#### A NETWORK OF COMMUNICATING LOGIC PROGRAMS AND ITS SEMANTICS

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#### **ABSTRACT**

In this paper, a network of communicating logic programs is proposed as a model for parallol/concrurrent programming based on logic programs. This network is regarded as an extension of Kahin's pure dataflow in the sense that nodes are logic programs which have atoms for receiving and sending messages as well as queues to accept and memorize them. The nodes' behaviour is unboundedly nondeterministic. On the assumption that the channels' denotations should be defined by using sequence domains, the main concern of the present paper is whether or not mathematical semantics is always well-defined for a given network even when it is of unbounded nondeterminism.

This paper will show that the proposed network is reduced to a dataflow when a kind of 'fairness' is asked for nondeterminism in which logic programs receive inputs and produce outputs based on their computations. The network, then, has a (least) fixpoint semantics; it is regarded as one of mathematical semantics of the network, since it can satisfy the recursive relations among the channels' denotations.

It is also stated that the network with fair merge operators is applicable to the realization of a computational mechanism for sequential ligic programs.

# 1. NIRODUCION

Kahn's pure dataflow network is one of the established models for parallel/concurrent programming in the sense that its mathematical semantics is clearly defined by the least fixpoint solution to the set of equations associated with

the channels of the network [4,5].

When establishing a model for parallel/concurrent programming based on Togic programs, Kalin's pure dataflow can be extended to a network whose nodes are not functional but relational in accordance with logic programs. Jf we establish a network according to such a model, there would be a strict question as to whether or not mathematical semantics of the network is well-defined even when it is of unbounded nondeterminism. The question arises from the standpoint that mathematical semantics should be made clear in order for operational semantics to be given so that implementation of the system is possible. In both PAROG (2,3] and Concurrent Prolog [10] the channels' semantics are indirectly defined, hence the communications between processes seem complicated.

In general, unbounded nondeterminism due to the computation of logic programs should be respected and mathematical semantics should be defined based on the relations among the denota-Present Affiliation: Department of Information Science,

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tions of channels. Hence the fixpoint theory in [4] is not appropriate in this case. Even theories concerning powerdomain constructions in [9,11] are unsuccessful for the network we intend to define, since, in essence, the network is described by relations among the denotations of the channels.

On the other hand, one way to extend Kahn's model to a more general one is to establish a class of nondeterministic computing networks whose nodes are either asynchronous, continuous functions or fair merge operators. Fixpoint semantics of such networks is well-defined in [8]. The 'fairness' applied to such networks might be useful to solve the question of mathematical semantics of the unbounded nondeterministic network of logic programs. However, it seems that nodes consisting of logic programs contain more nondeterminism than the fair merge operations, since the choices of outputs produced by their computations are not only due to the relative timings of inputs but due to the decisions concerning their relations. At the same time, there is a crucial problem of how 'fairness\* can be realized for nondeterminism in which logic programs are computed.

In this paper, we have a fair sequence in which any natural number occurs an arbitrary number of times. By means of the fair sequence it will be shown that fairness is well realized for nondeterministic receiving of messages, and sending of messages related to the computations of logic programs, in the network. The network consisting of communicating logic programs might be reduced to a dataflow when fairness due to the fair sequence is asked. Thus, the network has a (least) fixpoint semantics for the reduced dataflow, according to [8]. The fixpoint semantics could be regarded as one of the mathematical semantics for the original network. According to the fixpoint semantics, an operational semantics will be shown on the condition that logic programs in the network are fairly nondeterministic in receiving inputs and sending outputs. If the intended network has fair merge operators, it can express a computational mechanism for sequential logic programs. It means that the semantics of sequential logic programs might be defined by applying the semantics of our network with fair merge operators.

The outline of the network is as follows: The network consists of channels and processes. The channels are only ways by which the processes may communicate with each other. There may be channels for inputs and/or outputs. The message is transmitted one way through each channel within a sufficiently short time. Each process is a logic program consisting of definite clauses with atoms corresponding to receiving and sending messages as well as unbounded FIFO queues each of which is connected with an input channel transmitting messages. In each process, the messages in the nondeterministically chosen positions of the queues are received for the computation of the logic programs, simultaneously.

