SOME PROPERTIES OF SMALL NETWORKS OF RANDOMLY
CONNECTED ARTIFICIAL NEURONS

R. B. Guy,
Ess0 Chemical Ltd.
Portland House, Stag Place,
London S.W.I. England.

R. L. Beurle,
Department of Electrical and Electronic
Engineering, The University,
Nottingham, England.

The activity in small networks of randomly connected artificial neurons, simulated on a digital computer, is described. Networks with only excitatory connections, and networks with both excitatory and inhibitory connections are considered.

Introduction

Much of the work on brain models in the late 1950's and early 1960's comprised the construction, either in hardware, or more commonly by simulation on a digital computer, of networks of artificial neurons (hereafter the term 'neuron' will refer to artificial neurons, where necessary brain cells will be referred as 'natural neurons'). Historically the idea networks of neurons performing brain like functions stems from the work of McCulloch and Pitts.

Similar work was presented and expanded by Kleene , who showed that it is possible to represent any definite event (i.e. an event comprising a definite set of input sequences but not an event whose input sequences extend indefinitely into the past) in a finite automaton which could be constructed from McCulloch-Pitts neurons. Unfortunately this work is of little practical help to model builders, it points out some of the possibilities and limitations of the excercise without giving any firm guidelines as to the construction of such models.

The process of neuron modelling adopted by various workers has tended to follow a fairly fixed pattern, starting with a description of some of the generally assumed properties of natural neurons, and the interconnections between them, giving a framework for the construction of a network of neurons. These networks have then received discrete or continuous input stimulation and some record made of the resultant pattern of activity. There may or may not be any modification (adaption) of the network parameters to attempt to demonstrate learning by the network.

Generally assumed properties of neurons

Most model neurons incorporate all or many of the following features (see fig 1). The inputs to a neuron may be either excitatory, e_i, (positive) or inhibitory, i , (negative). Each input connection has a certain weight ascribed to it, if an impulse is received along a particular fibre an amount of excitation (positive or negative) proportional to the weight of the connection is given to the neuron. The neuron is a threshold device; it has an 'all or none' mode of operation and will only produce an out-put (fire) if its excitation exceeds a certain predetermined (threshold) value.

Thus the neuron will fire if

$$\sum_{i=1}^{n} e_i - \sum_{j=1}^{m} i_j \geq T \qquad (1)$$

where T is the threshold and e, is proportional to the weight of the ith. excitatory connection if an input pulse has just been received along ith. fibre, and zero otherwise (**sim. for** i_j**).**

The neuron does not transmit an output pulse as soon as the excitation exceeds the threshold value, there is a short delay - corresponding to the synaptic delay in natural neurons - before on output pulse is transmitted. Although in natural systems a neuron may apparently fire at any time, in simulation experiments it is only possible to deal with discrete time intervals, thus the synaptic delay time, t, is usually taken as the unit of time. Therefore ifat time t - 1 a neuron receives an amount of excitation greater than its threshold value it will transmit an output pulse at time t =0 to all neurons connected to its axon branches, this output becomes an input to these neurons at time t ■ 0, unless some speed of travel constraint is applied to the connections. Normally all cells with which an axon system connects receive inputs in the same time interval irrespective of their distance from the originating cell.

If a neuron does not fire in a particular time step after receiving some excitation then, in many models, its excitation does not immediately drop to zero but falls exponentially towards tero with a time constant usually of the order of 0.1 to 2 time intervals. Hence a neuron may fire after repeated bursts of stimulation, none of which by itself is sufficient to fire the cell; this is observed to occur in natural neurons.

Also, in order to comply with neuro-physiological data, immediately after a cell has fired its threshold becomes infinite, this state (the absolutely refractory period) usually lasts 1 to 3 time intervals. The threshold then decays towards its quiescent value, again it is convenient to assume an exponential decay with a typical time constant of 0.2 to 3 time steps. These points are illustrated in fig. 2.

The connections from cell to cell are unidirectional and independent (pulses may only travel as output from one cell via its axon

branches to provide inputs to other cells, but there is no reason why if an axon branch of cell a is connected to cell b that an axon branch of cell b should not be connected to cell a). The connections are distributed according to some rule, normally a function of the radial distance between cells.

Some feedback mechanism may be be applied to the network in an attempt to cause it to produce particular forms of output in response to particular inputs. Mechanisms of the form - if the required output is not obtained reduce the weights of all connections which contributed to the firing of cells and increase the weights of all those which fed pulses to cells which did not fire - have been used with some success. If inhibitory as well as excitatory connections have been used then some originally positive (excitatory) connections may become negative (inhibitory) by the use of such mechanisms. This has been the case in several models, but it is believed that this is not physically possible in natural systems, where a synapse appears to be always either excitatory or inhibitory. Uttley has suggested that this difficulty may be overcome by postulating that there are always two pathways connecting any two neurons, a direct excitatory pathway, and an indirect (via an Intermediate neuron) inhibitory pathway.

To summarise we have the following parameters for each cell.

