A. Organization of the Appendix

Appendix B describes the Greedy algorithm to solve Noisy **M**k**SC** and its approximation ratio.

Appendix C provides omitted proofs from Section 3.

Appendix D provides omitted proofs from Section 4.

Appendix E presents details of the sampling method for Influence Maximization with k topics.

B. Greedy Algorithm

In this part, we investigate performance guarantee of Greedy algorithm. Greedy has been proven to obtain an approximation ratio of 2 for **M**k**SC** with monotone non-noisy objective function (Ohsaka & Yoshida, 2015). We extend the authors' proof to **M**k**SC** under noise and show that Greedy is able to obtain approximation of $\frac{2+2\epsilon B}{1-\epsilon}$ when f is monotone and $\frac{(2+2\epsilon+4\epsilon B)(1+\epsilon)}{(1-\epsilon)^2}+1$ in case of non-monotonicity.

The pseudo code of Greedy is presented by Alg. 4

Algorithm 4 Greedy Algorithm

Input F, k, B

$\mathbf{s}^0 = \{\emptyset, \emptyset, ...\emptyset\}$
for $t = 1 \to B$ **do**
  $e, i = argmax_{e \in V; i \in [k]} F(\mathbf{s}^{t-1} \sqcup \langle e, i \rangle)$
  $\mathbf{s}^t = \mathbf{s}^{t-1} \sqcup \langle e, i \rangle$

Return $\mathbf{s}^B$ if f is **monotone**; $argmax_{\mathbf{s}^i; i \in \{1,...,B\}} F(\mathbf{s}^i)$ if f is **non-monotone**.

Theorem 5. Given an instance of Noisy **M**k**SC** with input V, k, B and F is an ϵ-estimation of the **monotone** k-submodular objective function f. If $\mathbf{s}$ is an output of the Greedy algorithm and $\mathbf{o}$ is an optimal solution, then

$$f(\mathbf{o}) \leq \frac{2+2\epsilon B}{1-\epsilon} f(\mathbf{s}) \tag{3}$$

Proof. Let e^j and i^j be a selection in iteration j of the Greedy algorithm, we construct a sequence $\{\mathbf{o}^j\}$ as follows:

- $\mathbf{o}^0 = \mathbf{o}$
- With $j > 0$, let $S^j = supp(\mathbf{o}^{j-1}) \setminus supp(\mathbf{s}^{j-1})$. Let $o^j = e^j$ if $e^j \in S^j$, otherwise let o^j to be an arbitrary element in S^j.
  - Define $\mathbf{o}^{j-1/2}$ as a k-set that $\mathbf{o}^{j-1/2}(e) = \mathbf{o}^{j-1}(e)\ \forall e \in V \setminus \{o^j\}$ and $\mathbf{o}^{j-1/2}(o^j) = 0$.
  - Define $\mathbf{o}^j$ as a k-set that $\mathbf{o}^j(e) = \mathbf{o}^{j-1/2}(e)\ \forall e \in V \setminus \{e^j\}$ and $\mathbf{o}^j(e^j) = i^j$
  - Define $\mathbf{s}^{j-1/2}$ as a k-set that $\mathbf{s}^{j-1/2}(e) = \mathbf{s}^{j-1}(e)\ \forall e \in V \setminus \{o^j\}$ and $\mathbf{s}^{j-1/2}(o^j) = i^j$.

It is trivial that $\mathbf{o}^B = \mathbf{s}^B$. We have:

$$f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \leq f(\mathbf{o}^{j-1}) - f(\mathbf{o}^{j-1/2}) \tag{4}$$
$$\leq f(\mathbf{s}^{j-1/2}) - f(\mathbf{s}^{j-1}) \tag{5}$$
$$\leq \frac{1}{1-\epsilon} F(\mathbf{s}^{j-1/2}) - f(\mathbf{s}^{j-1}) \tag{6}$$
$$\leq \frac{1}{1-\epsilon} F(\mathbf{s}^{j}) - f(\mathbf{s}^{j-1}) \tag{7}$$
$$\leq \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^{j}) - f(\mathbf{s}^{j-1}) \tag{8}$$

The inequality (4) is due to f is monotone, thus $f(\mathbf{o}^j) \geq f(\mathbf{o}^{j-1/2})$. The inequality (5) is from k-submodularity of f. The inequality (7) is due to greedy selection. Therefore:

$$f(\mathbf{o}) - f(\mathbf{s}) = \sum_{j=1}^{B} \Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \Big) \leq \sum_{j=1}^{B} \Big(\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - f(\mathbf{s}^{j-1}) \Big)$$
$$\leq \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}) + \sum_{j=1}^{B} \frac{2\epsilon}{1-\epsilon} f(\mathbf{s}^j) \leq \frac{1+\epsilon+2\epsilon B}{1-\epsilon} f(\mathbf{s})$$

Thus $f(\mathbf{o}) \leq \frac{2+2\epsilon B}{1-\epsilon} f(\mathbf{s})$, which completes the proof. □

Theorem 6. Given an instance of Noisy **MkSC** with input V, k, B and F is an ϵ-estimation of the **non-monotone** k-submodular objective function f. If $\mathbf{s}$ is an output of the Greedy algorithm and $\mathbf{o}$ is an optimal solution, then

$$f(\mathbf{o}) \leq \frac{3+\epsilon+4\epsilon B}{1-\epsilon} f(\mathbf{s}) \tag{9}$$

Proof. We use the same definition of $\mathbf{o}^j, \mathbf{o}^{j-1/2}, \mathbf{s}^j$ as in proof of Theorem 5.

