Supplementary Material

Robustness against Regularization Parameters

We analyze the robustness of the performance of Algorithm 1 against the regularization parameter θ_1 in Eq. (5), and the results averaged over 20 random graph instances are presented in Fig. S1. In general, in addition to the effect of $\rho(\beta\mathbf{G})$ discussed in the main text, we see a consistent pattern across the three graph models that link the values of θ_1 and θ_2 to the learning performance. Specifically, when θ_1 is smaller than around 10^2, there is a region where a certain ratio of θ_1 to θ_2 leads to optimal performance, suggesting that in this case, the second and third terms are the dominating factors in the optimization of Eq. (5). A phase transition takes place when θ_1 is larger than 10^2, where the performance becomes largely constant. The reason behind this behavior is as follows. When θ_1 increases, the Frobenius norm of $\mathbf{G}$ in the objective function of Eq. (5) tends to be small. Given a fixed element-wise L^1-norm of $\mathbf{G}$, this leads to a more uniform distribution of the off-diagonal entries. When θ_1 is large enough, the edge weights become almost the same, leading to a constant AUC measure.

Similarly, we present in Fig. S2 the performance of Algorithm 2 with respect to different values of θ_1 and θ_2 in Eq. (7). We see that the patterns are generally consistent with that in Fig. S1, with one noticeable difference being that there also seems to be a phase transition taking place around the value of 10^{-1} for θ_2. One possible explanation for this behavior is that, when θ_2 is large enough, the trace term in the objective function of Eq. (7) tends to be small, making the resulting graph with fewer edges but with larger weights. This contributes to an AUC score that is mostly constant.

Performance of Algorithm 2 with respect to Number of Games and Noise Intensity in Marginal Benefits

The performance of Algorithm 2 with respect to the number of games and noise intensity in marginal benefits analysed in Section 5.2 is presented in Fig. S3.

Performance of Algorithm 2 with respect to Strength of Homophily Effect

We analyze the influence of the strength of homophily on the learning performance of Algorithm 2. We consider three scenarios, i.e., weak, medium and strong homophily effect. To this end, we generate the marginal benefits $\mathbf{b}$ as linear combinations of the eigenvectors corresponding to the 1^{st}-5^{th}, 6^{th}-10^{th}, and 11^{th}-15^{th} smallest eigenvalues of the graph Laplacian. Due to the properties of the eigenvectors, these three sets lead to different quantities for the Laplacian quadratic form, hence corresponding to weak, medium and strong homophily effect, respectively. Notice that the presence of the homophily effect in $\mathbf{B}$ tends to imply homophily in $\mathbf{A}$ for the following reason. Regardless of the characteristics of the game, a higher marginal benefit $\mathbf{b}$ is more likely to incentivize higher activity level $\mathbf{a}$ due to the first term of the payoff function in Eq. (1). Therefore, homophily in $\mathbf{B}$ tends to lead to homophily in $\mathbf{A}$, hence revealing more information about the graph structure. As shown in Fig. S4, for all the three types of networks, the stronger the homophily in the marginal benefits, the better the learning performance.

Results in Section 5.2 for Algorithm 1

The performance of Algorithm 1 with respect to the factors analysed in Section 5.2 is presented in Fig. S5.