T	Threshold
e_i	Excitatory connections) Denoting cell
i_j	Inhibitory connections) ^{of} origin and weight
$f_e(r)$	Excitatory connectivity) r is the function) radial dist-
$f_i(r)$	Inhibitory connectivity) ance between function) cells.
t	Synaptic delay (basic unit of time)
E_o	Absolutely refractory period
E_1	Refractory period time constant
E_2	Excitation decay time constant
w_e	Excitatory weight modification function
w_i	Inhibitory weight modification function

7 x 7 Networks

Several networks were simulated and their performance examined on the Atlas computers at Manchester and Harwell. The type of neuron programmed incorporated most of the features described in section 1, the networks being conceived as plane square matrices normally having a side length of 7 cells.

The cells were connected randomly with a given, variable exponential probability distribution of connections (see fig 3). The number of connections per cell might vary from about 5 to 25 depending on the parameters fed into the program data. For each cell, for each possible connection either to every other cell or to all cells within a certain radial distance, a random number in the range 0 - 1 was generated (using a psaudo-random number generator sub-routine), if the probability of connection was greater than the random number produced then a connection was established between the two cells. Thus the constant 'A' (fig. 3) effectively controlled the number of connections per cell (for a given 'a*') and the constant 'a' the distance decay of probability of connection. The program was arranged such that a cell could not be connected to itself.

In many of these models the connection pattern for each cell was calculated and stored separately, as far as the author is aware in all other simulation studies a repetitive from of connection has been used. Some later networks were constructed with a repetitive connection pattern and are compared with the previous models in section 7. In the 7 x 7 models only excitatory connections of fixed weight were used. A quiescent threshold level, To, of 200 was chosen, and a value of 1000 to represent the initial point, Tm, for the decay of the refractory period. Two forms of refractory period were used:-

a) 1st. time step after firing, cell absolutely refractory

   2nd. time step after firing, threshold - $^3/2$ To

   3rd. time step after firing, threshold - To

b) 1st. time step after firing, cell absolutely refractory

   2nd. to nth. time steps governed by the expression

$$T = To + (Tm - To).E1^{(g - 1)} \qquad (2)$$

where El is a constant less than unity, g is the nuuber of time steps since the cell last fired

(n + 1)th. time step after firing, threshold = To

For type (b) temporal summation of excitation was effected by using the relation

$$E_t = E_{t-1}.E2 + \text{incoming excitation} \qquad (3)$$

where E is the excitation level at time t, and E2 Is a constant less than unity.

Values of El used were 0.4, 0.7

Values of E2 used were 0.5, 0

There was no temporal summation of stimuli for type (a) networks.

The advantages of using a pseudo-random number generator were that the same sequence of statistically random numbers could be produced repeatedly merely by starting the generation sequence with the same 'priming[1]' number, this enabled the same network to be set up on the computer on different occasions without having to read in the connection details as data; and also networks with the same connection parameters

(density of connections, and probability of connection distribution) could be set up and compared by simply regenerating the network using a different priming number.

For each time step a cell which has just fired is denoted by the symbol 1, and a cell which has not fired by the symbol 0.

Description of results - 1st series

7 x 7 networks of neurons were set up on the computer as described above, in this first series of experiments the inter neuron connections were calculated and stored separately for each cell in the network. The network was initially stimulated by exciting three or four of the nine central cells with one of the patterns shown in fig. 4.

The activity was allowed to continue for a number of time steps, usually 20 or 40, the activity was then stopped, all cells returned to their quiescent state with zero excitation, and a new pattern of activity initiated by stimulating the network with a different input pattern. Provision was made to output the activity pattern in the network at each time step, or for the final few time steps.

The type of activity which occurred in the networks fell into one of five categories. Examples of which are shown in figs. 5 - 9.

1. Attenuation. The connectivity of the network is not sufficiently 'strong' and the activity dies out in a few time steps, (fig. 5)

2. Unstable. The activity does not attenuate, but there is no discernible continuing cyclic pattern; at least in the short term, (fig. 6)

3. Virtually stable (a). With the number of time steps recorded the output has not satisfied one of the conditions 4 or 5 (see below), but it appears probable that it will lock in to one of these modes during the next few time steps, (fig. 7)

4. Virtually stable (b). The output pattern at time t is not identical with that at time t + 2, but is identical with that at some time t + n, where n is an integer greater than 2 (fig. 8)

5. Stable. The network produces an oscillatory output such that all cells which do not fire at time step t, fire at time step t + 1. All cells which fired at time step t are of course absolutely refractory at t + 1. The t th. pattern is therefore the inverse of the pattern of the previous time step, and identical with the (t + 2) th. pattern, (fig. 9)

From an examination of the outputs typified In figs. 5 - 9 several features emerge.

The distinction between modes 2 - 4 is somewhat arbitary and more one of degree than a precise division. Those networks classified as type 3 normally only lack stability in a few (say less than 6) 'outer square' cells. In type 2 networks there is a more random variation, frequently some cells change allegiance; i.e. if they were initially ON during an even time step they may, at a latter stage, be ON during an odd time step. In such unstable networks no two identical outputs have been noted over periods of 40 time steps (the exponential refractory period lasted 5 time steps). It is difficult to imagine that one of these networks if allowed to maintain activity for a sufficiently long time would not fall into a cycle, even if this were of many hundreds of time steps duration. An interesting feature of this type of network is the tendency for underlying patterns of activity (similar to type 4) to last for a few time steps, slowly disintegrating to form a new basic pattern which in turn builds up and then disintegrates - though a few cells may be common to all such transitory patterns.

