A. Proof that L is concave and positive.

We will use the notations previously introduced as well as:

$$z_p = U_n x_p \,.$$

As $L(W) = \sum_p \ell(W, z_p)$, we will simply study $\forall z$,

$$W \to \ell(W, z) = \|(\mathbf{I} - A)z\|^2 - \|A|Wz|\|^2 \,.$$

Observe first that if $\|\{W, A\}\| \leq 1$, then $\|A| \leq 1$, and:

$$\|Wz\| \leq \|(\mathbf{I} - A)z\|$$

Thus,

$$\||Wz|\| \leq \|(\mathbf{I} - A)z\|$$

and:

$$\|A|Wz|\| \leq \|(\mathbf{I} - A)z\| \,.$$

Consequently, $\ell(W, z) \geq 0, \forall z, \forall W \in \mathcal{C}$. Furthermore, let $W_1, W_2 \in \mathcal{C}$ two operators and $0 \leq \lambda \leq 1$. Then:

$$|\big(\lambda W_1 + (1 - \lambda)W_2\big)z| \leq \lambda|W_1|z + (1 - \lambda)|W_2|z$$

where for $x \in \mathbb{R}^n$, $x \geq 0$ iff $x_i \geq 0$. If $Ax > 0$ when $x > 0$, then:

$$A|\big(\lambda W_1 + (1 - \lambda)W_2\big)z| \leq \lambda A|W_1|z + (1 - \lambda)A|W_2|z| \,,$$

which implies (as all coordinates are non negative):

$$\|A|\big(\lambda W_1 + (1 - \lambda)W_2\big)z|\|^2 \leq \|\lambda A|W_1|z + (1 - \lambda)A|W_2|z|\|^2 \,,$$

yet one can use the fact that $z \to \|z\|^2$ is convex to conclude. Thus, $W \to \ell(W, z)$ is convex in W.

B. Proof of Proposition 3.5

Proof. Observe that $\mathcal{F}$ linearly conjugates $\mathcal{C}$ to $\{\hat{W} \in \mathbb{C}^{(2d+1)\times K}, \sum_{k=1}^{K} |\hat{W}^k[i]|^2 + |W^k[2d+1-i]|^2 + |\hat{A}[i]|^2 + |\hat{A}[2d+1-i]|^2 \leq 1, \forall i \leq d, \sum_{k=1}^{K} |W^k[2d+1]|^2 + |\hat{A}[2d+1]|^2 \leq 1\}$. The extremal points of the latter are simply $\mathcal{S}' = \{\hat{W} \in \mathbb{C}^{(2d+1)\times K}, \sum_{k=1}^{K} |\hat{W}^k[i]|^2 + |W^k[2d+1-i]|^2 + |\hat{A}[i]|^2 + |\hat{A}[2d+1-i]|^2 = 1, \forall i \leq d, \sum_{k=1}^{K} |W^k[2d+1]|^2 + |\hat{A}[2d+1]|^2 = 1\}$, which is conjugated by $\mathcal{F}^*$ to $\mathcal{S}$. But $\mathcal{S}'$ corresponds to the spectrum of an isometry, leading to the conclusion. □