After the logic program\*s finite computation, each process nondeterministically provides either one of possible outputs

or no output.

# 2 NEDVORK OP COMUNDATNG LOOG PROGRAMS (NOLP)

A logic program is a set of definite Horn clauses, A definite Horn clause is either an expression  $A \leftarrow B_1 \dots B_n$  or  $A \leftarrow$  for stoms A and  $B_1$ . An atom is an expression  $P(t_1, \dots, t_m)$ , where P is a predicate symbol and  $t_1, \dots, t_m$  are terms. If P(t) is an atom for a term t, P is called monadic. A term is defined recursively as follows: (1) A constant or a variable is a term. (2)  $f(t_1, ..., t_k)$  is a term if f is

a function symbol and t, ..., t are terms.

The atom on the left-hand side of <- in a definite Horn clause is called the head of the clause. The set of atoms (which might be empty) on the right-hand side of <- in a definite Horn clause is called its body.

A network of communicating logic programs (NCLP) is N=(C,L,I,O,Comb1,Comb0),

- where: (1) C= {C,...,C} is a set of channels.

  (2) I= {L,...,L} is a set of logic programs with queues.

  For 1 (jcn, 1, =(S,q), where
  - (i) S is a logic program such that
- (a) It contains the atoms whose predicate symbols are Se D (monadic) for D in C (D  $_{jk}$  n I is empty, k=1,...,n  $_{j}$  and D  $_{jk}$  n D is empty iff k#h) in the heads of some clauses in S  $_{j}$ , and
- (b) it might contain the atoms Re-C(x) for channels C and variables x in the bodies of some clauses in S, (ii) q is a tuple of unbounded queues.
- (3) I is a subset of C, called an input channel set.
- O is a subset of C, called an output channel set. (4) CombI is a function from L to 2 such that for j

is in CombO. From now on, CombO(L, D) is used to mean that (L,D) is in CombO.

# 3 DENUTATONAL SEMANTOS OF NOOP

It is assumed in the present paper that the logic programs in NOP are communicated based on their finite computations. On this assumption, the semants of logic programs are given by their minimal Halbard moots [3,12],

### 3.1. Semento Domain

(1) Hu denotes the Helbard universe constructed over the set of constants and function symbols  $i s_1, \dots, s_n \in d$  in the drames of I. (2) Frans the Habard base formed by the predicate syntor in such as and by the terms in Hu. (Note that hi is a hiation used in [81, which is to dende a time-delay.)

(4) Let peru (hi), and let D of dende the set of all finite and infinite sequences from D. A cardes the empty sequence in D. A partial order to on D of is defined by.

\*\*United But the sequences from D. A partial order to on D of is defined by.

\*\*United But the sequences from D. The sequence in D. A partial order to on D of its defined by.

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Note that any chain { u, + u, + ... + u, } has a least upper bound, which is denoted by fig. ... The partial order ∤ is extended hothe one on ເພ∞ා" by: (uj,...,uj) † (vj,...,vj) lift uj † vj for 15psn. O) dendes the set of natural numbers

# 3.2. Denotations of Charmets

For i=1,...,m,  $X_i$  in  $D^{\infty}$  means the denotation of  $C_i$ C, which is to be defined by the relations as shown in 3.4.

#### 3.3. Denotations of Logic Programs

For j=1,...,n,

- (1) (i) the denotation of q is  $d=q_j=(E(X_{j1}),\dots,E(X_{jh}))$  where (a)  $\{C_{j1},\dots,C_{jh}\}$ =CombI(L<sub>j</sub>), and (b) E: D  $\xrightarrow{\infty}$  Hu  $\xrightarrow{\infty}$  is recursively defined by:
- $E(\lambda) = \lambda$ , E(hi.u) = E(u), E(a.u) = a.E(u) for a  $\Phi i$ ,
- (ii)  $Q_j = C_{ji}$  means  $E(X_{ji})$  of  $d = q_j$ , for  $C_{ji}$  in  $CombI(L_j)$ ,
- (2) the denotation of S, for the inputs of length p, is defined by

