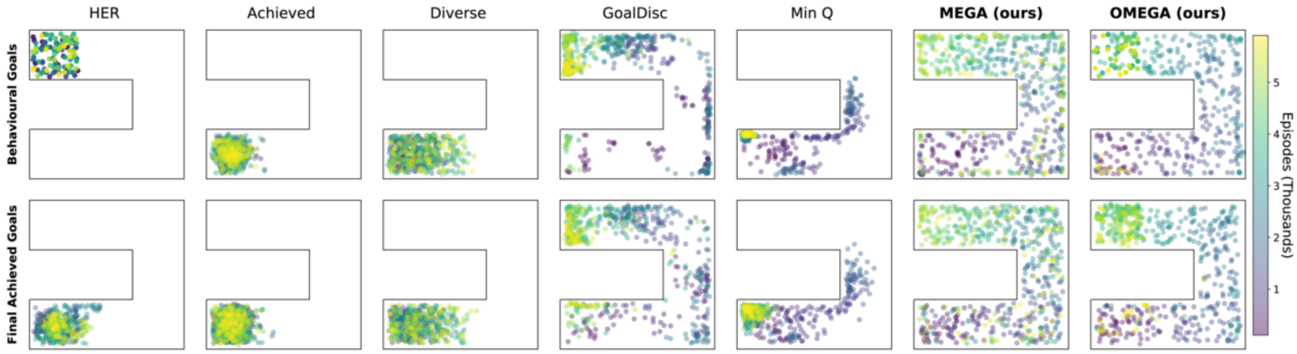

Figure 6. Visualization of behavioural (**top**) and terminal achieved (**bottom**) goals in `AntMaze`, colour-coded for over the course of training for several behavioural goal sampling methods. The only baseline that reached the desired goal in the top left was GoalDisc.

Visualization of achieved goals To gain intuition for how our method compares to the baselines, we visualize the terminal achieved goal at the end of the episodes throughout the training for `PointMaze` in Figure 4. Both MEGA and OMEGA set goals that spread outward from the starting location as training progresses, akin to a breadth-first search, with OMEGA eventually transitioning to goals from the desired goal distribution in the top right corner. Diverse sampling maintains a fairly uniform distribution at each iteration, but explores slowly as most goals are sampled from the interior of the support instead of the frontier. Achieved sampling oversamples goals near the starting location and suffers from a "rich get richer" problem. Difficulty-based GoalDisc and distance-based MinQ sampling explore deeply in certain directions, akin to a depth-first search, but ignore easier/closer goals that can uncover new paths.

A similar visualization for `AntMaze` is shown in Figure 6. Aside from our methods, the only baseline able to reach the desired goal area is GoalDisc. MEGA and OMEGA observe a higher diversity in achieved goals, which suggests the learned policy from our methods will be more robust than the GoalDisc policy if the desired goal distribution changes, but we did not directly test this hypothesis.

5. Other Related Work

Maximum entropy-based prioritization (MEP) While MEGA influences the entropy of the historical achieved goal distribution during SELECT, MEP (Zhao et al., 2019) reweighs experiences during OPTIMIZE to increase the entropy of the goals in an agent's training distribution. Unlike MEGA, MEP does not set intrinsic goals and does not directly influence an agent's exploratory behavior. As a result, MEP is limited to the support set of the observed achieved goals and must rely on the generalization of the neural network model to cross long-horizon gaps. As MEGA and MEP can be applied simultaneously, we compared using HER and O/MEGA, with and without MEP in `PointMaze` and `FetchPickAndPlace`. As shown in Figure 7, applying MEP to HER helps the agent achieved some success in the `FetchPickAndPlace`, but is unable to help in the `PointMaze` where the long horizon gap is more severe. Combining MEGA and MEP has limited effect, possibly because a MEGA agent's achieved goal distribution is already close to uniform. See Appendix C ("MEP") for details.

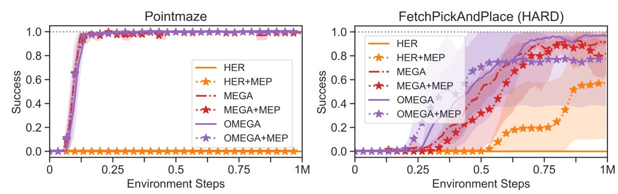

Figure 7. MEP (Zhao et al., 2019) maximizes the entropy of training goals in the OPTIMIZE method. While MEP can help the function approximator generalize, and allows HER to achieve some success in `FetchPickAndPlace` (hard), it does not directly help the agent explore and cross long horizon gaps.

Curiosity Maximizing entropy in the goal space is closely related to general RL (not multi-goal) algorithms that seek to maximize entropy in the state space (Hazan et al., 2018; Lee et al., 2019) or grant the agent additional reward based on some measure of novelty, surprise or learning progress (Kolter & Ng, 2009; Schmidhuber, 2010; Lopes et al., 2012; Bellemare et al., 2016; Ostrovski et al., 2017; Tang et al., 2017; Pathak et al., 2017; Burda et al., 2019). Two key differences should be noted. First, MEGA uses a low-dimensional, abstract goal space to drive exploration in meaningful directions (Baranes & Oudeyer, 2010). This focuses the agent on what matters, and avoids the "noisy-TV" problem (Burda et al., 2019). As this requires a known, semantically meaningful goal space, future work might explore how one can automatically choose a good goal space (Lee et al., 2020). Second, MEGA agents learn and use a goal-conditioned policy, which makes MEGA exploration more "active" than exploration based on intrinsic reward (Shyam et al., 2018). It is reasonable to interpret the low density region of an agent's achievable goal space as its "frontier", so that MEGA exploration is a form of *frontier exploration* (Yamauchi, 1997; Holz et al., 2010; Ecoffet et al., 2019). Recent work in this family includes Badia et al. (2020), Bharadhwaj et al. (2020) and Zhang et al. (2020). Since the agent's entire policy changes with the goal, goal-conditioned exploration is somewhat similar to noise-conditioned (Plappert et al., 2017; Osband et al., 2017) and variational exploration algorithms (next paragraph), a key difference being that MEGA agents *choose* their goal.