The difference between modes 5 and 4 is presumed to be a lack of specific connections to a particular cell or cells, such that they only fire every 4th. or 6th. time step rather than every 2nd. step. It is also possible for a cell to fire every 3rd. time step giving a mode 4 output with a cycle time of 6 time steps.

The lock in time before a stable output was produced was affected by the type of input pattern used to stimulate the network. Networks stimulated with patterns i and ii tended generally to stabilise more rapidly than iii and iv type inputs due to their having a greater probability of two or more active connections to the same cell during the first time step. For example in patterns i and ii there are six cells which are only one unit distance from two stimulated cells, in iii and iv there are only four such cells. In v there are nine such cells, but insufficient runs were made to satisfactorily compare the tendency of this input to stabilize quickly compared with types i - iv.

The lock in time is also dependent on the constant 'A' being shorter for large values of A. For sufficiently large A the network will stabilize instantly to repeat the input pattern unchanged in alternate time steps (fig. 10). This 'over connected' network gives rise to an interesting and possibly useful mode of activity which it is convenient to designate as mode 6 - the saturated stable mode.

To summarise, dependent on the network parameters the flow of activity in the network will give rise to one of six possible modes. These are:-

1. Attenuation   4. Virtually stable (b)

2. Unstable   5. Stable

3. Virtually stable (a) 6. Saturated stable

Comments on results - 1st series

The results of this first series of experiments raised several points on the behaviour of these and more complicated networks. Many of these were in the form of a more accurate assessment of the variation in output with the various network parameters, e.g.

1. What is the effect of the threshold/connectivity strength ratio on the stability of the network - perhaps expressed in terms of the lock in time and the period of the cyclic activity.

2. A quantitative analysis of the effects of •A', 'a^f, the length of the absolutely refractory period, the refractory period decay constant and the excitation decay constant on the network.

3. The effect of the variation of input pattern sise and shape on the tendency to produce different modes of operation.

The most interesting form of activity which the networks produced was their ability to give a stabilised output of short cycle length. As far as the author is aware this property has not been previously demonstrated. Presumably because of the shortage of computing storage and time other workers have tended only to output the frequency of firing of cells, or monitor the gross activity of the network. Hence they may not have realised that the nature of the activity - even when they observed stability - was such a fine measure of the input pattern.

The stabilised pattern is always characteristic of the particular input pattern initiating the activity (fig. 11), though it is possible that two different input patterns could give rise to the same output activity, but this has not been observed to occur. With a lightly connected network the output pattern for a given input pattern varies with the connectivity, i.e. it is dependent on the particular random number generator (b) used (fig. 12). However, when there are sufficient intercell connections a pattern may be set up which duplicates the original input pattern in alternate time steps Independent of the particular connectivity generator.

This latter form of activity is interesting for it shows that a very simple network, having neurons with only excitatory connections to other cells, can exhibit a useful memory function. The requirement is that there should be a large number of such connections but these may be made at random, the activity pattern being independent of the actual connectivity pattern. Once stimulated the network is unaffected by additional excitatory inputs; the pattern once established cannot be altered except by inhibiting some of the cells from firing in a particular time step. This is so because every cell which is not absolutely refractory fires as soon as it has emerged from its absolutely refractory period. Thus we have a mechanism which can be used to

store an input pattern indefinitely and which is unaffected by subsequent excitatory inputs to the network. The network may be returned to its quiescent state by applying blanket inhibition over the whole network.

These networks therefore possess some possibly useful properties, in order to further evaluate these properties a second series of networks was simulated.

Second Simulation

A major drawback to the method used to construct the original networks - calculating the connectivity pattern for each cell - was the large amount of computer storage which had to be allocated for this purpose, also causing a consequent increase in computing time. This was so severe that practically it was impossible to run the simulation for networks of much more than 100 cells, and potentially restrictive on the number of runs which might be made to evaluate the properties of 7 x 7 or similar networks. Accordingly a modified program was written in which a more regular connectivity pattern could be adopted, but hopefully have no significant effect on the performance of the network compared with the original model. A program of this type was essential if larger networks with inhibitory and excitatory connections were to be simulated.

Comments on 7 x 7 networks - 2nd, series

In order to evelute the performance of 7 x 7 networks various network parameters were varied in turn, holding all other parameters constant. There being several variables it is difficult to adequately portray the performance in graphical form. Nevertheless figs. 13 & 14 give an impression of the behaviour of the networks.

It will be recalled that the connectivity of the network Is governed by the expression

$$y(r) - A.\exp(-r/a) \qquad (4)$$

where A, a are constants, and r is the radial distance between cells, and that a connection is made if

$$y(r) \qquad x \qquad (5)$$

where x is a randomly generated number on the range 0 - 1.