$$R_{j}^{(p,d-q_{j})} = U_{r_{1},...,r_{h},p} v_{j}^{(r_{1},...,r_{h},d-q_{j})},$$

where  $V_j(r_1,...,r_h,d-q_j)=(\min I)T_1(r_1,...,r_h,d-q_j,I)$  for

set 2HB, WY: the cardinal number of a set Y), defined by

$$\begin{split} & \overset{\mathsf{T}_{\mathbf{j}}(x_1, \dots, x_h, d-\mathbf{q}_{\mathbf{j}}, 1)}{= \{ & \mathsf{A}\theta & \text{in } \mathsf{B}_{\mathbf{B}} \mid \mathsf{A} \leftarrow \mathsf{B}_1 \dots \mathsf{B}_{\mathbf{x}} \text{ is in } \mathsf{S}_{\mathbf{j}}, \text{ and} \\ & \{ & \mathsf{B}_{\mathbf{j}}\theta , \dots, \mathsf{B}_{\mathbf{x}}\theta \} \mathbf{c} \mathbf{I} \cup \bigcup_{\mathsf{C}_{\mathbf{j}} \in \mathsf{in } \mathsf{Comb} \mathbf{I}(\mathsf{L}_{\mathbf{j}})} \{ & \mathsf{Re-c}_{\mathbf{j}}(Q_{\mathbf{j}} - \mathsf{c}_{\mathbf{j}}(x_{\mathbf{j}})) \} \} \end{split}$$

and for

$$(\min \ 1) T_j(x_1, ..., x_h, d-q_j, 1) = \Lambda(\ J \mid T_j(x_1, ..., x_h, d-q_j, J) \in J)$$
.

#### 3.4. Relations among Denotations of Channels

For 
$$1 \le j \le n$$
,  
(1) let  $W_{ik}(r_1, \dots, r_h, d-q_j)$  be

$$\{ t \text{ in Ho } \big| Se^{k} D_{jk}(t) \text{ is in } V_{j}(r_{1},...,r_{h},d-q_{j}) \}$$

$$(k=1,...,n_{-}),$$

(2) for each  $C_i$  in  $D_{ik}$  such that  $CombO(L_i, D_{ik})$ , and for any  $r_1, \dots, r_h$  in  $\omega$ ,

$$\mathbf{x}_{i}^{(p)}$$
 is in  $\mathbf{U}_{\mathbf{r}_{1},\dots,\mathbf{r}_{h} \stackrel{c}{\leftarrow} \mathbf{p}} \mathbf{w}_{jk}^{(\mathbf{r}_{1},\dots,\mathbf{r}_{h},\mathbf{d}-\mathbf{q}_{j})}$ 

if  $|E(X_n)| \ge p$  ( | u | means the length of u) for some e such that C is in CombI(L) or if #CombI(L)=0, and

X<sub>i</sub>(p)=hi otherwise,

(3) for each C, in I, 
$$X_i$$
= the element in  $D^{\infty}$  , provided through  $C_i$ .

The semantics of the network N is  $(E(X_1),...,E(X_n))$ for  $(X_1, \dots, X_m)$  which is in

$$Sem(N) = \left\{ \begin{array}{l} (X_1, \dots, X_m) & X_1, \dots, X_m \text{ satisfy the relations} \\ & \text{in (2) and (3)} \end{array} \right\}.$$

Note that  $V_j(r_1,\dots,r_h,d-q_j)$  and  $W_{jk}(r_1,\dots,r_h,d-q_j)$  depend on the denotation of  $q_j$ 

Note that for  $c_i$  in  $p_{ik}$  such that  $CombI(L_i, p_{ik})$ , there are 1+#(  $U_{r_1,...,r_k,p}$   $w_{jk}(r_1,...,r_k,d-q_j)$  ) choices for X, (p).

Such nondeterminism is due to

- (1) what numbers x,...,r are chosen smong{ 1,...,p}, to indicate the positions of queues of q on which the denotations of 'Re-C '-atoms depend, and
- (2) what element is chosen in  $W_{jk}(r_1,...,r_h,d-q_j)^{M}$  (hi ), to indicate the denotation of 'Se-D<sub>jk</sub>'-stoms.

