Chapter 2

The Making of the Standard Theory

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1. Introduction

The construction of the Standard Model, which became gradually the Standard Theory of elementary particle physics, is, probably, the most remarkable achievement of modern theoretical physics. In this Chapter we shall deal mostly with the weak interactions. It may sound strange that a revolution in particle physics was initiated by the study of the weakest among them (the effects of the gravitational interactions are not measurable in high energy physics), but we shall see that the weak interactions triggered many such revolutions and we shall have the occasion to meditate on the fundamental significance of "tiny" effects. We shall outline the various steps, from the early days of the Fermi theory to the recent experimental discoveries, which confirmed all the fundamental predictions of the Theory. We shall follow a phenomenological approach, in which the introduction of every new concept is motivated by the search of a consistent theory which agrees with experiment. As we shall explain, this is only part of the story, the other part being the requirement of mathematical consistency. Both went in parallel and the real history requires the understanding of both. In fact, as we intend to show, the initial motivation was not really phenomenological. It is one of these rare cases in which a revolution in physics came from theorists trying to go beyond a simple phenomenological model, not from an experimental result which forced them to do so. This search led to the introduction of novel symmetry concepts which brought geometry into physics. It is this exciting story which will be presented here.

2. Prehistory

2.1. *The electron spectrum in β-decay*

Chadwick vs Hahn and Meitner — Ellis and Wooster settle the issue.

In the early years of the 20th century the only known weak interaction was nuclear β-decay, which was believed to be a two-body decay of the form $N_1 \to N_2 + e^-$. The first revolution came from the study of the electron spectrum, which, for a two-body decay, is expected to be mono-energetic. The measurements were performed by two groups: (i) Otto Hahn and Lise Meitner[1] in Berlin were measuring the electron energy by looking at the penetration depth in various materials. The precision was not very good and their assumptions were rather crude, but the results could be interpreted as compatible with mono-energetic rays. (ii) James Chadwick, working in the group of Hans Geiger, also in Berlin, used a magnetic spectrometer and an electron counter. His results,[2] first published in 1914, showed instead a continuous spectrum, incompatible with a two-body decay. This was the first *energy crisis* in β-decay.

Hahn and Meitner attempted to explain the continuous spectrum by the re-scattering of the electron in nuclear matter, before its ejection from the nucleus. (Remember, they were still using the old nuclear model of a nucleus being a bound state of protons and electrons.) The issue was settled by a calorimetry experiment performed by Charles Drummond Ellis and William Alfred Wooster in 1927. They measured the total energy released during a certain number of decays N and they found[3] that $E_{\text{tot}} = NE_{\text{mean}}$, the *mean* energy of the electron spectrum, while, if Hahn and Meitner were right, they should have found $E_{\text{tot}} = NE_{\text{max}}$. The electrons were indeed emitted with a continuous spectrum. This was the second, and the most serious, *energy crisis*. Meitner, who declared having felt "a great shock" reading the paper, repeated the experiment and confirmed the result,[4] but she proposed no explanation. Resolving the energy crisis was left to the theorists.

2.2. *Enter the neutrino*

Pauli vs Bohr — Bohr looses the battle.

In the meantime the crisis was getting worse with new evidence showing that not only energy, but also angular momentum was not conserved and the Pauli exclusion principle was violated. Faced with such challenges many prominent physicists were ready to abandon the validity of all conservation laws in the new physics. The most important among them was no lesser man than Niels Bohr who, already in 1924,[5] had a scheme in which energy was conserved only *in the mean*. Heisenberg, and even Einstein and Dirac among others, toyed with this idea for a while.