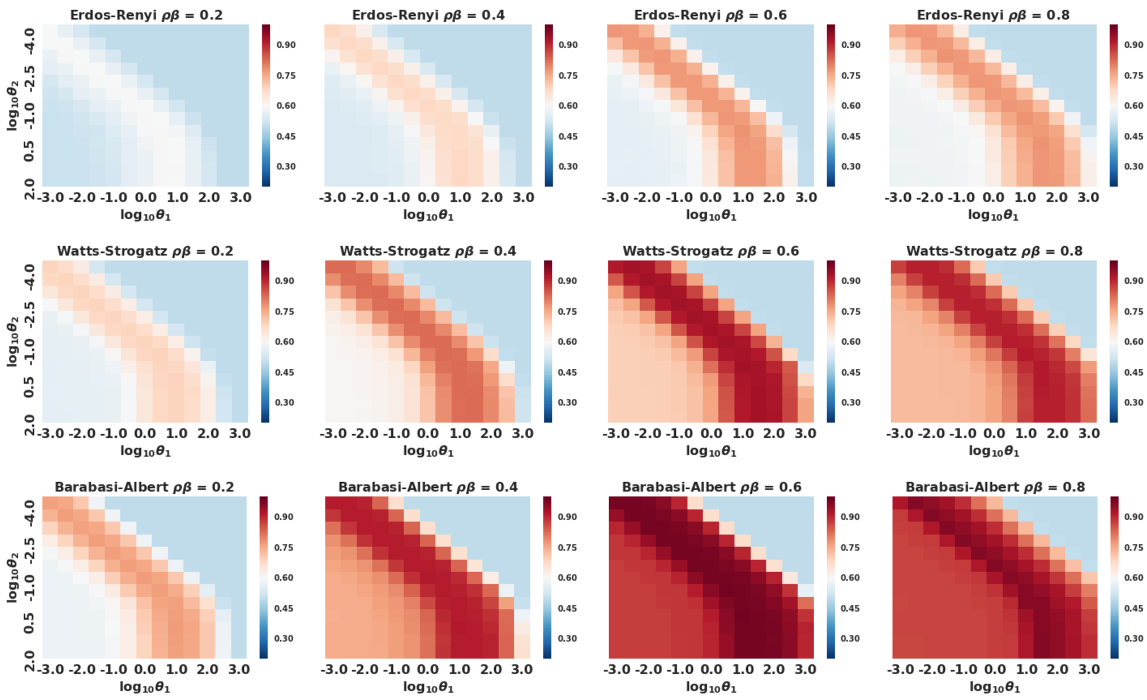

Figure S1: Performance (AUC) of Algorithm 1 with respect to $\rho(\beta\mathbf{G})$, θ_1, and θ_2.

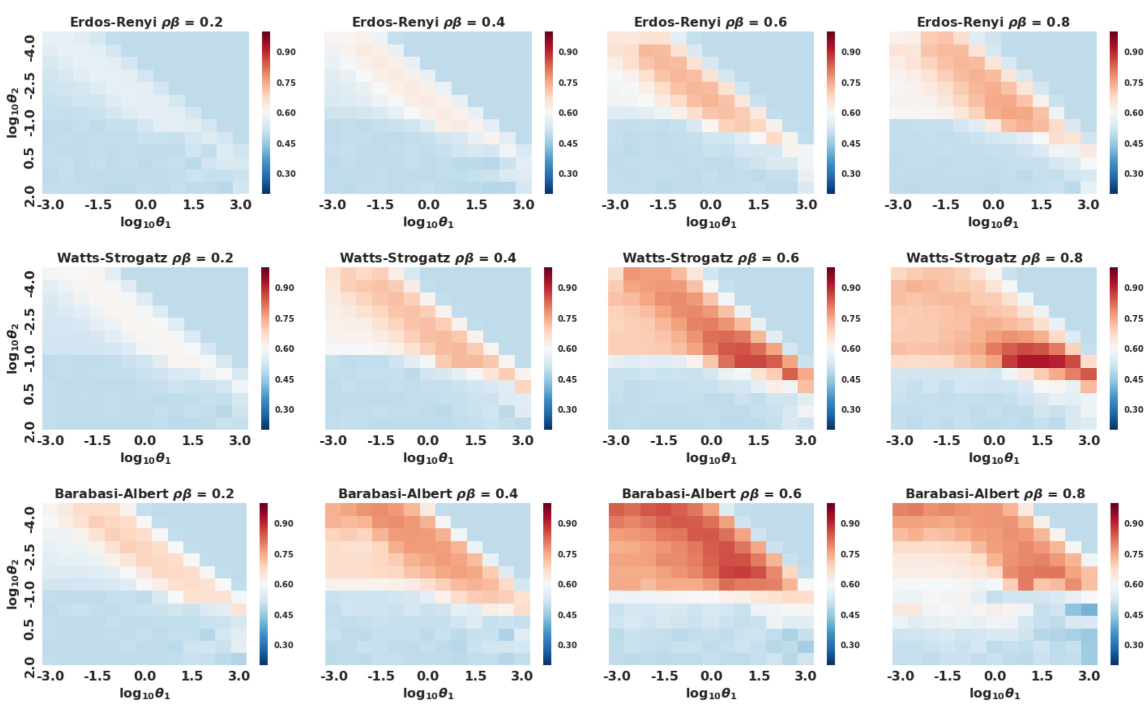

Figure S2: Performance (AUC) of Algorithm 2 with respect to $\rho(\beta\mathbf{G})$, θ_1, and θ_2.

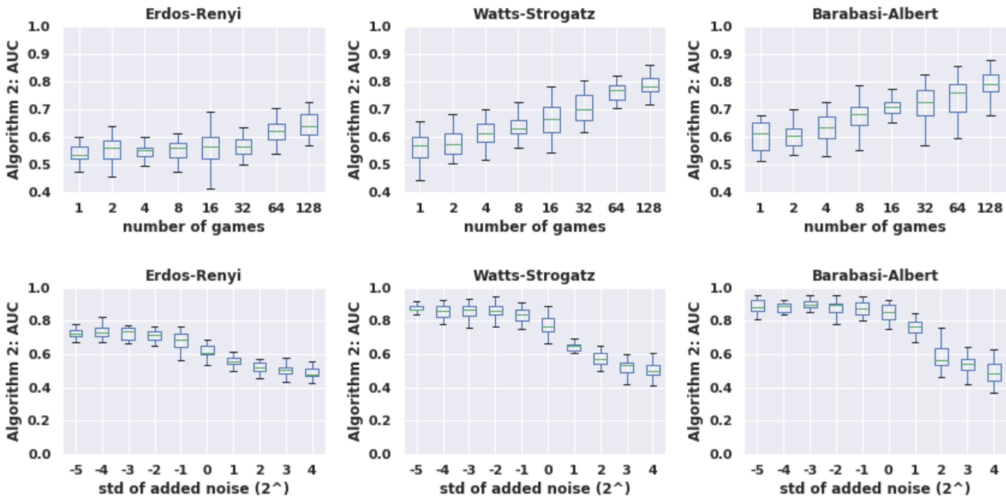

Figure S3: Performance of Algorithm 2 versus number of games (top) and noise intensity in marginal benefits (bottom).