Empowerment Since the agent's off-policy, goal relabeling learning algorithm can be understood as minimizing the conditional entropy of (on-policy) achieved goals given some potential goal distribution p_g (not necessarily the behavioural goal distribution p_{bg}), simultaneously choosing p_{bg} to maximize entropy of historical achieved goals (the MEGA objective) results in an *empowerment*-like objective: $\max_{p_{bg}} H[p_{ag}] - H[\mathrm{AG}(\tau \,|\, p_g) \,|\, p_g] \approx \max_{p_g} I[p_g; \mathrm{AG}(\tau \,|\, p_g)]$, where equality is approximate because the first max is with respect to p_{bg}, and also because $H[p_{ag}]$ is historical, rather than on-policy.

Empowerment (Klyubin et al., 2005; Salge et al., 2014; Mohamed & Rezende, 2015) has gained traction in recent years as an intrinsic, unsupervised objective due to its intuitive interpretation and empirical success (Eysenbach et al., 2018; Hansen et al., 2020). We can precisely define empowerment in the multi-goal case as the *channel capacity* between goals and achieved goals (Cover & Thomas, 2012):

$$\mathcal{E}(s_0) = \max_{p_g} \mathbb{E}_{p(\tau|g,s_0)p_g(g)} I[p_g; \mathrm{AG}(\tau \,|\, p_g)], \tag{8}$$

where s_0 represents the initial state distribution. To see the intuitive appeal of this objective, we reiterate the common argument and write: $I[p_g; \mathrm{AG}(\tau \,|\, p_g)] = H[p_g] - H[p_g \,|\, \mathrm{AG}(\tau \,|\, p_g)]$, where H is entropy. This now has an intuitive interpretation: letting $H[p_g]$ stand for the size of the goal set, and $H[p_g \,|\, \mathrm{AG}(\tau \,|\, p_g)]$ for the uncertainty of goal achievement, maximizing empowerment roughly amounts to maximizing the *size of the achievable goal set*.

The common approach to maximizing empowerment has been to either fix or parameterize the distribution p_g and maximize the objective $I[p_g; \mathrm{AG}(\tau \,|\, p_g)]$ *on-policy* (Gregor et al., 2016; Warde-Farley et al., 2019; Pong et al., 2019). We can think of this as approximating (8) using the behavioural goal distribution $p_{bg} \approx \arg\max_{p_g} I[p_g; \mathrm{AG}(s_T \,|\, p_g)]$. A key insight behind our work is that there is no reason for an off-policy agent to constrain itself to pursuing goals from the distribution it is trying to optimize. Instead, we argue that for off-policy agents seeking to optimize (8), the role of the behavioural goal distribution p_{bg} should be to produce useful empirical data for optimizing the true *off-policy* empowerment (8), where the maximum is taken over all possible p_g. Practically speaking, this means exploring to maximize entropy of the historical achieved goal distribution (i.e,. the MEGA objective), and letting our off-policy, goal relabeling algorithm minimize the conditional entropy term. Future work should investigate whether the off-policy gain of MEGA over the on-policy Diverse sampling can be transferred to general empowerment maximizing algorithms.

6. Limitations and Future Work

The present work has several limitations that should be addressed by future work. First, our experiments focus on environments with predefined, semantically meaningful, and well-behaved goal spaces. In the general case, an agent will have to learn its own goal space (Warde-Farley et al., 2019; Pong et al., 2019) and it will be interesting to see whether MEGA exploration extends well to latent spaces. A foreseeable problem, which we did not experience, is that differential entropy is sensitive to reparameterizations of the feature space; this implies that either (1) a MEGA agent's goal space needs to be, to a degree, "well-behaved", or (2) the MEGA objective needs to be recast or extended so as to be robust to parameterization. We hypothesize that a major reason for MEGA's success is that the goal spaces in our chosen tasks are semantically meaningfully and directly relevant to the tasks being solved; an interesting direction for future research involves the automatic discovery of such low-dimensional abstractions (Lee et al., 2020). A second limitation of our work is the approach to achievability, which is required in order for our minimum density heuristic to be sensible. Presently, we rely on a combination of buffer-based generation, a cutoff mechanism that eliminates goals with low Q-values, and the ability of our off-policy learning algorithm to generalize. But even so, our `FetchStack2` results show that the MEGA agent's performance begins to diverge after about 5 million steps. This is because the table (on which the blocks are being stacked) is not enclosed, and the agent begins to pursue difficult to achieve goals that are off the table. Future work should explore better ways to measure off-policy achievability (Thomas et al., 2015). Finally, the behavior of MEGA on `FetchStack2` suggests that an unconstrained, intrinsically motivated agent may start to set unsafe goals, which has implications for safety (Garcıa & Fernández, 2015).

7. Conclusion

This paper proposes to address the long-horizon, sparse reward problem in multi-goal RL by having the agent maximize the entropy of the historical achieved goal distribution. We do this by setting intrinsic goals in sparsely explored areas of the goal space, which focuses exploration on the frontier of the achievable goal set. This strategy obtains results that are more than 10 times more sample efficient than prior approaches in four long-horizon multi-goal tasks.

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