The form of activity sustained in the network as A and a are varied is shown in fig. 13. For large A and a saturated stable activity (mode 6) occurs, as the value of these constants is reduced this activity gives way to the stable mode (5), initially with immediate lock in to the mode, and only one or two cells in addition to

the originally stimulated cells firing in 'odd'
time steps. As A and a are further reduced the
following progression of activity takes place:
the lock in time to mode 5 gradually increases,
and more cells fire in odd time steps, when the
lock in time is about five or six time steps and
about half the cells are ON in each time step,
the activity changes to mode 4 with immediate
lock in time. At this stage the mode 4 activity
will generally have a cycle time of six time
steps, or occasionally four time steps. Then the
cycle and lock in times gradually increase until
no purely cyclic activity is noted over a period
of 20 or so time steps, mode 3. Though in these
cases the central 25 cells usually cycle with a
short cycle time, and instability is confined to
the outer ring of cells. Where longer simulations
have been made these networks tend to have very
long lock in and cycle times, the latter were
usually a multiple of 6, which was the total
length of the refractory period used. A very
narrow band of unstable (mode 2) activity occurs
between mode 3 and mode 1 (attenuation). In this
region the pattern in the central core of cells
changes and the instability is no longer confined
to the outer ring of cells. Though some repeated
patterns have occasionally been noted in simu-
lations of 40 or more time steps no tendency to
cyclic activity has been noted. The activity in
this mode is either a very extreme case of mode
3 with exceedingly long, probably greater than
100 time steps cycle time (c.f. reference 8), or
alternatively activity which will ultimately
attenuate; a case has been observed where the
activity attenuated after some 20 time steps.
Near the virtually stable region activity
attenuates (mode 1), initially activity lasts for
a short period, as A and a are reduced this
period decreases until the activity dies out
immediately after stimulation. There is a gradual
drift of the mode division curves towards the
origin as the number of cells initially stim-
ulated rises to half the total number of cells,
but this effect is very slight for small numbers
of cells initially stimulated, the curves for 3
and 7 cells being virtually identical. All the
mode division curves will become asymptotic to
the A and a axes, since for both A = 0 and a ∎ 0
there can be no connections to other cells, but
for a value of one variable near zero, and a very
large value for the other sustained activity
is possible.

Figs. 14 a - d show the effects of
varying both the quiescent threshold and the
maximum threshold following the absolutely
refractory period for different values of a, all
the graphs being drawn for a value of A ∎ 1. The
threshold decay was controlled by equation 2.

As the quiescent threshold rises it becomes
more and more difficult to initiate sustained
activity, when the connectivity is strong enough
for sustained activity to occur there is a
corresponding increasing tendancy for it to be
stable, and the region (modes 2, 3 and 4) of
virtually stable and unstable activity becomes
very narrow (fig. 14d). For larger values of A

the mode division boundaries will move upwards
and to the left. In the same manner as fig. 13,
there is a gradual change from one form of
activity to another, but the mode division
boundaries are surprisingly clear, very rarely is
a network found which does not respond in a manner
which could be predicted from the graphs. In
these rare cases where the activity has not been
in the predicted group it has, except in one case,
been of the form of the nearest adjacent region.

Different random number generators, b's,
have little effect on the form of figs. 13 and 14,
occasionally a network, whose parameters place it
very near a mode division boundary, will change
from one mode to the other for different gen-
erators, but normally the activity may be acc-
urately predicted. Thus the value of 'b' used
does not seriously affect the prediction, it
merely adds the additional constraint that for
certain critical values of the network parameters
the activity mode will be dependent on the actual
network connections.

This second series of experiments has shown
that the specification of the parameters A, a,
T, and T is sufficient to predict the behaviour
or the network, and that the particular random
number generator will usually only affect the
form of the output pattern, as in fig. 12.

Modified simulation program

The method of generating the connectivity
pattern used in the experiments so far discussed
is satisfying in that a unique set of connections
is generated for each cell, within the same
statistical constraints as the connections to
other cells. However, this method is wasteful in
terms of computer storage, and to a lesser extent
time. Therefore in the modified program instead
of generating the connection pattern separately
for each cell four possible connection patterns
were generated; being the same as four arbitrary
patterns which might have been generated by the
original programme. Each cell in the network
was assigned one of these connection patterns at
random, the connectivity thus being a compromise
between the original connection method and a
purely repetitive method.

A series of 7 x 7 networks was set up using
this connection technique and the performance
of the networks compared with the original net-
works as depicted in fig. 13. It was found that
the performance was in general very similar, but
that tne results were not quite so consistent.
With the original networks, given the parameters
A, a, a network*s behaviour may be accurately
predicted. In the modified system there was a
far greater probability, perhaps 0.25 that the
form of the activity would not lie within the
regional boundaries of fig. 13, compared with
about an 0.05 probability for the original net-
works. This is not altogether a surprising
result, since if any one of the four connectivity
patterns gives a 'poor' distribution of connec-

tions this will affect about a quarter of the cells, whereas the odd poor distribution in the original program will have little effect. However, within this restriction the second program gives a satisfactory simulation method.

Possible extensions of 7 x 7 networks

The stability of neuron has been discussed by several authors 1, 2, 6, 7, with the conclusion that either some fairly complicated feedback mechanism monitoring the level of activity in the network and holding it within certain limits is necessary to prevent saturation. Or, as was then not such a popular idea with neurophysiologists but is now generally accepted as being the case, inhibitory as well as excitatory connections are necessary. To some extent in a small network, such as the 7 x 7 networks, these mechanisms are unnecessary since the absolutely refractory period ensures that all the cells can never fire at the same time - unless they were originally all stimulated together, in which case the activity will extinguish itself immediately if the excitation of a cell falls to sero once it has fired. Alternatively, if the excitation is merely reduced by its normal decrement, the network may exhibit saturated stable oscillation with all cells on in one time step, and none in the following step. Else, if the connectivity were strong enough to allow activity to continue but not to allow the saturated mode to occur the network would settle in one of the modes 3 - 5 .