Therefore, there may be as an infinitely many number of choices as  $\,\omega_{\, \nu}\,$  when  $\, X_{\, \nu}\,$  is fixed.

We shall have a class of fair sequences in  $\omega^{\,\,\omega}$  as oracles in order that the choices for the positions on the queues and for the atoms' demotations may be made fairly under such nondeterminism. If the oracles are used, the denotations of channels can be described by a continuous, asynchronous function from a direct product of Doo to itself.

# 4.1. Fair Segmence in www

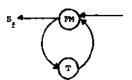
**DESTINITION** 1 u in  $w^{\omega}$  is fair iff for any k in  $\omega$  and for any h in  $\omega$  , k occurs in u, h times.

DEFINITION 2 Let  $S_f$  denote one of sequences in  $\omega^{\omega}$ , provided by the network as shown below, where:

(1) Τ: ω co is defined by

 $T(\lambda) = \lambda$ ; T(a.u)=(a+1).T(u) for u in ω<sup>®</sup>.

(2) FM:  $\omega^{\infty} \times \omega^{\infty} \rightarrow \omega^{\infty}$  means a fair merge operator, that is, FM(u,v) w iff w is a sequence obtained by interleaving u and v in a fairly nondeterministic way. (For the fair merge, see [6,7,8]).



# Fig. 1 A Network to Provide Fair Sequences

DEFINITION 3 A function F: D \*\* -> Hu \*\* is asynchronous if F(E(u))=F(u) for any u in D ... F is continuous if F(  $\uparrow$   $\mu$   $\nu$ ) =  $\uparrow$  p F( $\mu$ ) for any chain  $\{u_1 \uparrow u_2 \uparrow \dots \uparrow u_n \uparrow \dots\}^p$ .

#### PROPOSITION 1

- (1) S is fair in www.
- (2) S<sub>e</sub>(j)+1<j.

<u>Proof</u> (1) Note that  $S_z = 0^T$ , u for some u in  $\omega^{(i)}$ . Now let Cocur(k,h) mean that k occurs h times in u.

- It is seen that Occur(0,h) for any h in W , because  ${\bf 0}^{\;\;\text{LL}}$  is an input of PM and thus 0 appears as its output an arbitrary number of times.
- (ii) Assume that for some h, Occur(n,h) for nck. Then, since k+1 should be at the same time an output of T and hence an input of FM, Occur(k+1,h). By mathematical induction, for some h in W , Occur(k,h) (for any k).

4 A FAPONT STANTS AND OPRIONAL STANTS OF INTERIOR Occur(0,1) from (1), it follows from (2) that Occur(k,h) for any n in (). Suppose that Occur(k,h) for any k in w and h≤m. Since k should be an output of T, k-1 should be an input of T, that is, k-1 occurs in u as many times as k. Thus Occur(k-1,h). Finally Occur(0,h). It follows from (1) that Occur(0,h+1). Therefore there is no case that Occur(k,h+1) does not hold. This completes the induction step of methematical induction. Thus Occur(k,m) for any k and for any m.

0 1 5 (2) (i) If j=1, the proposition holds, since (ii) Assume that S<sub>c</sub>(h)+1≤h for h≤k. Since one of the inputs of PM is  $S_{k+1} = 0$ ,  $E(S_{k+1}) = E(S_{k+1}) = 0$ . Thus  $S_{k+1} = S_{k+1} = 0$ . Thus  $S_{k+1} = S_{k+1} = 0$ . This completes the induction step.

# 4.2. A Pixpoint Sementics of NCLP

By means of fair sequences in  $\omega^{\omega}$  as oracles, the denotations of channels can be defined as determinant for each logic programs.