This confusion brings us to December 1930. A Conference was organised in Tübingen to debate all the relevant issues. Pauli was invited but he decided not to attend. He sent a letter instead,[6] written in an inimitable style, in which he makes a bold suggestion: nuclear β-decay *is not a two-, but a three-body decay!* The electron is accompanied by a light, neutral, weakly interacting particle which carries away part of the energy. If, in addition, it is assumed to have spin 1/2, all problems are solved. Bohr did not give up immediately and was considering energy violating theories as late as 1936, but, by and large, the new particle was generally accepted. Pauli had called it "neutron", but when Chadwick discovered our neutron in 1932, Fermi coined the name "neutrino". In Fermi's theory we shall see next, the neutrino is a particle like any other. The first direct observation of the neutrino had to wait until 1956[7] with Frederick Reines and Clyde Lorrain Cowan. Their first announcement of the discovery was a telegram to Pauli.[8]

2.3. *Fermi's Tentativo*

Quantum Field Theory becomes the language of particle physics.

Already in 1926, before the introduction of the Schrödinger wave equation, Fermi had published two papers with the rules of counting particles which established the Fermi quantum statistics and gave fermions their name.[9] In 1933 he came back with one of the most influential papers in particle physics in which he proposes a field theory model for the β-decay of neutrons. Even today, when this theory has been superseded by the Standard Model of weak interactions, Fermi's theory is still used as a good low energy approximation.

This paper contains many revolutionary ideas. Fermi was one of the first physicists who believed in the physical existence of the neutrino. Contrary to Heisenberg, in the Bohr–Pauli controversy Fermi sided clearly with the latter. But he went further and broke completely with the prevailing philosophy, according to which particles which come out from a nucleus ought to be present inside it.[a] In his paper he formulated the full quantum field theory for fermion fields and introduced for them the formalism of creation and annihilation operators. It was the first time that quantised fermion fields appeared in particle physics. The paper appeared at the beginning of 1934[11] in Italian[b] under the title *Tentativo di una teoria della emissione di raggi* β.

Fermi's starting point was an analogy with the electromagnetic interactions in which the current j_μ produced by the charged particles acts as the source of the electromagnetic potential A^μ. This simple idea influenced all further developments we shall review in this book. He introduced quantised fermion fields for the electron, $\psi_{(e)}(x)$, and the neutrino, $\psi_{(\nu)}(x)$. He did not do the same for the nucleons because there was still a confusion regarding the magnetic moment of the proton and it

[a] Similar ideas had been expressed before by D. Iwanenko, in 1932 and Francis Perrin in 1933. The latter wrote: "... The neutrino ... does not preexist in atomic nuclei, it is created when emitted, like the photon",[10] but Fermi was the first to show how such a thing could actually happen.

[b] An english version had been submitted earlier to *Nature*, but it was rejected "because it contained speculations too remote from reality to be of interest to the reader".

was not clear whether the Dirac equation was applicable to them. He bypassed this difficulty by considering a static density for the nucleons and used the isospin operators $\tau_{\pm}$ Heisenberg had introduced earlier for the nuclear forces. The result was the following expression for the β-decay interaction Hamiltonian:

$$H_I = \frac{G_F}{\sqrt{2}}[\tau_- \psi^\dagger_{(e)}(x)\psi_{(\nu)}(x) + \tau_+ \psi^\dagger_{(\nu)}(x)\psi_{(e)}(x)] \tag{1}$$

where † means "hermitian adjoint" and $\frac{G_F}{\sqrt{2}}$ is a coupling constant, F stands for Fermi.[c] As we see, the effort to understand the nuclear forces went in parallel to that of the weak interactions and they influenced each other considerably. By 1936 the use of Dirac fields quantised *à la* Fermi for the nucleons became common in describing all nuclear interactions and the β-decay Hamiltonian of Eq. (1) took the form:

$$H_I = \frac{G_F}{\sqrt{2}}\sum_{i=1}^{5}\bar{\Psi}_{(p)}(x)O_i\Psi_{(n)}(x)\bar{\psi}_{(e)}(x)O_i\psi_{(\nu)}(x) \tag{2}$$

where the sum runs over the five Dirac invariants. It is the form under which the Fermi theory is known.[d]

2.4. *The high energy behaviour*

Infinities are never good.