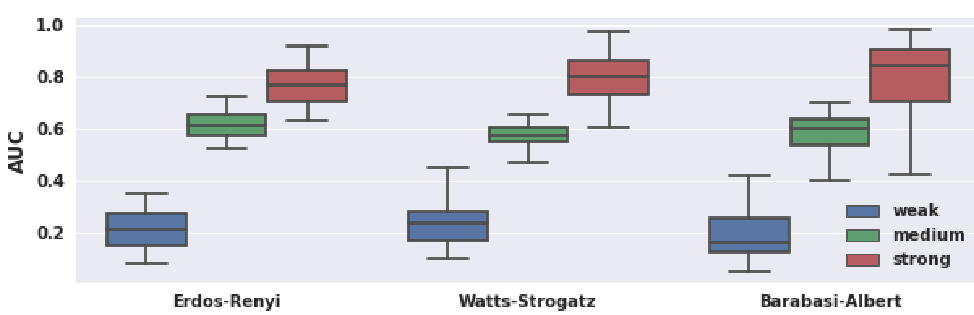

Figure S4: Performance of Algorithm 2 versus strength of homophily in the marginal benefits.

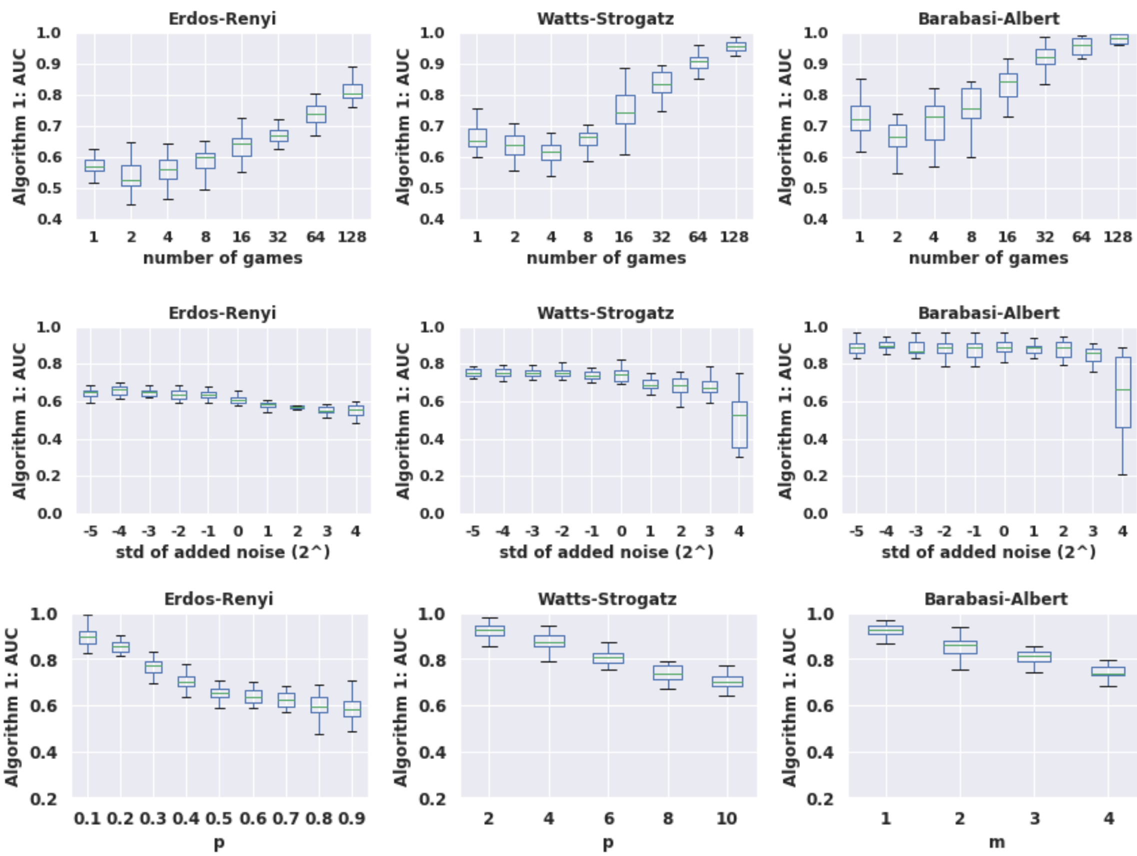

Figure S5: Performance of Algorithm 1 versus number of games (top), noise intensity in marginal benefits (middle), and structural properties of the network (bottom).