For j=1,...,n, let  $P_{jk}(p)$ :  $W_{jk}(I_{j}(p),d-q_{j})\rightarrow \omega$  p in W ) be a one-to-one function such that (for

for t in  $W_{jk}(I_{j}(p),d-q_{j})$ , where k=1,...,h and  $I_{j}:\omega->\omega^{-h}$ is a bijection such that any element of I (p) p. Let  $\mathbf{S}_{\mathbf{p}}\text{-}\mathbf{j}$  denote a sequence in  $\boldsymbol{\omega}^{(k)}$  , given by DEPINITION 2. Also let

$$\operatorname{Ord}(S_{\mathbf{f}}^{-}\mathbf{j}(p))=\emptyset \{ \ r \ | \ S_{\mathbf{f}}^{-}\mathbf{j}(r)=S_{\mathbf{f}}^{-}\mathbf{j}(p) \ \text{and} \ r\leq p \} \ .$$

For k=1,...,n, and each C in D such that CombO(L,D,D), we define

if  $| E(X_e) | \ge p$  for some e such that  $C_e$  is in  $CombI(L_i)$ or if #CombI(L\_) = O, and  $X_{j}(p)$ =hi otherwise.

$$X_i = Y_i (X_1, \dots, X_m)$$
.

Proof Since either C is in I or C is in D for some D in 2 such that CombO(L ,D  $_j$  ). X is defined by either a given element in D  $_i^\infty$  through C or by the above formula.

Since  $P_{ik}(S_f^{-j}(p)+1)^{-1}$  is a one-to-one function from to  $W_{ik}(\hat{T}_i(\hat{s}_f-j(p)+1),d-q_i)$ , and the denotation of d-q is defined by  $\hat{\mathbf{x}}$  for e such that C is in CombI(L), we see  $X_{i}(p)$  is a function of  $X_{i}(p)$ . (By  $X_{i}(p)$ , the first sequence of length p in X is meant.) Thus, we could gut

 $x_i = x_i(x_1, ..., x_m)$  for  $x_i : (D^{\infty})^m \to D^{\infty}$ . It follows from  $d = (E(X_{j1}), ..., E(X_{jh}))$  that  $F_i$  are asymchronous. Since  $X_i[p]=F_i(X_1[p],...,X_m[p]) \uparrow X_i=F_i(X_1,...,X_m)=$  $\mathbf{F}_{\mathbf{i}}( \uparrow_{\mathbf{p}} \mathbf{X}_{\mathbf{i}}[\mathbf{p}], \dots, \uparrow_{\mathbf{p}} \mathbf{X}_{\mathbf{m}}[\mathbf{p}]),$ 

$$\underset{p}{+} \underset{i}{F_{i}}(X_{i}[p], \dots, X_{m}(p)) + F_{i}(X_{1}, \dots, X_{m}).$$
 On the other hand, for any p in  $(x_{i}, x_{i}, y_{i}) = F_{i}(X_{1}, \dots, X_{m})[p] + F_{i}(X_{i}, \dots, X_{m})[p] + F_{i}(X_{i}$ 

 $F_{i}(X_{1}^{(p)},...,X_{m}^{(p)})$ , since  $X_{i}^{(p)}$  is determined by  $X_{1}^{(p)},...,X_{n}^{(p)}$ . Therefore

$$\mathbf{F}_{\mathbf{i}}(\mathbf{X}_{1},\ldots,\mathbf{X}_{m}) = \mathbf{X}_{\mathbf{i}} = \mathbf{A}_{\mathbf{p}} \mathbf{X}_{\mathbf{i}}[\mathbf{p}] + \mathbf{A}_{\mathbf{p}} \mathbf{F}_{\mathbf{i}}(\mathbf{X}_{1}[\mathbf{p}],\ldots,\mathbf{X}_{m}[\mathbf{p}]),$$

This completes the proof of the continuity of F.

This proposition states that the network is reducible to a dataflow by means of fair sequences in  $\omega^{(ij)}$ . Finally we have the following proposition:

#### PROPOSITION 3

For the network N=({C<sub>1</sub>,...,C<sub>m</sub>},{L<sub>1</sub>,...,L<sub>n</sub>},I,O,Comb1,CombO) there exists a function  $F_n: (D^{\infty}) \to (D^{n_0})^m$  such that the (least) fixpoint of  $F_n$  is in Sem(N).