These forms of activity only suggest possibly useful applications in a small network, in a large homogeneous network (one with the same connectivity function throughout the network) if the connectivity is sufficiently strong such that activity does not attenuate then the activity will slowly spread throughout the network and will probably set up modes of activity very similar to those in 7 x 7 networks, but with much greater lock in times. Fig. 15 shows how the activity has spread through a 50 x 50 network 15 time steps after being stimulated with a three cell input pattern.

It is useful to ask at this point whether the form of stabilised output which is easily demonstrated in a small 7 x 7 network can be found to exist locally in much larger networks, without the activity spreading through the whole network as in fig. 15. One possibility is that in addition to the short range excitatory connections so far postulated we add a number of long range inhibitory connections, thus each local area when stimulated Is surrounded by a barrier of inhibited cells (fig. 16). In principle we may now construct a model with this form of connectivity, stimulate a small area of the network and observe if stable local activity occurs. If this is so we may further note the number of such stable systems which can be operated in a given sice of network. It is not being proposed that distinct excitatory and inhibitory areas are set up in the network, but that the network is homogeneous with short range excitatory connec-

tions.

Networks with inhibitory and excitatory connections

Using the modified program a small number of 28 x 28 networks with inhibitory as well as excitatory connections were simulated. The networks had a homogeneous connection pattern, the excitatory connections being precisely as described above. Only one pattern of inhibitory connections was used, being common to all cells, again this was of the usual form governed by equation 4, except that no inhibitory connections were made to cells less than five cells distant. The connectivity constants (now denoted by A , a , and A , a , for inhibitory and excitatory connections) could be varied independently.

From the results obtained it seems likely that the form of activity suggested earlier - a stable excited area surrounded by an inhibitory barrier - may be produced. Fig. 17 shows the activity in such a network after the 9th. and 10th. time steps. The activity in the 10th. step is identical with that of the 6th., and differs in only four cells from that of the 8th. The network is apparently exhibiting mode 4 activity, but this activity is limited to one area of the network. The tendancy for the activity to concentrate in a horisontal band appears to be a function of the particular connectivity generator used. With longer range excitatory connections, and weaker inhibition, pockets of activity may be stimulated in other parts of the network beyond the inhibitory barrier. In these cases activity will presumably spread throughout the network as in fig. 15, but at a much slower rate. No attempt has been made to evaluate the properties of the network in the manner of the 7 x 7 networks, or to initiate activity in two independent areas.

References

1. Allanson, J. T. 1956
   Some properties of a randomly connected neural network, in Third London Symposium on Information Theory (Cherry, Ed.), Butterworth, London.

2. Beurle, R. L. 1956
   Properties of a mass of cells capable of regenerating pulses, Phil. Trans. Roy. Soc. of London, Vol. B 240, pp 56-94.

3. Farley, B. G., & Clark, W. A. 1954
   Simulation of self organising systems by digital computer, I.R.E. Trans., Vol. PGIT 4, pp 76-84.

4. Kleene, S. C. 1956
   Representation of events in nerve nets and finite automata, in Automata studies - Annals of mathematics studies No. 34 (Shannon & McCarthy, Eds.), Priceton Univ. Press.

5. MmCulloch, W. S., & Pitts, W. 1943
   A logical calculus of the ideas immanent in nervour activity, Bull Math. Biophys., Vol. 5, pp 115-133.

6. Milner, P. M. 1956
    The cell assembly theory Mark 11,
    Psych. Review, Vol. 64, pp 242-252.

7. Rochester, N., et. al 1956
    Tests on a cell assembly theory of the
    action of the brain, using a large digital
    computer, I.R.E. Trans., Vol PGIT 2, No. 3,
    pp 80-92.

8. Smith, D. R., & Davidson, C. H. 1962
    Activity levels and oscillation modes in
    neural nets, in Biological prototypes and
    synthetic systems (Bernard & Rare, Eds.),
    Plenum Press, New York.

9. Uttley, A, M. 1966
    The transmission of information and the
    effect of local feedback in theoretical and
    neural networks, Brain Research, Vol. 2,
    pp 21-50.

Probability of connection, y

$y = A \cdot e^{-r/a}$

A

0 1 2 3 4 5

Radial separation of cells (r)

r = 1
r = 2
r = 3, etc.

Fig. 3   Neural network connectivity function

neuron cell with threshold T

outputs to other cells

axon

e_j excitatory inputs
i_j inhibitory inputs

e_1
e_i
e_m
i_j
i_n
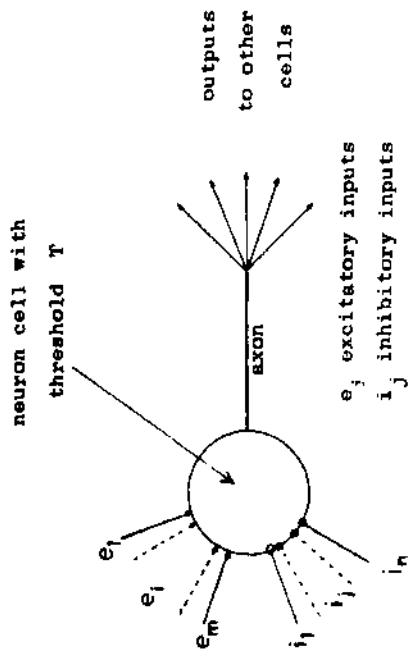
Fig. 1   Artificial neuron cell