Although f is non-monotone, f is *pairwise-monotone* due to k-submodularity (Ward & Živnỳ, 2016). To be specific, given a k-set $\mathbf{x}$ and $e \notin supp(\mathbf{x})$, we have

$$\Delta_{e,i} f(\mathbf{x}) + \Delta_{e,j} f(\mathbf{x}) \geq 0 \ \forall i, j \in [k] \text{ and } i \neq j \tag{10}$$

Let consider a value of $j \in [1, B]$, due to pairwise-monotonicity, there should exist $i' \in [k]$ that $f(\mathbf{s}^{j-1} \sqcup \langle e^j, i' \rangle) \geq f(\mathbf{s}^{j-1})$. Moreover, due to greedy selection, $f(\mathbf{s}^{j-1} \sqcup \langle e^j, i' \rangle) \leq \frac{1}{1-\epsilon} F(\mathbf{s}^{j-1} \sqcup \langle e^j, i' \rangle) \leq \frac{1}{1-\epsilon} F(\mathbf{s}^j) \leq \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j)$. Thus $\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) \geq f(\mathbf{s}^{j-1})$. We consider 2 following cases:

- $\mathbf{o}^j = \mathbf{o}^{j-1}$. In this case $f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) = 0 \leq \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - f(\mathbf{s}^{j-1})$
- $\mathbf{o}^j \neq \mathbf{o}^{j-1}$. There would be 2 sub-cases:
  - $\mathbf{o}^{j-1}(e^j) = 0$. Then let $i' \in [k]$ be an arbitrary number and $i' \neq i^j$. Then

$$f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) = f(\mathbf{o}^{j-1}) + f(\mathbf{o}^{j-1/2} \sqcup \langle e^j, i' \rangle) - 2f(\mathbf{o}^{j-1/2}) - \Big(f(\mathbf{o}^{j-1/2} \sqcup \langle e^j, i' \rangle) + f(\mathbf{o}^j) - 2f(\mathbf{o}^{j-1/2}) \Big) \tag{11}$$
$$\leq f(\mathbf{o}^{j-1}) + f(\mathbf{o}^{j-1/2} \sqcup \langle e^j, i' \rangle) - 2f(\mathbf{o}^{j-1/2}) \tag{12}$$
$$\leq f(\mathbf{s}^{j-1/2}) + f(\mathbf{s}^{j-1} \sqcup \langle e^j, i' \rangle) - 2f(\mathbf{s}^{j-1}) \tag{13}$$
$$\leq \frac{1}{1-\epsilon} F(\mathbf{s}^{j-1/2}) + \frac{1}{1-\epsilon} F(\mathbf{s}^{j-1} \sqcup \langle e^j, i' \rangle) - 2f(\mathbf{s}^{j-1}) \tag{14}$$
$$\leq \frac{2}{1-\epsilon} F(\mathbf{s}^j) - 2f(\mathbf{s}^{j-1}) \leq 2\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - 2f(\mathbf{s}^{j-1}) \tag{15}$$

  The inequality 12 is due to pairwise-monotonicity of f. The inequality 13 is from k-submodularity and inequality 15 is due to greedy selection.
  - $\mathbf{o}^{j-1}(e^j) \neq i^j$. Then:

$$f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) = 2f(\mathbf{o}^{j-1}) - 2f(\mathbf{o}^{j-1/2}) - \Big(f(\mathbf{o}^{j-1}) + f(\mathbf{o}^j) - 2f(\mathbf{o}^{j-1/2}) \Big)$$
$$\leq 2f(\mathbf{o}^{j-1}) - 2f(\mathbf{o}^{j-1/2}) \leq 2f(\mathbf{s}^{j-1/2}) - 2f(\mathbf{s}^{j-1}) \leq \frac{2}{1-\epsilon} F(\mathbf{s}^{j-1/2}) - 2f(\mathbf{s}^{j-1})$$
$$\leq \frac{2}{1-\epsilon} F(\mathbf{s}^j) - 2f(\mathbf{s}^{j-1}) \leq 2\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - 2f(\mathbf{s}^{j-1})$$

Both cases imply that $f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \leq 2\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - 2f(\mathbf{s}^{j-1})$. Therefore:

$$f(\mathbf{o}) - f(\mathbf{s}) = \sum_{j=1}^{B} \Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \Big) \leq 2 \sum_{j=1}^{B} \Big(\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - f(\mathbf{s}^{j-1}) \Big)$$
$$\leq 2 \times \Big(\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^B) + \sum_{j=1}^{B} \frac{2\epsilon}{1-\epsilon} f(\mathbf{s}^j) \Big) \leq \frac{2+2\epsilon+4\epsilon B}{1-\epsilon} \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s})$$

Thus, $f(\mathbf{o}) \leq \big(\frac{(2+2\epsilon+4\epsilon B)(1+\epsilon)}{(1-\epsilon)^2} + 1 \big) f(\mathbf{s})$, which completes the proof. □

C. DSTREAM Algorithm

In this part, we present in detail omitted proofs of DSTREAM's approximation ratio. There would be 2 separate sub-parts: one is for when f is **monotone** - which we describe omitted proofs of claims in Proposition 1; the other is for when f is **non-monotone** - which we would fully prove the Theorem 2.

C.1. Approximation ratio of DSTREAM when f is monotone

Proof of Claim 1. Denote $\langle e^j, i^j \rangle$ as j-th addition of Alg. 1, i.e $\mathbf{s}^j = \mathbf{s}^{j-1} \sqcup \langle e^j, i^j \rangle$. Define:

$$\mathbf{o}^{j-1/2} = (\mathbf{o} \sqcup \mathbf{s}^j) \sqcup \mathbf{s}^{j-1}$$

Furthermore, define $\mathbf{s}^{j-1/2}$ as:

- If $e^j \in supp(\mathbf{o})$, let $i' = \mathbf{o}(e^j)$. Then $\mathbf{s}^{j-1/2} = \mathbf{s}^{j-1} \sqcup \langle e^j, i' \rangle$
- Otherwise, $\mathbf{s}^{j-1/2} = \mathbf{s}^{j-1}$

We have:

$$f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \leq f(\mathbf{o}^{j-1}) - f(\mathbf{o}^{j-1/2}) \tag{16}$$
$$\leq f(\mathbf{s}^{j-1/2}) - f(\mathbf{s}^{j-1}) \tag{17}$$
$$\leq \frac{1}{1-\epsilon} F(\mathbf{s}^{j-1/2}) - f(\mathbf{s}^{j-1}) \tag{18}$$
$$\leq \frac{1}{1-\epsilon} F(\mathbf{s}^{j}) - f(\mathbf{s}^{j-1}) \tag{19}$$
$$\leq \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^{j}) - f(\mathbf{s}^{j-1}) \tag{20}$$