An important property of the Fermi Hamiltonian was discovered in 1936 by Markus Fierz,[13] who computed the cross section for neutrino scattering and found that, at high energies, it increases with the neutrino energy:

$$d\sigma(\bar{\nu} + p \to n + e^+) = \frac{G_F^2}{2\pi^2}p_\nu^2 d\Omega \tag{3}$$

where p_ν is the neutrino momentum in the centre-of-mass system and $d\Omega$ is the element of the solid angle of the positron momentum. Similar conclusions were reached also by Heisenberg for the inelastic cross sections. One could guess these results by a simple dimensional analysis, taking into account the fact that the coupling constant G_F has the dimensions of inverse mass square. It became immediately obvious that such a behaviour is unacceptable because, at sufficiently high energies, the higher order terms will exceed this lowest order result, which means that an expansion in powers of G_F is meaningless. This problem haunted weak interactions for many years and led to the formulation of the new theory whose exposition is the subject of this book.

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[c] This is the modern notation. Fermi used simply the symbol g.

[d] It is not clear whether Fermi ever wrote this form. It is possible that he had introduced it in one of his lectures. It appeared for the first time in a review article by H. A. Bethe and R. F. Bacher in 1936.[12]

3. Thirty Years of Unconcern, Thirty Years of Doubt

3.1. *Fermi's theory as the most successful phenomenology*

Elegance is the name of the game.

Following an impressive series of experimental and theoretical investigations, the form (2) was gradually reduced to a superposition of only the vector and pseudo-vector parts in the V-A combination which violates maximally the invariance under space inversions. Even a superficial history of the subject should include some of the early experiments, which count among the most significant and beautiful ones in physics. They include the discovery of parity violation in the β-decay of ^{60}Co by Chien-Shiung Wu in 1956[14] and the measurement of the neutrino helicity by Maurice Goldhaber in 1957.[15] On the other hand, the main theoretical ideas were: (i) The β-decay $\leftrightarrow$ μ-decay universality. (ii) The Conserved Vector Current (CVC) hypothesis. (iii) The Partial Conservation of the Axial Current (PCAC) hypothesis. (iv) The Cabibbo extension to $SU(3)$ currents and the generalised universality condition. All these gave rise to the current$\times$current theory, with the current being a sum of a leptonic and a hadronic part, of the form:

$$H_I = \frac{G_F}{\sqrt{2}} J^\mu(x) J^\dagger_\mu(x) \quad ; \quad J^\mu(x) = l^\mu(x) + h^\mu(x). \tag{4}$$

The leptonic part was written, as Fermi had done it in the early thirties, in terms of the fields of known leptons:[e]

$$l^\mu(x) = \bar{\nu}_{(e)}(x)\gamma^\mu(1+\gamma_5)e(x) + \bar{\nu}_{(\mu)}(x)\gamma^\mu(1+\gamma_5)\mu(x) \tag{5}$$

while the explicit form of the hadronic part depended on the assumptions regarding the strong interactions, in some sense coming back to the original Fermi formula (1). This simple and elegant form, not only described all weak interaction phenomena known at the time, but also led to the discovery of several fundamental symmetry properties in particle physics. It was a very satisfactory model, especially if one compared it with the situation in strong interactions, for which we had neither a successful phenomenology, nor an elegant form.

3.2. *Fermi's theory as the most inspiring model*

Chiral symmetry — Current algebra.