Threshold and Excitation

absolutely refractory period

exponential recovery of threshold

excitation

T_m

T_0

-4   0   4   8   Time steps
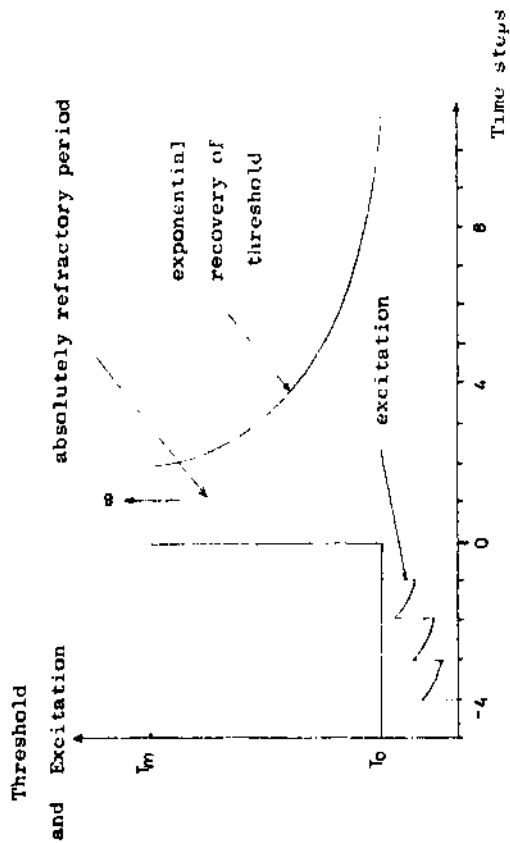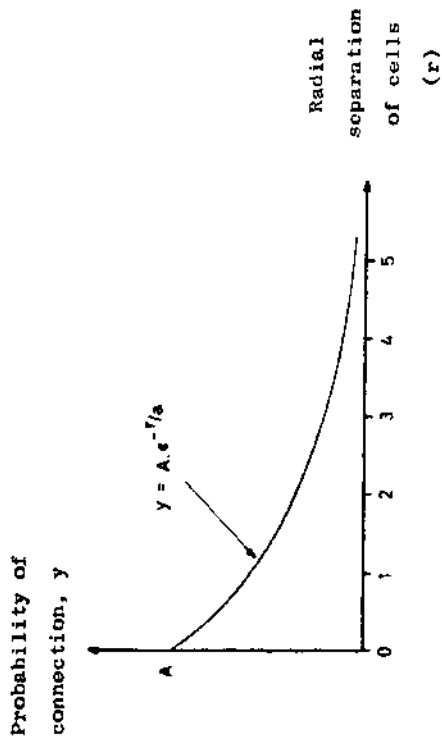
Fig. 2   Cell fires at time step 0 after receiving four bursts of excitation

(i)   (ii)   (iii)   (iv)   (v)

$A = 2.5$ $a = .667$ $b = 5$ stimulated cells   24 25 26

Fig. 4   Nonovert stimulation patterns

Fig. 5   Mode 1, attenuated activity

A = 1   a = 2   b = 4   stimulated cells   17   to   31   33

Fig. 7   Mode 3, virtually stable a

A = 1   a = 1.35   b = ?   stimulated cells   17   to   31   33

Fig. 6   Mode 2, unstable

A = 1 a = 2 b = 3   stimulated cells   17 19 31 33

A = 1 a = 4 , b = 5   stimulated cells   17 19 31 33

Fig. 8   Mode 4, virtually stable b,
cycle time = 6, lock in time = 3

Fig. 9   Mode 5, stable activity
lock in time = 3

A = 1   a = 2   b = 5   stimulated cells   17 18 19 23 25 32 33

stabilises

giving

mode 5

output

A = 1   a = 2   b = 5   stimulated cells   17 18 23 25 32 39 46

stabilises

giving

mode 5

output

A = 1   a = 2   b = 5   stimulated cells   1   9   17 25 33 40 41 49

stabilises

giving

mode 5

output

A = 2   a = 4   b = 5   stimulated cells   24 25 26

A

saturated stable mode

stable
mode

stable
mode

unstable and
virtually stable

2 modes

attenuated
mode

a

Fig. 13   Variation of activity with connectivity
constants A, a. τ_m = 1000, T_0 = 200