The Inequality 16 is due to monotonicity of f; the inequality 17 is due to its k-submodularity and the inequality 19 is from selection of Alg. 1 that guarantees $i^j = argmax_{i \in [k]} F(\mathbf{s}^{j-1} \sqcup \langle e^j, i \rangle)$. Therefore:

$$f(\mathbf{o}) - f(\mathbf{o}^t) = \sum_{j=1}^{t} \Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \Big) \leq \sum_{j=1}^{t} \Big(\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - f(\mathbf{s}^{j-1}) \Big)$$
$$= \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}) + \sum_{i=1}^{t-1} \Big(\frac{1+\epsilon}{1-\epsilon} - 1 \Big) f(\mathbf{s}^i) \leq \frac{1+\epsilon+2B\epsilon}{1-\epsilon} f(\mathbf{s})$$

which completes the proof.

Proof of Claim 2. As $\mathbf{s} \sqsubseteq \mathbf{o}^t$, denote $\{\langle u_1, j_1 \rangle, ... \langle u_r, j_r \rangle\}$ as a set of elements and their placement in $\mathbf{o}^t$ that are not in $\mathbf{s}$.

For each $\langle u_i, j_i \rangle$, denote $\mathbf{s}_i$ as $\mathbf{s}$ when Alg. 1 encounters u_i. As u_i was not added into $\mathbf{s}$, $\frac{F(\mathbf{s}_i \sqcup \langle u_i, j_i \rangle)}{1-\epsilon} \leq (|\mathbf{s}_i| + 1)\frac{o}{M}$. While $\frac{F(\mathbf{s}_i)}{1-\epsilon} \geq |\mathbf{s}_i| \frac{o}{M}$, we have:

$$\frac{F(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle)}{1-\epsilon} - \frac{F(\mathbf{s}_i)}{1+\epsilon} \leq \frac{o}{M} + \frac{2\epsilon F(\mathbf{s}_i)}{1-\epsilon^2}$$

Denote $\mathbf{u}_i = \mathbf{s} \sqcup \{\langle u_1, j_1\rangle, ...\langle u_i, j_i\rangle\}$. We have

$$\begin{aligned} f(\mathbf{o}^t) - f(\mathbf{s}) &= \sum_{i=1}^{B-t} \Big(f(\mathbf{u}_i) - f(\mathbf{u}_{i-1}) \Big) \leq \sum_{i=1}^{B-t} \Big(f(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle) - f(\mathbf{s}_i) \Big) \\ &\leq \sum_{i=1}^{B-t} \Big(\frac{1}{1-\epsilon} F(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle) - \frac{1}{1+\epsilon} F(\mathbf{s}^i) \Big) \leq \sum_{i=1}^{B-t} \left(\frac{o}{M} + \frac{2\epsilon}{1-\epsilon^2} F(\mathbf{s}_i) \right) \leq \frac{1}{M} f(\mathbf{o}) + \frac{2\epsilon B}{1-\epsilon} f(\mathbf{s}) \end{aligned}$$

which complete the proof.

C.2. Approximation of DSTREAM when f is non-monotone

Proof of Theorem 2 We still use definition of $\mathbf{o}^j, \mathbf{o}^{j-1/2}, e^j, i^j, \mathbf{s}^j, \mathbf{s}^{j-1/2}$ as in the monotone proof. Also, for simplicity, we first prove the approximation ratio of Alg. 1 (assume $f(\mathbf{o})$ is known).

If $t = B$, $f(\mathbf{s}) \geq \frac{1}{1+\epsilon} \max_{i \in \{1,..,B\}} F(\mathbf{s}^i) \geq \frac{1}{1+\epsilon} F(\mathbf{s}^B) \geq \frac{1-\epsilon}{1+\epsilon} B \frac{o}{M} \geq \frac{1-\epsilon}{1+\epsilon} \frac{1}{(1+\gamma)M} f(\mathbf{o})$. The rest of the proof will focus on case $t < B$.

Due to pairwise-monotonicity of a k-submodular function, for any $j \in \{1, ..., t\}$, there exists no pair $i_1 \neq i_2 \in [k]$ that $\Delta_{e^j, i_1} f(\mathbf{s}^{j-1}) < 0$ and $\Delta_{e^j, i_2} f(\mathbf{s}^{j-1}) < 0$. Therefore $\max_{i \in [k]} f(\mathbf{s}^{j-1} \sqcup \langle e^j, i\rangle) \geq f(\mathbf{s}^{j-1})$. Thus

$$f(\mathbf{s}^j) \geq \frac{F(\mathbf{s}^j)}{1+\epsilon} = \frac{\max_{i\in[k]} F(\mathbf{s}^{j-1} \sqcup \langle e^j, i\rangle)}{1+\epsilon} \geq \frac{1-\epsilon}{1+\epsilon} \max_{i \in [k]} f(\mathbf{s}^{j-1} \sqcup \langle e^j, i\rangle) \geq \frac{1-\epsilon}{1+\epsilon} f(\mathbf{s}^{j-1}) \tag{21}$$

Let's consider relation between $\mathbf{o}^{j-1}$ and $\mathbf{o}^j$, there are 2 cases:

- $|supp(\mathbf{o}^{j-1})| < |supp(\mathbf{o}^j)|$, which means $e^j \notin supp(\mathbf{o})$. We randomly pick $i' \in [k]$ and $i' \neq i^j$, we have

$$\begin{aligned} f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) &= f(\mathbf{o}^{j-1} \sqcup \langle e^j, i'\rangle) - f(\mathbf{o}^{j-1}) - \big(f(\mathbf{o}^j) + f(\mathbf{o}^{j-1} \sqcup \langle e^j, i'\rangle) - 2f(\mathbf{o}^{j-1}) \big) \\ &\leq f(\mathbf{o}^{j-1} \sqcup \langle e^j, i'\rangle) - f(\mathbf{o}^{j-1}) \leq f(\mathbf{s}^{j-1} \sqcup \langle e^j, i'\rangle) - f(\mathbf{s}^{j-1}) \\ &\leq \frac{1}{1-\epsilon} F(\mathbf{s}^{j-1} \sqcup \langle e^j, i'\rangle) - f(\mathbf{s}^{j-1}) \leq \frac{1}{1-\epsilon} F(\mathbf{s}^j) - f(\mathbf{s}^{j-1}) \\ &\leq \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - f(\mathbf{s}^{j-1}) \end{aligned}$$