The leptonic part of the weak interaction current being determined, the effort was concentrated on the hadronic part $h^\mu(x)$. In a typical semi-leptonic weak interaction one needs the matrix elements $\langle a|h^\mu|b\rangle$, where $|a\rangle$ and $|b\rangle$ are hadronic states. So, when we say "to determine h^μ", we really mean "to identify it with a known operator of the strong interactions". The only operators whose properties are supposed

[e]Today we should add a third term for the τ-lepton.

to be known independently of the details of a particular dynamical model, are the currents of whichever symmetries one assumes for strong interactions. This way the effort to understand the structure of the weak interaction Hamiltonian helped discovering fundamental symmetries of the strong interactions. The important steps were the following:

- CVC. The vector part of the strangeness conserving weak current is the charged component of the isospin current.[16]
- The Cabibbo universality condition. Nicola Cabibbo generalised the universality and the CVC conditions to include the strangeness changing currents.[17] He wrote the weak hadronic current as:

$$h_\mu(x) = \cos\theta h_\mu^{(\Delta S=0)}(x) + \sin\theta\ h_\mu^{(\Delta S=1)}(x) \tag{6}$$

  with θ "the Cabibbo angle". This way the weak current became a member of an $SU(3)$ octet. Cabibbo checked that this hypothesis fits the experimental results on the decays of strange particles.
- PCAC, or Partially Conserved Axial Current. The CVC hypothesis answered the question of determining the vector part of the weak hadronic current by identifying it with the $SU(3)$ symmetry current. Could an analogous determination be extended to the axial part?[18] At first sight the answer is no, because no approximate axial symmetry seems to be present in the spectrum of hadrons. Yoichiro Nambu[19] gave the correct explanation: strong interaction dynamics is approximately invariant under chiral transformations, but the symmetry is spontaneously broken. The pions, or the octet of 0^- bosons if we extend the idea to $SU(3)$, are the corresponding Nambu–Goldstone bosons.[20]
- The Algebra of Currents. These considerations led Murray Gell-Mann[21] to postulate an algebraic scheme which translated the approximate symmetries of strong interactions. The symmetry group is assumed to be:

$$U(3) \times U(3) \sim U(1) \times U(1) \times SU(3) \times SU(3). \tag{7}$$

  To this symmetry correspond 18 conserved, or approximately conserved, currents out of which we can construct 18 charges, the generators of the group transformations. It is convenient to write them as follows: (i) Q_V for the vector $U(1)$. (ii) Q_A for the axial $U(1)$. (iii) Q_R^a for the right-hand $SU(3)$ and (iv) Q_L^a for the left-hand part, $a = 1, 2, \ldots, 8$. In the limit of exact symmetry they satisfy the commutation relations:

$$[Q_R^a\ , Q_R^b] = if^{abc}Q_R^c\ \ ;\ \ [Q_L^a\ ,\ Q_L^b] = if^{abc}Q_L^c\ \ ;\ \ a, b, c = 1, 2, \ldots, 8 \tag{8}$$

  with all other commutators vanishing. f^{abc} are the structure constants of $SU(3)$ and a sum over repeated indices is understood. It is instructive to see the fate of the various factors in (7): (i) The vector $U(1)$ remains as an exact symmetry and the corresponding conservation law is that of baryon number. (ii) The axial

$U(1)$ puzzled people for a long time and it took some years before it was finally understood that, at the quantum level, the symmetry is broken by a phenomenon we shall see later, called "the axial anomaly". (iii) The $SU(3) \times SU(3)$ part is spontaneously broken as:

$$SU(3)_R \times SU(3)_L \ \rightarrow \ SU(3)_V \tag{9}$$

with $SU(3)_V$ being the diagonal subgroup of the chiral $SU(3)_R \times SU(3)_L$ which contains only vector currents. It is called "the flavour group". As we said before, the corresponding Nambu–Goldstone bosons form the 0^- octet of flavour $SU(3)$. (iv) There was a hierarchy of strong interactions: the "very strong interactions" were invariant under the full $SU(3)_V$. The "medium strong interactions" were assumed to break explicitly $SU(3)_V$ and leave invariant only the isospin subgroup.

3.3. *Fermi's theory as a an effective field theory*

Where is the cut-off, or the vital importance of precision measurements.