A = 1.5   a = 2   b = 3   stimulated cells   24 25 26

stabilises

giving

mode 5

output

A = 1.5   a = 2   b = 5   stimulated cells   24 25 26

stabilises

giving

mode 5

output

Fig. 12   Output patterns for the same network parameters,
but differing random number generators
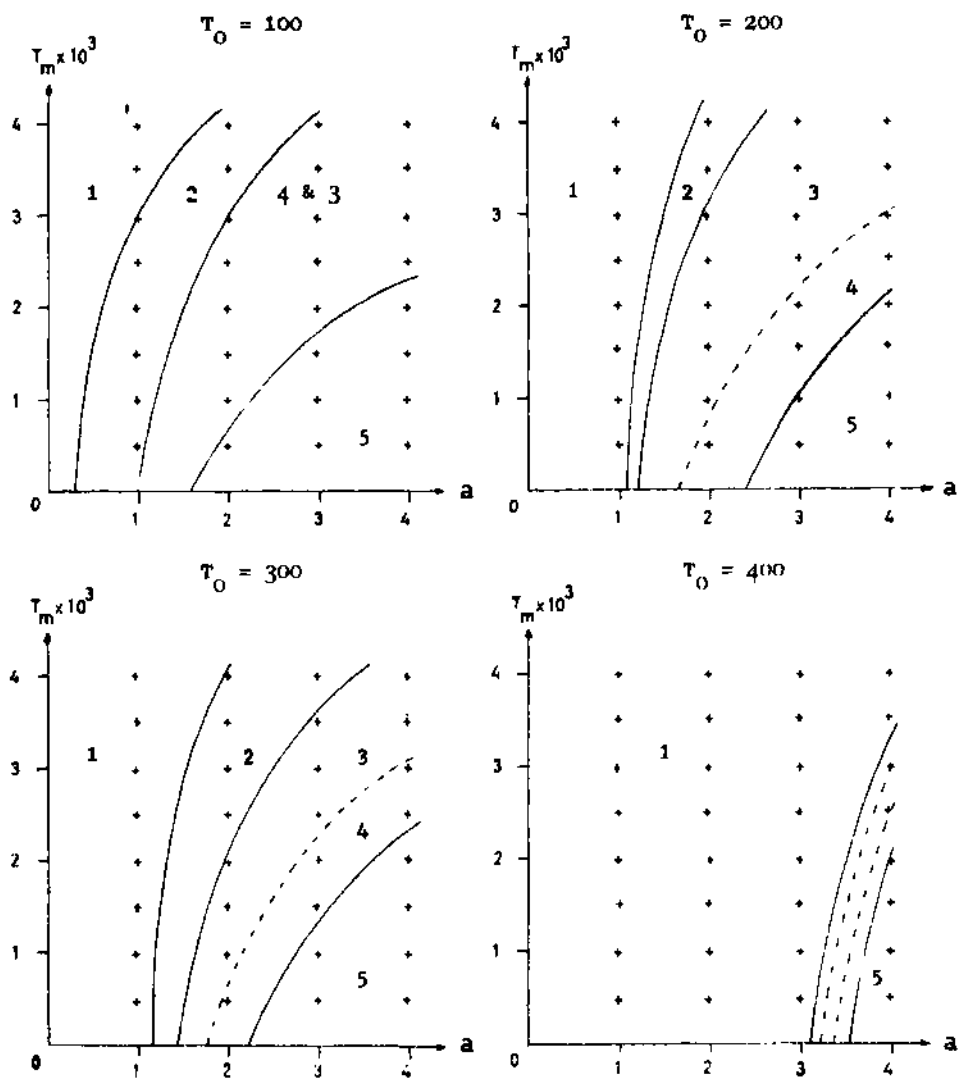
Fig. 14   Variation of activity with threshold and
           connectivity, A = 1 in all cases

```
0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 0 1 0 1 1 1 1 0 0 0 0 1 1 1 1 0 1 0 0 0 0 0 0
0 0 0 0 0 0 1 1 1 0 1 0 0 0 0 0 0 0 0 1 1 1 1 0 1 0 0 0 0 1 0 1 1 1 1 0 0 0 0 0 0 1 1 0 1 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 1 0 1 1 1 0 0 0 1 1 0 1 1 1 1 1 0 0 0 0 1 0 1 1 1 0 1 0 1 0 0 0
1 0 0 0 0 0 1 1 1 1 1 0 1 0 1 1 0 1 0 1 1 1 1 1 1 1 0 1 1 0 1 1 0 0 0 0 0 0 1 0 1 1 0 0 0 1 0 0 1 0
1 0 1 1 0 0 1 0 1 0 1 0 0 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 1 1 1 1 1 0 0 0 0 0 1
1 0 0 1 1 0 1 0 1 0 1 0 1 1 1 0 1 1 1 0 1 0 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 1 0 1 1 1 0 0 0 1 0 1 0
0 0 1 1 1 0 0 1 0 1 0 1 1 0 1 1 0 0 0 1 0 1 0 0 0 1 1 1 1 1 1 0 0 1 1 1 1 0 1 1 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1 1 0 1 1 0 1 1 0 0 0 1 0 1 0 1 0 0 0 0 1 0 1 1 1 1 1 0 0 1 1 1 1 1 0 1 1 0 0 0 0 0 1 0 1 0
1 1 0 0 0 0 0 1 0 0 1 1 0 0 1 1 1 0 1 0 0 0 1 1 1 1 1 0 0 0 1 0 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1
1 0 0 0 0 0 0 1 1 0 1 0 1 1 0 0 0 1 0 0 1 0 1 1 1 0 1 0 0 0 0 1 1 1 1 0 1 0 0 0 1 1 1 1 1 1 1 1 0
1 1 1 0 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 0 1 0 0 1 0 0 0 0 0 0 1 0 0 1 1 0 1 0 0 1 0 1 1 1 1 0 0 0 0 1
1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0 0 1 0 1 1 1 0 1 0 0 1 1 0 1 1 1 1 0 0 0 1 0 1 1 1
1 0 0 0 1 0 1 1 1 1 0 0 0 0 0 0 1 1 0 1 1 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0 1 0 0 0 0 0 0 1 0 0 1 1 0 0 1 0 0
0 1 0 0 0 0 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 0 1 0 0 0 1 0 0 1 0 0 0 0 1 0 1 1 1 1 1 1 0 1 1 1 1 0 0 0 1
0 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 0 0 1 0 0 1 0 1 1 1 0 1 1 1 0 1 0 1 0 1 0 0 1 1 0 0 0 0 1 1 1 1 0 1 1
0 0 0 0 1 1 0 1 0 1 1 0 0 0 1 0 0 0 0 0 1 1 1 0 1 0 0 1 1 1 0 0 1 1 1 0 0 0 1 0 0 0 1 0 1 1 0 1 0
1 1 1 1 1 0 0 1 1 0 1 0 0 1 1 1 1 0 1 1 1 0 0 0 0 0 1 0 0 0 1 1 0 1 1 1 0 0 0 0 0 1 0 0 0 0 0 0 0 1
0 1 0 1 1 0 1 0 1 0 0 0 0 1 1 1 1 0 1 1 0 0 0 1 1 0 0 0 0 1 1 0 1 0 1 0 1 1 1 0 0 1 0 1 0 0 0 1 1 0
0 1 0 0 0 1 0 1 0 1 0 1 1 1 1 0 0 0 0 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 1 1 1 0 0 0 1 0 1 1 0 1 0 1 1 1 1
0 0 0 0 0 1 0 0 1 0 1 0 0 1 0 0 1 1 0 1 0 1 0 0 1 0 1 1 1 1 0 0 1 1 1 0 0 0 1 1 1 0 1 0 1 0 1 0 1 1 1 0
1 0 1 0 1 0 1 0 1 0 0 1 0 0 1 1 1 1 1 0 1 0 1 0 0 1 0 1 1 0 0 0 0 0 0 0 0 1 0 1 1 1 0 1 1 1
0 1 1 1 1 0 0 1 1 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 0
1 1 1 1 0 0 0 0 0 0 1 0 1 1 0 1 1 0 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 1 0 1 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 1 1 1 1 0 0 0 1 0 0 0 1 0 1 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

Fig. 15a   Output from a 50 x 50 network after 15 time steps, rows 1 - 25, rows 26 - 50 overleaf