- $|supp(\mathbf{o}^{j-1})| = |supp(\mathbf{o}^j)|$, which means $e^j \in supp(\mathbf{o})$. We have 2 sub-cases
  - $\mathbf{o}^{j-1}(e^j) = i^j$. Then $f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) = 0 \leq \frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - f(\mathbf{s}^{j-1})$
  - $\mathbf{o}^{j-1}(e^j) \neq i^j$, we have

$$\begin{aligned} f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) &= 2f(\mathbf{o}^{j-1}) - 2f(\mathbf{o}^{j-1/2}) - \big(f(\mathbf{o}^{j-1}) + f(\mathbf{o}^j) - 2(\mathbf{o}^{j-1/2}) \big) \\ &\leq 2f(\mathbf{o}^{j-1}) - 2f(\mathbf{o}^{j-1/2}) \leq 2f(\mathbf{s}^{j-1/2}) - 2f(\mathbf{s}^{j-1}) \leq 2\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - 2f(\mathbf{s}^{j-1}) \end{aligned}$$

Therefore, in overall $f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \leq 2\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - 2f(\mathbf{s}^{j-1})$. We have

$$f(\mathbf{o}) - f(\mathbf{o}^t) = \sum_{j=1}^{t} \Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) \Big) \leq \sum_{j=1}^{t} 2\Big(\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^j) - f(\mathbf{s}^{j-1}) \Big) \tag{22}$$

$$= 2\frac{1+\epsilon}{1-\epsilon} f(\mathbf{s}^t) + 2\sum_{i=1}^{t-1} \frac{2\epsilon}{1-\epsilon} f(\mathbf{s}^i) \leq \frac{2+2\epsilon+4\epsilon B}{1-\epsilon} \max_{i \leq t} f(\mathbf{s}^i) \leq \frac{(1+\epsilon)(2+2\epsilon+4\epsilon B)}{(1-\epsilon)^2} f(\mathbf{s}) \tag{23}$$

Also, similar to monotone case, as $\mathbf{s}^t \sqsubseteq \mathbf{o}^t$, denote $\{\langle u_1, j_1\rangle, ...\langle u_r, j_r\rangle\}$ as a set of elements and their placement in $\mathbf{o}^t$ that are not in $\mathbf{s}^t$. For each $\langle u_i, j_i\rangle$, denote $\mathbf{s}_i$ as $\mathbf{s}^t$ when Alg. 1 encounters u_i. Denote $\mathbf{u}_i = \mathbf{s}^t \sqcup \{\langle u_1, j_1\rangle, ...\langle u_i, j_i\rangle\}$. We have

$$f(\mathbf{o}^t) - f(\mathbf{s}^t) = \sum_{i=1}^{B-t}\Big(f(\mathbf{u}^i) - f(\mathbf{u}^{i-1})\Big) \leq \sum_{i=1}^{B-t}\Big(f(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle) - f(\mathbf{s}_i)\Big) \tag{24}$$

$$\leq \sum_{i=1}^{B-t}\left(\frac{F(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle)}{1-\epsilon} - \frac{F(\mathbf{s}_i)}{1+\epsilon}\right) \leq \sum_{i=1}^{B-t}\left(\frac{o}{M} + \frac{2\epsilon}{1-\epsilon^2}F(\mathbf{s}^i)\right) \tag{25}$$

$$\leq \frac{1}{M}f(\mathbf{o}) + \frac{2\epsilon B}{1-\epsilon^2}F(\mathbf{s}) \leq \frac{1}{M}f(\mathbf{o}) + \frac{2\epsilon B}{1-\epsilon}f(\mathbf{s}) \tag{26}$$

Combining Equ. 23 and 26, we have

$$(1-\frac{1}{M})f(\mathbf{o}) \leq \Big(\frac{(1+\epsilon)(2+2\epsilon+4\epsilon B)}{(1-\epsilon)^2} + \frac{2\epsilon B}{1-\epsilon} + \frac{1+\epsilon}{1-\epsilon}\Big)f(\mathbf{s}) = \frac{3+4\epsilon+6\epsilon B+\epsilon^2+2\epsilon^2 B}{(1-\epsilon)^2}f(\mathbf{s})$$

The approximation ratio of DSTREAM when discarding assumption of known $f(\mathbf{o})$ is trivially follows as in the proof of the monotone case.

D. RSTREAM Algorithm

In this part, we present in detail omitted proofs of RSTREAM's approximation ratio. There would be 2 separate sub-parts: one is for when f is **monotone** - which we describe omitted proofs of claims in Theorem 3; the other is for when f is **non-monotone** - which we would fully prove the Theorem 4.

Note that in proofs related to RSTREAM, we abuse the notation and simply write t and $\mathbf{s}^i$ to indicate $t_{j,\epsilon'}$ and $\mathbf{s}^i_{j,\epsilon'}$ in Alg. 3 when o is estimated by $M(1+\gamma)^j$ that satisfies $f(\mathbf{o}) \geq oB \geq \frac{f(\mathbf{o})}{1+\gamma}$; and $\epsilon' = \epsilon$.