Fermi's theory cannot be viewed as a fundamental theory because, in technical terms, it is *non-renormalisable*. In practical terms this means that, if we write any physical amplitude as a power series in the Fermi coupling constant G_F, every term in the expansion requires the introduction of a cut-off parameter Λ. In a renormalisable theory, such as quantum electrodynamics, there exists a well-defined prescription to take the limit $\Lambda \rightarrow \infty$ and obtain unambiguous results, but to a non-renormalisable theory the prescription does not apply. The cut-off must remain finite and its value determines the energy scale above which the theory cannot be trusted. This is the definition of *an effective theory.*

Can we estimate an order of magnitude for the cut-off? A very simple method is the following: Ordinary dimensional analysis tells us that a physical quantity $\mathcal{A}$, for example a weak decay amplitude, can be written in a series expansion as:

$$\mathcal{A} = A_1 G_F \left(1 + \sum_{n=2}^{\infty} A_n (G_F \Lambda^2)^{n-1} \right) \tag{10}$$

where, in every order of the expansion, we have kept only the highest power in Λ. We see that the expression $g_{\text{eff}} = G_F \Lambda^2$ acts as an effective, dimensionless coupling constant. The expansion will become meaningless when $g_{\text{eff}} \sim 1$, which, for the numerical value of G_F, gives $\Lambda \sim 300\,\text{GeV}$, a value which, for the accelerators of the 1960s, was essentially infinite.

It was B. L. Ioffe and E. P. Shabalin,[23] from the Soviet Union, who first remarked that, in fact, one can do much better. Let us go back to the expansion (10) and consider also the sub-dominant terms in powers of Λ. We can rephrase their argument and write any physical quantity as a double expansion in $g_{\rm eff}$ and G_F:

$$\mathcal{A} = \sum_{n=0}^{\infty} A_n^{(0)} g_{\rm eff}^n + G_F M^2 \sum_{n=0}^{\infty} A_n^{(1)} g_{\rm eff}^n + (G_F M^2)^2 \sum_{n=0}^{\infty} A_n^{(2)} g_{\rm eff}^n + \cdots \tag{11}$$

where the quantities $A_n^{(i)}$ may contain powers of the logarithm of Λ. M is some mass parameter, which, for a typical quantity in particle physics, is of the order of 1 GeV. The first series contains the terms with the maximum power of Λ for a given power of G_F, they are called *the leading divergences.* Similarly, the second series contains all the *next-to-leading divergences*, the third the *next-to-next-to-leading divergences*, etc. Following Ioffe and Shabalin, let us choose for $\mathcal{A}$ a quantity in strong interactions, for example the energy levels in a nucleus. The leading divergences represent the weak interaction corrections to this quantity. But weak interactions violate parity and/or strangeness, therefore the high precision with which such effects are known to be absent in nuclear physics gives a much more stringent bound for Λ, of the order of 2–3 GeV. Similarly the next-to-leading divergences contribute to "forbidden" weak interaction processes, such as $\Delta S{=}2$ transitions (the $K_L^0 - K_S^0$ mass difference), or $K_L^0 \to \mu^+\mu^-$ decays. Again, the precision measurements of such quantities give the same 2–3 GeV limit for Λ.

4. Gauge Theories

4.1. *Gauge invariance in classical physics*

From electrodynamics to general relativity.

Classical electrodynamics is traditionally formulated in terms of the electric and magnetic fields which form a redundant set of variables. A first step towards a more reduced system was the introduction of the vector potential during the first half of the nineteenth century, either implicitly or explicitly, by several authors independently. It appears in some manuscript notes by Carl Friedrich Gauss as early as 1835 and it was fully written by Gustav Kirchhoff in 1857, following some earlier work by Franz Neumann.[22] It was soon noticed that it still carried redundant variables and several "gauge conditions" were used. The condition, which in modern notation is written as $\partial_\mu A^\mu = 0$, was proposed by the Danish mathematical physicist Ludvig Valentin Lorenz in 1867. However, the profound geometric interpretation of gauge invariance was not noticed until much later.