 $\begin{array}{llll} \underline{Proof} & F_N \text{ is qiven by } (F_1,\ldots,F_m), \text{ where } F_1 \text{ are asynchronous,} \\ \underline{continuous functions.} & \underline{Thus there exists a (least) fixpoint} \\ \text{of } F_N & [8], & F_1 \text{ are in accordance with the formula we have} \\ \text{already obtained. Hence } X_1 \text{ satisfy the relations as shown} \\ \text{in 3.4 (2).} & \underline{Finally the fixpoint of } F_N \text{ is in Sem(N).} \end{array}$ 

#### 4.3. Operational Sementics of NCLP

In this section, we have an operational semantics of NCLP based on its fixpoint semantics given in the previous section. In operational semantics, the historn his used in denotational semantics, corresponds to the states of waiting messages and/or producing nothing. The denotations of 'Re-C '-atoms correspond to receiving messages and those of 'Se-D  $_{jk}^{\alpha-1}$ -atoms correspond to sending messages.

#### 4.3.1. Assumptions

- (1) The messages sent to each channel C should be stored in the unbounded FIFO q of L, when C is in Combf(L), (2) When a logic program L wants a longer queue, in
- (2) When a logic program L wants a longer queue, in order to generate (send) the next output, it must wait for the necessary messages arriving at its queue.
- (3) Unless a logic program can get something by its computations, it never sends any message to channels.

<u>DEFINITION 4</u> For a nonempty set A, FuirChoice(A) means some b in A is chosen at any time such that the sequence of chosen elements is fair in  $\lambda^{(i)}$ . (A sequence is fair in  $\lambda^{(i)}$  if any b in A occurs an arbitrary number of times in the sequence.)

<u>DEFINITION 5</u> For a set S of definite clauses, Comput(S) denotes the set of ground atoms, which is the minimal Herbrand model of S.

#### 4.3.2. Procedure for NCLP

## DEFINITION 5

Let  $N=(\{c_1,...,c_n\},\{L_1,...,L_n\},1,0,Comb1,CombO),$ 

Let  $q_j^{=(Hu^{\infty})}$  \*\*Combl( $L_j^{-1}$ ), that is, a tuple of queues. The length of  $q_j^{-1}$  is the maximum length among lengths of sequences stored in  $q_j^{-1}$ .  $Q_j^{-1}C_j^{-1}$  means the content of queue of  $q_j^{-1}$ , connected to the channel  $C_j^{-1}$  such that  $C_j^{-1}$  is in Combl( $L_j^{-1}$ ). For p in W,  $Q_j^{-1}C_j^{-1}(p)$  denotes the p-th content it holds.

The procedure  $f_{\frac{1}{2}}$  for  $L_{\frac{1}{2}}$  is given as follows:

$$P_{i}^{-1}$$

while p<sub>i</sub> in to do

begin

$$(r_{i1}, ..., r_{jh}) \leftarrow I_{j} (\min \{ p_{j}, \text{FairChoice}(\omega) + 1 \} ),$$

$$\begin{array}{c} \frac{u_{j}(r_{j1},...,r_{jh})}{c_{j1}(r_{j1},...,r_{jh})} \\ \frac{1}{\sqrt{\left\{|t| \text{ in Hu}|\right\}}} \frac{1}{Se^{k}} c_{jk}(t) \text{ is in}} \\ = comput(s_{j} \cup \left\{|\text{Re-c}_{j1}(Q_{j} - c_{j1}(r_{j1}))\right\})} \\ = \cdots \\ \cup \left\{|\text{Re-c}_{jh}(Q_{j} - c_{jh}(r_{jh}))\right\}|\right\}; \end{array}$$

$$\begin{array}{l} \underline{if} \ B_j(r_{j1},\ldots,r_{jh}) \ \ \text{is not empty} \\ \underline{then} \ \ \text{send t in PairChoice}(B_j(r_{j1},\ldots,r_{jh})) \\ \text{to each channel of D}_{jk} \\ \underline{else \ send \ nothing;} \end{array}$$

end

end

(Note:  $B(r_{j1},...,r_{jh})$  is initially defined as empty for each tuple of  $r_{j1},...,r_{jh}$ .)

The procedure for N is defined as follows:

#### program N;

#### 5. APPLICATION OF NOLP

This section is an overlook of the application of NOLP to the transformation of sequential logic programs to dataflow networks with fair merge operators. Any sequential logic program denotes an NOLP with fair merge operbors. It follows from the results of the present paper and [8] that the denotational semantics of a logic program might be defined as a fixpoint over sequence domains.