Similar to DSTREAM's proof, denote $\langle e^j, i^j\rangle$ as j-th addition of Alg. 3, i.e $\mathbf{s}^j = \mathbf{s}^{j-1} \sqcup \langle e^j, i^j\rangle$. Define:

$$\mathbf{o}^{j-1/2} = (\mathbf{o} \sqcup \mathbf{s}^j) \sqcup \mathbf{s}^{j-1}$$

Furthermore, define $\mathbf{s}^{j-1/2}$ as:

- If $e^j \in supp(\mathbf{o})$, let i' be an index of a set containing e^j in $\mathbf{o}$. Then $\mathbf{s}^{j-1/2} = \mathbf{s}^{j-1} \sqcup \langle e^j, i'\rangle$
- Otherwise, $\mathbf{s}^{j-1/2} = \mathbf{s}^{j-1}$

D.1. Approximation ratio of RSTREAM when f is monotone

Proof of Lemma 1 We have:

$$F(\mathbf{s}) = F(\mathbf{s}^B) = \sum_{i=1}^{B}\Big(F(\mathbf{s}^i) - F(\mathbf{s}^{i-1})\Big) = \sum_{i=1}^{B}\Big(F(\mathbf{s}^i) - \frac{1-\epsilon}{1+\epsilon}F(\mathbf{s}^{i-1})\Big) - \frac{2\epsilon}{1+\epsilon}\sum_{i}^{B-1}F(\mathbf{s}^i)$$

$$\geq (1-\epsilon)\sum_{i=1}^{B}\frac{o}{M} - 2\epsilon\sum_{i}^{B-1}f(\mathbf{s}^i) \geq \frac{(1-\epsilon)f(\mathbf{o})}{(1+\gamma)M} - 2\epsilon Bf(\mathbf{s})$$

Therefore, $\frac{(1+\gamma)(1+\epsilon+2\epsilon B)M}{1-\epsilon}f(\mathbf{s}) \geq f(\mathbf{o})$

Proof of Claim 3 Let's consider Alg. 3 before adding e^j into $\mathbf{s}^{j-1}$. Denote $d_i = \frac{1}{1-\epsilon}F(\mathbf{s}^{j-1} \sqcup \langle e^j, i\rangle) - \frac{1}{1+\epsilon}F(\mathbf{s}^{j-1})$. We consider the following cases:

- $|supp(\mathbf{o}^{j-1})| = |supp(\mathbf{o}^j)|$, which means $e^j \in supp(\mathbf{o})$. Let $p = \mathbf{o}^{j-1}(e^j)$.

  Let $I \subseteq [k]$ be a set of values of i that $d_i > \frac{o}{M}, T = |I|$.

  We define $\mathbf{o}_i^j$ as a k-set that $\mathbf{o}_i^j(e) = \mathbf{o}^j(e)\ \forall e \in V \setminus \{e^j\}$ and $\mathbf{o}_i^j(e^j) = i$. Define $\mathbf{o}^{j-1/2}$ as a k-set that $\mathbf{o}^{j-1/2}(e) = \mathbf{o}^j(e)\ \forall e \in V \setminus \{e^j\}$ and $\mathbf{o}^{j-1/2}(e^j) = 0$.

  Let $\mathbf{s}_i^j = \mathbf{s}^{j-1} \sqcup \langle e^j, i \rangle$. We have two following sub-cases:

  - $d_p \geq \frac{o}{M}$, then if $T = 1$, $\mathbf{o}^{j-1} = \mathbf{o}^j$ and $f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j) = 0 \leq \left(1 - \frac{1}{k}\right)\left(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\right)$. Thus, we assume $T > 1$, in which we have

$$f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] = \frac{1}{D}\sum_{i \in I \setminus \{p\}} \Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}_i^j)\Big) d_i^{T-1} \leq \frac{1}{D}\sum_{i \in I \setminus \{p\}} \Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}^{j-1/2})\Big) d_i^{T-1} \tag{27}$$

$$\leq \frac{1}{D}\sum_{i \in I \setminus \{p\}} \Big(f(\mathbf{s}_p^j) - f(\mathbf{s}^{j-1})\Big) d_i^{T-1} \leq \frac{1}{D}\sum_{i \in I \setminus \{p\}} d_p d_i^{T-1} \tag{28}$$

$$\leq \frac{1}{D}\Big(1 - \frac{1}{T}\Big)\sum_{i \in I} d_i^T \tag{29}$$

$$= \frac{1}{D}\Big(1 - \frac{1}{T}\Big)\sum_{i \in I}\Big(\frac{1}{1-\epsilon}F(\mathbf{s}_i^j) - \frac{1}{1+\epsilon}F(\mathbf{s}^{j-1})\Big) d_i^{T-1} \tag{30}$$

$$\leq \frac{1}{D}\Big(1 - \frac{1}{T}\Big)\sum_{i \in I}\Big(\frac{1+\epsilon}{1-\epsilon}f(\mathbf{s}_i^j) - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\Big) d_i^{T-1} \tag{31}$$

$$\leq \Big(1 - \frac{1}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\Big) \tag{32}$$

    Inequality 29 comes from AM-GM inequality, defined as:

    Theorem 7. (Hirschhorn, 2007) Given n non-negative numbers $x_1, ...x_n$

$$x_1 + ... + x_n \geq n\sqrt[n]{x_1 \times ... \times x_n}$$

    Thus, directly apply the theorem given us $d_p d_i^{T-1} \leq \frac{1}{T}(d_p^T + (T-1)d_i^T)$.

  - $d_p < \frac{o}{M}$, which also means $d_p \leq \frac{o}{M} \leq \mathrm{E}[d_{i^j}] \leq \frac{1+\epsilon}{1-\epsilon}E[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})$. So:

$$\begin{aligned} f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] &\leq f(\mathbf{s}_p^j) - f(\mathbf{s}^{j-1})] \leq d_p \leq \Big(1 - \frac{1}{k}\Big)\mathrm{E}[d_{i^j}] + \frac{1}{k}\frac{o}{M} \\ &\leq \Big(1 - \frac{1}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\Big) + \frac{1}{k}\frac{o}{M} \end{aligned}$$

- $|supp(\mathbf{o}^{j-1})| < |supp(\mathbf{o}^j)|$, then $e^j \notin supp(\mathbf{o})$. Due to monotonicity of f.

$$f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] \leq 0 \leq \Big(1 - \frac{1}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\Big)$$

Therefore, in overall $f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] \leq \left(1 - \frac{1}{k}\right)\left(\frac{1+\epsilon}{1-\epsilon}E[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\right) + \frac{o}{kM}$