```
1 0 0 1 0 0 1 1 1 1 0 0 0 0 0 1 1 1 1 0 1 1 1 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 1 1 1 1 1 0 1 0 1 0 1 0 0 1 0 1 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 1 0 1 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 1 0 0 0 1 1 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 1 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 0 0 1 0 0 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 1 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 0 0 0 1 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 1 1 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

Fig. 15b   Output from a 50 x 50 network, showing initially stimulated cells and extent of activity flow through the network
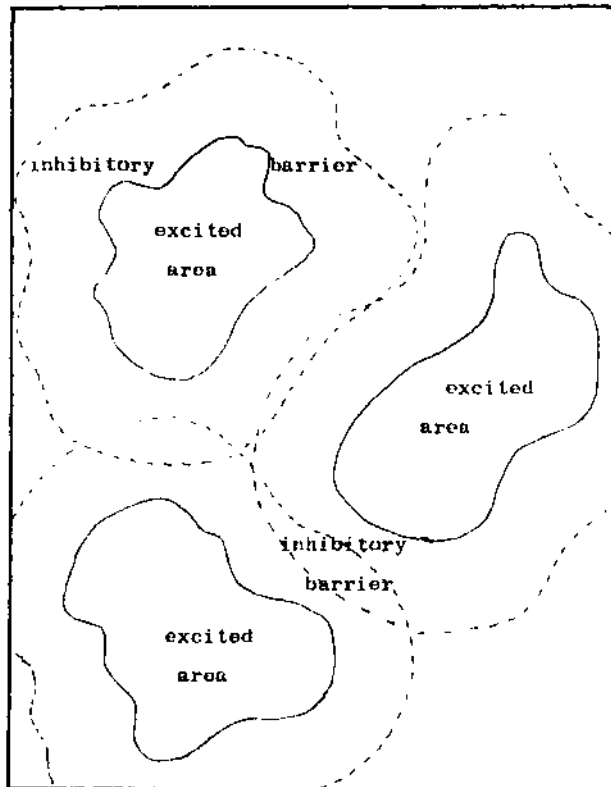
Fig. 16   Possible homogeneous neural network supporting
          several local stable oscillating networks

excitation connection strength = 100
inhibitory connection strength = 100
excitation decay constant = .5
inhibition decay constant = .8

Tm = 1000   To = 200 A_e = 2 A_i = 3 a_e = 4 a_i = 2
b = 5   stimulated cells   18   45   46   47   74

excitation connection strength = 100
inhibitory connection strength = 100
excitation decay constant = .5
inhibition decay constant = .8

Tm = 1000   To = 200 A_e = 2 A_i = 3 a_e = 4 a_i = 2
b = 5   stimulated cells   18   45   46   47   74

Fig. 17a   Cell, time step output from a network with
inhibitory and excitatory connections

Fig. 17b   Cell, time step output from a network with
inhibitory and excitatory connections