A. Appendix

A.1. Proof of Theorem 3.3

Proof of Theorem 3.3. Since $\mathcal{A}^i$ satisfies inequality 3, we have that

$$\sum_{t=1}^{T}\left(\ell_t^i(u_{t-H+1},...,u_t)-\ell_t^i(u^f_{t-H+1},...,u^f_t)\right) \leq R(T)+2T\varepsilon. \quad (9)$$

Denote $j^- = t-j+1$ for brevity. Define for any $i \in [N]$, $t \in [T]$, and any $\ell_t \in \mathcal{L}'$ loss function encountered by the booster,

$$\Delta_{t,i} \triangleq \ell_t(\mathbf{0}, u^i_{H^-}, ..., u^i_t) - \ell_t(\mathbf{0}, u^f_{H^-}, ..., u^f_t).$$

Denote $\Delta_i = \sum_t \Delta_{t,i}$. Notice that by α-strongly convexity 5 of ℓ_t, as long as we choose $\eta_i \leq \frac{\alpha}{\beta}$, we have

$$\ell^i_t(u^f_{t-H+1},...,u^f_t) = \sum_{j=1}^{H}\frac{\eta_i\beta}{2}\|u^f_{t-H+j}-u^{i-1}_{t-H+j}\|^2 + \sum_{j=1}^{H}\left(\nabla_j^\top(u^f_{t-H+j}-u^{i-1}_{t-H+j})\right)$$
$$\leq \ell_t(\mathbf{0}, u^f_{t-H+1},...,u^f_t) - \ell_t(\mathbf{0}, u^{i-1}_{t-H+1},...,u^{i-1}_t).$$

Thus by summing them up we get

$$\sum_{t=1}^{T}\ell^i_t(u^\pi_{t-H+1},...,u^\pi_t) \leq -\Delta_{i-1} \quad (10)$$

Consider the following calculations for $\Delta_{t,i}$:

$$\Delta_{t,i} = \ell_t\Big(\mathbf{0}, u^{i-1}_{H^-}+\eta_i(\mathcal{A}^i(x_{H^-})-u^{i-1}_{H^-}),\ldots,$$
$$u^{i-1}_t+\eta_i(\mathcal{A}^i(x_t)-u^{i-1}_t)\Big) - \ell_t(\mathbf{0}, u^\pi_{H^-},...,u^\pi_t)$$

(by substituting u^i_t as in line 7 of Algorithm 2)

$$\leq \ell_t(\mathbf{0}, u^{i-1}_{H^-},...,u^{i-1}_t) - \ell_t(\mathbf{0}, u^\pi_{H^-},...,u^\pi_t) + \sum_{j=1}^{H}\Big(\eta_i\nabla_j^\top(\mathcal{A}^i(x_{t-H+j})-u^{i-1}_{t-H+j}) + \frac{\eta_i^2\beta}{2}\|\mathcal{A}^i(x_{t-H+j})-u^{i-1}_{t-H+j}\|^2\Big)$$

(by convexity and β-smoothness of ℓ_t)

By summing $\Delta_{t,i}$ over $t \in [T]$, we have that

$$\Delta_i \leq \sum_{t=1}^{T}\Big(\big(\ell_t(\mathbf{0}, u^{i-1}_{H^-},...,u^{i-1}_t) - \ell_t(\mathbf{0}, u^\pi_{H^-},...,u^\pi_t)\big) + \eta_i\sum_{j=1}^{H}\Big(\nabla_j^\top(\mathcal{A}^i(x_{t-H+j})-u^{i-1}_{t-H+j}) + \frac{\eta_i\beta}{2}\|\mathcal{A}^i(x_{t-H+j})-u^{i-1}_{t-H+j}\|^2\Big)$$

$$= \sum_{t=1}^{T}\Big(\big(\ell_t(\mathbf{0}, u^{i-1}_{H^-},...,u^{i-1}_t) - \ell_t(\mathbf{0}, u^\pi_{H^-},...,u^\pi_t)\big) + \eta_i\ell^i_t(\mathcal{A}^i(x_{H^-}),...,\mathcal{A}^i(x_{1^-}))\Big)$$

$$\leq \sum_{t=1}^{T}\Big(\big(\ell_t(u^{i-1}_{H^-},...,u^{i-1}_t) - \ell_t(u^\pi_{H^-},...,u^\pi_t)\big) + \eta_i\ell^i_t((u^\pi_{t-H+1},...,u^\pi_t)\Big) + \eta_i(R(T)+2T\varepsilon)$$

(by the weak-controller regret bound 9)

$$\leq (1-\eta_i)\Delta_{i-1} + \eta_i(R(T)+2T\varepsilon) \quad \text{(by inequality 10)}$$

Choosing $\eta_i = \frac{\alpha}{\beta}$, then by noticing Δ_i is always upper bounded by a convex combination of Δ_0 and $(R(T)+2T\varepsilon)$, we have

$$\Delta_i \leq \left(1-\frac{\alpha}{\beta}\right)^i\Delta_0 + \left(1-\left(1-\frac{\alpha}{\beta}\right)^i\right)(R(T)+2T\varepsilon)$$
$$\leq \left(1-\frac{\alpha}{\beta}\right)^i 2GT + R(T) + 2T\varepsilon$$

plugging $i = N$ in finishes our proof. ∎