Proof of Claim 4 We have

$$\begin{aligned} f(\mathbf{o}) - \mathrm{E}[f(\mathbf{o}^t)] &= \sum_{j=1}^{t}\mathrm{E}\Big[f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j)\Big] \leq \sum_{j=1}^{t}\Big(\big(1 - \frac{1}{k}\big)\big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}\mathrm{E}[f(\mathbf{s}^{j-1})]\big) + \frac{o}{kM}\Big) \\ &\leq \Big(1 - \frac{1}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s})] + \sum_{j=1}^{t-1}\frac{4\epsilon}{1-\epsilon^2}\mathrm{E}[f(\mathbf{s}^j)]\Big) + \frac{oB}{kM} \\ &\leq \Big(1 - \frac{1}{k}\Big)\frac{(1+\epsilon)^2 + 4B\epsilon}{1-\epsilon^2}\mathrm{E}[f(\mathbf{s})] + \frac{1}{kM}f(\mathbf{o}) \end{aligned}$$

which completes the proof.

Proof of Claim 5 Denote $\{\langle u_1, j_1\rangle, ...\langle u_r, j_r\rangle\}$ as a set of elements and their placements in $\mathbf{o}^t$ that are not in $\mathbf{s}$. For each $\langle u_i, j_i\rangle$, denote $\mathbf{s}_i$ as $\mathbf{s}$ when Alg. 3 encounters u_i. As u_i was not added into $\mathbf{s}_i$, $\frac{1}{1-\epsilon}F(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle) - \frac{1}{1+\epsilon}F(\mathbf{s}_i) < \frac{o}{M}$ Let $\mathbf{u}_i = \mathbf{s} \sqcup \{\langle u_1, j_1\rangle, ...\langle u_i, j_i\rangle\}$. We have

$$\begin{aligned}\mathrm{E}[f(\mathbf{o}^t)] - \mathrm{E}[f(\mathbf{s})] &= \sum_{i=1}^{B-t} \mathrm{E}\Big[f(\mathbf{u}_i) - f(\mathbf{u}_{i-1})\Big] \le \sum_{i=1}^{B-t} \mathrm{E}\Big[f(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle) - f(\mathbf{s}^i)\Big] \\ &\le \sum_{i=1}^{B-t} \mathrm{E}\Big[\frac{1}{1-\epsilon}F(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle) - \frac{1}{1+\epsilon}F(\mathbf{s}_i)\Big] \le \sum_{i}^{B-t} \frac{o}{M} \le \frac{1}{M}f(\mathbf{o})\end{aligned}$$

which completes the proof.

D.2. Approximation ratio of RSTREAM when f is non-monotone

Proof of Theorem 4 Similar to monotone case, the key of our proof is to show that:

$$\max\big(\alpha(M), \beta(M)\big)\mathrm{E}[f(\mathbf{s}^t)] \ge f(\mathbf{o}) \tag{33}$$

If $t = B$, we have:

$$f(\mathbf{s}) \ge \frac{F(\mathbf{s})}{1+\epsilon} \ge \frac{F(\mathbf{s}^B)}{1+\epsilon} = \frac{1}{1+\epsilon}\sum_{i=1}^{B}\Big(F(\mathbf{s}^i) - F(\mathbf{s}^{i-1})\Big) \tag{34}$$

$$= \frac{1}{1+\epsilon}\sum_{i=1}^{B}\Big(F(\mathbf{s}^i) - \frac{1-\epsilon}{1+\epsilon}F(\mathbf{s}^{i-1})\Big) - \frac{2\epsilon}{(1+\epsilon)^2}\sum_{i}^{B-1}F(\mathbf{s}^i) \tag{35}$$

$$\ge \frac{1-\epsilon}{1+\epsilon}\sum_{i=1}^{B}\frac{o}{M} - \frac{2\epsilon B}{(1+\epsilon)^2}F(\mathbf{s}) \ge \frac{(1-\epsilon)f(\mathbf{o})}{(1+\epsilon)(1+\gamma)M} - \frac{2\epsilon B}{1+\epsilon}f(\mathbf{s}) \tag{36}$$

Therefore, with $t = B$, $\frac{(1+\epsilon+2\epsilon B)(1+\gamma)M}{1-\epsilon}f(\mathbf{s}) \ge f(\mathbf{o})$. The rest of the proof would focus on when $t < B$. We re-use notations $\mathbf{o}_i^j, \mathbf{s}_i^j$ as in the monotone proof and still compare $\mathbf{o}^{j-1}$ and $\mathbf{o}^j$, we have two following cases.

- $|supp(\mathbf{o}^{j-1})| < |supp(\mathbf{o}^j)|$, which means $e^j \notin supp(\mathbf{o})$. We consider 2 sub-cases:
  - If $T < k$, which means there exists $i' \in [k]$ such that $\frac{F(\mathbf{s}^j_{i'})}{1-\epsilon} - \frac{F(\mathbf{s}^{j-1})}{1+\epsilon} < \frac{o}{M} \le \frac{\mathrm{E}[F(\mathbf{s}^j)]}{1-\epsilon} - \frac{F(\mathbf{s}^{j-1})}{1+\epsilon}$. Then

$$\begin{aligned}f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] &= f(\mathbf{o}^j_{i'}) - f(\mathbf{o}^{j-1}) - \Big(\mathrm{E}[f(\mathbf{o}^j)] + f(\mathbf{o}^j_{i'}) - 2f(\mathbf{o}^{j-1})\Big) \\ &\le f(\mathbf{o}^j_{i'}) - f(\mathbf{o}^{j-1}) \le f(\mathbf{s}^j_{i'}) - f(\mathbf{s}^{j-1}) \le \frac{F(\mathbf{s}^j_{i'})}{1-\epsilon} - \frac{F(\mathbf{s}^{j-1})}{1+\epsilon} \\ &\le \frac{\mathrm{E}[F(\mathbf{s}^j)]}{1-\epsilon} - \frac{F(\mathbf{s}^{j-1})}{1+\epsilon} \le \frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\end{aligned}$$