At the beginning of the twentieth century the development of the General Theory of Relativity offered a new paradigm for a gauge theory. The fact that it can be written as the theory invariant under local translations was certainly known to Hilbert, hence the name of *Einstein–Hilbert action.* The two fundamental forces known at that time, namely electromagnetism and gravitation, were thus found to obey a gauge principle. It was, therefore, tempting to look for a unified theory. Today we know the attempt by Theodor Kaluza, completed by Oscar Benjamin

Klein, which is often used in supergravity and superstring theories. These authors consider a theory of General Relativity formulated in a five-dimensional space–time (1+4). They remark that if the fifth dimension is compact the components of the metric tensor along this dimension may look to a four-dimensional observer as those of an electromagnetic vector potential. What is less known is that the idea was introduced earlier by the Finnish Gunnar Nordström who had constructed a scalar theory of gravitation. In 1914 he wrote a five-dimensional theory of electromagnetism[24] and showed that, if one assumes that the fields are independent of the fifth coordinate, the assumption made later by Kaluza, the electromagnetic vector potential splits into a four-dimensional vector and a four-dimensional scalar, the latter being identified to his scalar field of gravitation, in some sense the mirror theory of Kaluza and Klein.

4.2. *Gauge invariance in quantum mechanics*

The phase of the wave function.

The transformations of the vector potential in classical electrodynamics are the first example of an *internal symmetry* transformation, namely one which does not change the space–time point x. However, the concept, as we know it today, belongs really to quantum mechanics. It is the phase of the wave function, or that of the quantum fields, which is not an observable quantity and produces the internal symmetry transformations. The local version of these symmetries are the gauge theories we study here. The first person who realised that the invariance under local transformations of the phase of the wave function in the Schrödinger theory implies the introduction of an electromagnetic field was Vladimir Aleksandrovich Fock in 1926, just after Schrödinger wrote his equation. Fock noticed[25] that Schrödinger's equation, together with the normalisation condition of the wave function,

$$i\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = -\frac{1}{2m}\Delta\Psi(\mathbf{x},t)\ \ ;\ \ \int|\Psi(\mathbf{x},t)|^2 d\mathbf{x} = 1 \tag{12}$$

are invariant under the transformation $\Psi(\mathbf{x},t) \to e^{i\theta}\Psi(\mathbf{x},t)$, with θ a constant phase. Fock asked the question of what happens if the transformation becomes *local*, i.e. if we replace the constant θ by an arbitrary function of space and time. The answer is that we can restore invariance if we introduce a vector and a scalar potential $\mathbf{A}(\mathbf{x},t)$ and $A_0(\mathbf{x},t)$ and replace in the Eq. (12) the derivative operators by the covariant derivatives:

$$\left(\frac{\partial}{\partial t}\right)_{\text{cov}} = \frac{\partial}{\partial t} + ieA_0(\mathbf{x},t)\ \ ;\ \ (\nabla)_{\text{cov}} = \nabla - ie\mathbf{A}(\mathbf{x},t) \tag{13}$$

where e is a constant introduced only for convenience. The new equation:

$$i\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = \left[-\frac{1}{2m}(\nabla - ie\mathbf{A}(\mathbf{x},t))^2 + eA_0(\mathbf{x},t)\right]\Psi(\mathbf{x},t) \tag{14}$$

is invariant under local phase transformations, provided the potentials transform as:

$$A_0(\mathbf{x},t) \to A_0(\mathbf{x},t) + \frac{1}{e}\frac{\partial\theta(\mathbf{x},t)}{\partial t} \;\; ; \;\; \mathbf{A}(\mathbf{x},t) \to \mathbf{A}(\mathbf{x},t) - \frac{1}{e}\nabla\theta(\mathbf{x},t). \tag{15}$$

This equation describes the motion of a charged particle in an external electromagnetic field. The electromagnetic interactions are generated by a gauge principle. In 1929 Hermann Klaus Hugo Weyl[f] extended this work to the Dirac equation.[26] In this work he introduced many concepts which have become classic, such as the Weyl two-component spinors and the vierbein and spin-connection formalism. Although the theory is no more scale invariant, he still used the term *gauge invariance*, a term which has survived ever since.

