Pensieve header: The Cayley distance kernel following arXiv://2105.0287 by Corfield, Sati, and Schreiber.

In[∘]:=
```
n = 3;
S = PermutationCycles /@ Permutations@Range@n
```

Out[∘]=
```
{Cycles[{}], Cycles[{{2, 3}}], Cycles[{{1, 2}}],
 Cycles[{{1, 2, 3}}], Cycles[{{1, 3, 2}}], Cycles[{{1, 3}}]}
```

In[∘]:=
```
Clear[d];
d[p_] := d[p] = Total[Length[#] - 1 & /@ p[[1]]];
d[p1_, p2_] := d[PermutationProduct[p1, InversePermutation[p2]]];
d /@ S
```

Out[∘]=
```
{0, 1, 1, 2, 2, 1}
```

In[∘]:=
```
A = Table[T^(-d[p1,p2]), {p1, S}, {p2, S}];
A // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{1}{T} & \frac{1}{T} & \frac{1}{T^2} & \frac{1}{T^2} & \frac{1}{T} \\ \frac{1}{T} & 1 & \frac{1}{T^2} & \frac{1}{T} & \frac{1}{T} & \frac{1}{T^2} \\ \frac{1}{T} & \frac{1}{T^2} & 1 & \frac{1}{T} & \frac{1}{T} & \frac{1}{T^2} \\ \frac{1}{T^2} & \frac{1}{T} & \frac{1}{T} & 1 & \frac{1}{T^2} & \frac{1}{T} \\ \frac{1}{T^2} & \frac{1}{T} & \frac{1}{T} & \frac{1}{T^2} & 1 & \frac{1}{T} \\ \frac{1}{T} & \frac{1}{T^2} & \frac{1}{T^2} & \frac{1}{T} & \frac{1}{T} & 1 \end{pmatrix}$$

In[∘]:=
```
n = 6;
S = PermutationCycles /@ Permutations@Range@n;
Clear[d];
d[p_] := d[p] = Total[Length[#] - 1 & /@ p[[1]]];
d[p1_, p2_] := d[PermutationProduct[p1, InversePermutation[p2]]];
A = Table[T^(-d[p1,p2]), {p1, S}, {p2, S}];
Plot[T^(n-1) Min[Eigenvalues[A]], {T, 0.5, n}]
```

Out[∘]=