- If $T = k$, define a permutation $\pi : [k] \to [k]$ such that $\pi(i) \neq i$ for all $i \in [k]$. Then

$$\begin{aligned}
f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] &= \frac{1}{D}\sum_{i\in[k]}\Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j_i)\Big)d_i^{T-1}\\
&= \frac{1}{D}\sum_{i\in[k]}\Big(f(\mathbf{o}^j_{\pi(i)}) - f(\mathbf{o}^{j-1}) - \big(f(\mathbf{o}^j_{\pi(i)}) + f(\mathbf{o}^j_i) - 2f(\mathbf{o}^{j-1})\big)\Big)d_i^{T-1}\\
&\leq \frac{1}{D}\sum_{i\in[k]}\Big(f(\mathbf{o}^j_{\pi(i)}) - f(\mathbf{o}^{j-1})\Big)d_i^{T-1} \leq \frac{1}{D}\sum_{i}\Big(f(\mathbf{s}^j_{\pi(i)}) - f(\mathbf{s}^{j-1})\Big)d_i^{T-1}\\
&\leq \frac{1}{D}\sum_{i\in[k]} d_{\pi(i)}d_i^{T-1} \leq \frac{1}{D}\sum_{i\in[k]} d_i^T = \frac{1}{1-\epsilon}\mathrm{E}[F(\mathbf{s}^j)] - \frac{1}{1+\epsilon}F(\mathbf{s}^{j-1})\\
&\leq \frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})
\end{aligned}$$

- $|supp(\mathbf{o}^{j-1})| = |supp(\mathbf{o}^j)|$. We re-use notation $p, I, \mathbf{o}^{j-1/2}$ as in the monotone case and consider other two sub-cases:
  - If $d_p \geq \frac{o}{M}$, then

$$\begin{aligned}
f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] &= \frac{1}{D}\sum_{i\in I\setminus\{p\}}\Big(f(\mathbf{o}^{j-1}) - f(\mathbf{o}^j_i)\Big)d_i^{T-1}\\
&= \frac{1}{D}\sum_{i\in I\setminus\{p\}}\Big(2f(\mathbf{o}^j_p) - 2f(\mathbf{o}^{j-1/2}) - \big(f(\mathbf{o}^j_p) + f(\mathbf{o}^j_i) - 2f(\mathbf{o}^{j-1/2})\big)\Big)d_i^{T-1}\\
&\leq \frac{2}{D}\sum_{i\in I\setminus\{p\}}\Big(f(\mathbf{s}^j_p) - f(\mathbf{s}^{j-1})\Big)d_i^{T-1} \leq \frac{2}{D}\sum_{i\in I\setminus\{p\}} d_p d_i^{T-1} \leq \frac{2}{D}\Big(1-\frac{1}{T}\Big)\sum_{i\in I} d_i^T\\
&\leq \Big(2-\frac{2}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\Big)
\end{aligned}$$

  - If $d_p = 0$, which also means $\frac{1}{1-\epsilon}F(\mathbf{s}^j_p) - \frac{1}{1+\epsilon}F(\mathbf{s}^{j-1}) < \frac{o}{M} \leq \frac{1}{1-\epsilon}\mathrm{E}[F(\mathbf{s}^j)] - \frac{1}{1+\epsilon}F(\mathbf{s}^{j-1})$. So:

$$\begin{aligned}
f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] &= 2f(\mathbf{o}^j_p) - 2f(\mathbf{o}^{j-1/2}) - \Big(f(\mathbf{o}^j_p) + \mathrm{E}[f(\mathbf{o}^j)] - 2f(\mathbf{o}^{j-1/2})\Big)\\
&\leq 2f(\mathbf{o}^j_p) - 2f(\mathbf{o}^{j-1/2}) \leq 2f(\mathbf{s}^j_p) - 2f(\mathbf{s}^{j-1})]\\
&\leq \frac{2}{1-\epsilon}F(\mathbf{s}^j_p) - \frac{2}{1+\epsilon}F(\mathbf{s}^{j-1}) \leq \Big(2-\frac{2}{k}\Big)\Big(\frac{\mathrm{E}[F(\mathbf{s}^j)]}{1-\epsilon} - \frac{F(\mathbf{s}^{j-1})}{1+\epsilon}\Big) + \frac{2}{k}\frac{o}{M}\\
&\leq \Big(2-\frac{2}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\Big) + \frac{2o}{kM}
\end{aligned}$$

Since $k \geq 2$, in overall, $f(\mathbf{o}^{j-1}) - \mathrm{E}[f(\mathbf{o}^j)] \leq \Big(2-\frac{2}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}E[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\Big) + \frac{2o}{kM}$. We have

$$f(\mathbf{o}) - \mathrm{E}[f(\mathbf{o}^t)] = \sum_{j=1}^{t}\Big(\mathrm{E}[f(\mathbf{o}^{j-1})] - \mathrm{E}[f(\mathbf{o}^j)]\Big) \leq \sum_{j=1}^{t}\Big(\big(2-\frac{2}{k}\big)\big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^j)] - \frac{1-\epsilon}{1+\epsilon}f(\mathbf{s}^{j-1})\big) + \frac{2o}{kM}\Big) \tag{37}$$

$$= \Big(2-\frac{2}{k}\Big)\Big(\frac{1+\epsilon}{1-\epsilon}\mathrm{E}[f(\mathbf{s}^t)] + \sum_{i=1}^{t-1}\frac{4\epsilon}{1-\epsilon^2}\mathrm{E}[f(\mathbf{s}^i)]\Big) + \frac{2oB}{kM} \tag{38}$$

$$\leq \Big(2-\frac{2}{k}\Big)\frac{(1+\epsilon)^2 + 4\epsilon B}{1-\epsilon^2}\max_{i\leq t}\mathrm{E}[f(\mathbf{s}^i)] + \frac{2}{kM}f(\mathbf{o}) \tag{39}$$

$$\leq \Big(2-\frac{2}{k}\Big)\frac{(1+\epsilon)^2 + 4\epsilon B}{(1-\epsilon)^2}\mathrm{E}[f(\mathbf{s})] + \frac{2}{kM}f(\mathbf{o}) \tag{40}$$