Now let L be a l c program {  $n_1,\ldots,n_n$  } , r e each  $n_i$  is a definite clause  $n_i \leftarrow n_i$  for atoms

$$\mathbf{A_{i}} = \mathbf{P_{i}}(\mathbf{t_{i1}}, \dots, \mathbf{t_{im_{i}}}) \text{ and } \mathbf{B_{ij}} = \mathbf{Q_{ij}}(\mathbf{u_{ij1}}, \dots, \mathbf{u_{ijk_{ii}}}).$$

Let ->> be a subset of L × L such that (H,H) is in ->> (which is denoted by  $H_1 \rightarrow >H_2$ ) iff  $P_1$  is equal to  $Q_{jk}$  for some  $1 \le j \le n$  and  $1 \le k \le n$ . NOND: L->>u is defined by: NOND( $H_1$ )=# {  $H_1$  |  $H_2$  |  $H_3$  |  $H_4$  |

For each H, in L, we define

where

$$(c_{i11}, ..., c_{i1k_{i1}})$$
 is an input-tuple of channels for  $S(H_i)$ ,

(C 
$$in_i 1, ..., C in_i k$$
 ) is an input-tuple of channels for  $S(H_i)$ , and

$$S(H_{\underline{i}})$$
, and  $(C_{\underline{H}}, ..., C_{\underline{H}})$  is an output-tuple of channels for  $S(H_{\underline{i}})$ .

Let  $\mathbf{L}_i = (\mathbf{S}(\mathbf{H}_i), \mathbf{q}_i)$ , where  $\mathbf{q}_i$  is a tuple of queues connected with all the input channels for  $\mathbf{S}(\mathbf{H}_i)$ .

Next, whenever  $H_1^{->>H_1}$ , the output-tuple of channels for  $S(H_1)$  is to be connected with some appropriate input-tuples for  $S(H_1)$ , via an  $FM(NOND(H_1), m_1)$ . The appropriate input-tuple of channels is chosen on the condition that  $P_1$  is the same as the predicate symbol in the body of a definite clause in  $S(H_1)$ , which has  ${}^{\dagger}Re^{-1}$ -atoms corresponding to the input channels for  $S(H_1)$ .

The total network constructed for L in the way mentioned above is denoted by N(L). Then N(L) is regarded as an MCLP with fair merge operators. Let us define the denotational mensions of channels with L corresponding to  $S(H_1)$  and their operational behaviour as the sense as those of the MCLP—N(L). Since each  $S(H_1)$  contains only  $H_1$  as a definite clause without "Se-"-atoms and "Re-"-atoms, if we assume fair nondeterminism for choosing the positions of queues in each  $L_1$  to get messages and provide outputs, then the output-tuple of sequences from each  $S(H_1)$  is an asynchronous, continuous

function of input-tuples of sequences. FM(p,q) is an extention of an ordinary fair merge operator. Thus N(L) is regarded as the network presented and discussed by Park [8]. This means that the semantics of N(L) is well-defined. This is a theoretical guarantee from the point of semantics for regarding the computation mechanism of logic programs as dataflows with fair merge operators.

It might be concluded that by using the NOLP, we can transform any logic program to a network of dataflows with fair merge operators. This is an interesting and significant aspect of NOLP.

#### 6. CONCLUDIG FEMARIS

The NOLP is reduced to a dataflow when fair nondeterminism is asked for receiving and sending messages. In such a case, there is a (least) fixpoint semantics of a function associated with the NOLP. As we have seen, this is one of the denotational semantics defined in Section 3. Thus, the denotational semantics of NOLP is well-defined even when it has unbounded nondeterminism.

The operational semantics of NQP based on its fixpoint semantics was defined in Section 4. The implementation of 'FairChoice' can be done by the fair sequences in  $W^{\text{W}}$ .

The expressiveness of the NQP contains interesting aspects compared with other systems.

It is interesting to extend the NOLP so that each process can rewrite the contents of its queues.

It was briefly mentioned that the NOLP with fair merge operators can express a computation mechanism for sequential logic programs. This is a significant aspect of the proposed network as well as the nodeterministic computing network in [8].

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