Also, similar to monotone case, as $\mathbf{s}^t \sqsubseteq \mathbf{o}^t$, denote $\{\langle u_1, j_1\rangle, ...\langle u_r, j_r\rangle\}$ as a set of elements and their placement in $\mathbf{o}^t$ that are not in $\mathbf{s}^t$. For each $\langle u_i, j_i\rangle$, denote $\mathbf{s}_i$ as $\mathbf{s}^t$ when Alg. 1 encounters u_i. Denote $\mathbf{u}_i = \mathbf{s}^t \sqcup \{\langle u_1, j_1\rangle, ...\langle u_i, j_i\rangle\}$. We have

$$\mathrm{E}[f(\mathbf{o}^t)] - \mathrm{E}[f(\mathbf{s}^t)] = \sum_{i=1}^{B-t}\Big(\mathrm{E}[f(\mathbf{u}^i) - f(\mathbf{u}^{i-1})]\Big) \leq \sum_{i=1}^{B-t}\Big(\mathrm{E}[f(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle) - f(\mathbf{s}_i)]\Big) \tag{41}$$

$$\leq \sum_{i=1}^{B-t}\mathrm{E}\left[\frac{F(\mathbf{s}_i \sqcup \langle u_i, j_i\rangle)}{1-\epsilon} - \frac{F(\mathbf{s}_i)}{1+\epsilon}\right] \leq \sum_{i=1}^{B-t}\frac{o}{M} \leq \frac{1}{M}f(\mathbf{o}) \tag{42}$$

Combining Equ. 40 and 42, we have

$$f(\mathbf{o}) \leq \frac{M}{kM-k-2}\frac{(3k-2)(1+\epsilon)^2 + (8k-8)\epsilon B}{(1-\epsilon)^2}\mathrm{E}[f(\mathbf{s})]$$

The approximation ratio of RSTREAM when discarding assumption of known $f(\mathbf{o})$ trivially follows as in the proof of the monotone case.

E. Sampling Method for Influence Maximization with k topics

In this part, we would present the sampling method that helps obtaining $F(\mathbf{s})$ satisfying $(1-\epsilon)\mathbb{I}(\mathbf{s}) \leq F(\mathbf{s}) \leq (1+\epsilon)\mathbb{I}(\mathbf{s})$ with high probability. We adopt a concept of Reverse Influence Sampling (RIS) (Borgs et al., 2014) to the problem as follows:

Given a social network $G = (V, E)$ and $w^i_{u,v}$ is a weight of edge (u, v) on topic i, a random Reverse Reachable (RR) sample $\mathcal{R} = \{R_1, ..., R_k\}$ is generated from G by: (1) selecting a random node $v \in V$; (2) generating sample graph $\{g_1, g_2, ...g_k\}$ from G, where g_i is generated from weights $\{w^i_{u,v}\}$ for all $(u, v) \in E$; (3) return $\mathcal{R} = \{R_1, ...R_k\}$ where R_i is a set of nodes that can reach v in g_i. A k-set $\mathbf{s} = \{S_1, ...S_k\}$ would activate the random sample $\mathcal{R}$ iff there exists $i \in [k]$ that $S_i \cap R_i \neq \emptyset$. For simplicity, denote $\mathbf{s} \triangleright \mathcal{R}$ as an indicator variable whether $\mathbf{s}$ activates $\mathcal{R}$. Using a similar proof as **Observation 3.2** (Borgs et al., 2014), we have $\mathbb{I}(\mathbf{s}) = |V| \cdot \Pr_{\mathcal{R}}[\mathbf{s} \triangleright \mathcal{R}]$.

To estimate $\mathbb{I}(\mathbf{s})$, we generates multiple samples $\mathcal{R}_1, ...\mathcal{R}_n$ and let $F(\mathbf{s}) = \frac{|V|}{n}\sum_{i=1}^{n}\mathbf{s} \triangleright \mathcal{R}_i$. We apply Chernoff Bound [2] to bound the number of samples, which guarantee $F(\mathbf{s})$ is an ϵ-estimate of $\mathbb{I}(\mathbf{s})$ with high probability. The Chernoff Bound theorem is stated as follows.

Theorem 8. (Chernoff bound) Suppose $X_1, ...X_n$ are independent random variables taking values in $\{0, 1\}$. Let X denote their sum and let $\mu = \mathrm{E}[X]$. Then for $\epsilon \in [0, 1]$ we have

$$Pr\Big(X \leq (1-\epsilon)\mu\Big) \leq e^{-\frac{\epsilon^2\mu}{2}}$$
$$Pr\Big(X \geq (1+\epsilon)\mu\Big) \leq e^{-\frac{\epsilon^2\mu}{3}}$$

Therefore, the number of samples that helps us obtaining ϵ-estimate of $\mathbb{I}(\mathbf{s})$ is stated in the following lemma.

Lemma 3. Given a seed set $\mathbf{s}$, by generating at least $n = \frac{3|V|}{\epsilon^2|\mathbf{s}|}\ln\frac{1}{1-\sqrt{\lambda}}$ RR samples, $F(\mathbf{s})$ is ϵ-estimate of $\mathbb{I}(\mathbf{s})$ with probability at least λ.

Proof. A slight change in algebra in Chernoff bound helps us obtaining:

$$\Pr\Big((1-\epsilon)\mathbb{I}(\mathbf{s}) \leq F(\mathbf{s}) \leq (1+\epsilon)\mathbb{I}(\mathbf{s})\Big) \geq \big(1 - e^{-\frac{\epsilon^2\mathbb{I}(\mathbf{s})n}{3|V|}}\big)^2 \tag{43}$$

Since $\mathbb{I}(\mathbf{s}) \geq |\mathbf{s}|$, $n \geq \frac{3|V|}{\epsilon^2|\mathbf{s}|}\ln\frac{1}{1-\sqrt{\lambda}}$ guarantees $F(\mathbf{s})$ is ϵ-estimate of $\mathbb{I}(\mathbf{s})$ with probability at least λ □

In experiment, we set $\lambda = 0.8$ whenever the algorithms query $\mathbb{I}(\cdot)$.

[2] http://math.mit.edu/ goemans/18310S15/chernoff-notes.pdf