4.3. *From general relativity to particle physics*

The direct road is not always obvious.

Naturally, one would expect non-Abelian gauge theories to be constructed following the same principle immediately after Heisenberg introduced the concept of isospin in 1932. But here history took a totally unexpected route.

The first person who tried to construct the gauge theory for $SU(2)$ is Oskar Klein[27] who, in an obscure conference in 1938, he presented a paper with the title: *On the theory of charged fields*. The most amazing part of this work is that he follows an incredibly circuitous road: He considers general relativity in a five-dimensional space and compactifies *à la* Kaluza–Klein. Then he takes the limit in which gravitation is decoupled. In spite of some confused notation, he finds the correct expression for the field strength tensor of $SU(2)$. He wanted to apply this theory to nuclear forces by identifying the gauge bosons with the new particles which had just been discovered, (in fact the muons), misinterpreted as the Yukawa mesons in the old Yukawa theory in which the mesons were assumed to be vector particles. He considered massive vector bosons and it is not clear whether he worried about the resulting breaking of gauge invariance.

The second work in the same spirit is due to Wolfgang Pauli[28] who, in 1953, in a letter to Abraham Pais, developed precisely this approach: the construction of the $SU(2)$ gauge theory as the flat space limit of a compactified higher-dimensional theory of general relativity. He had realised that a mass term for the gauge bosons breaks the invariance and he had an animated argument during a seminar by Yang in the Institute for Advanced Studies in Princeton in 1954.[29] What is surprising is that Klein and Pauli, fifteen years apart one from the other, decided to construct the $SU(2)$ gauge theory for strong interactions and both choose to follow this totally counter-intuitive method. It seems that the fascination

[f]He is more known for his 1918 unsuccessful attempt to enlarge diffeomorphisms to local scale transformations.

which general relativity had exerted on this generation of physicists was such that, for many years, local transformations could not be conceived independently of general coordinate transformations. Yang and Mills[30] were the first to understand that the gauge theory of an internal symmetry takes place in a fixed background space which can be chosen to be flat, in which case general relativity plays no role.

4.4. *Yang–Mills and weak interactions*

With, or without, electromagnetism?

In particle physics we put the birth of non-Abelian gauge theories in 1954, with the fundamental paper of Chen Ning Yang and Robert Laurence Mills. It is the paper which introduced the $SU(2)$ gauge theory and, although it took some years before interesting physical theories could be built, it is since that date that non-Abelian gauge theories became part of high energy physics. It is not surprising that they were immediately named *Yang–Mills theories*. Although the initial motivation was a theory of the strong interactions, the first semi-realistic models aimed at describing the weak and electromagnetic interactions. In fact, following the line of thought initiated by Fermi, the theory of electromagnetism has always been the guide to describe the weak interactions.

Already in 1957, Julian Schwinger had conjectured[31] that the theory (4) should be modified with the introduction of an *intermediate vector boson* (IVB) $W_\mu^\pm$:

$$H_I = gJ^\mu(x)W_\mu^-(x) + \mathrm{hc} \tag{16}$$

with g a new dimensionless coupling constant. This way weak interactions looked pretty much like the electromagnetic ones, a vector boson coupled to a current, but with some very important differences: (i) The photon is massless and the electromagnetic interactions are long ranged. The weak interactions are known to be short ranged, so the W's must be massive. (ii) The photon is neutral, the W's are charged. (iii) The electromagnetic current is conserved, the weak current is not. It was soon clear that these differences implied that the theory (16) was in fact as hopelessly non-renormalisable as (4).

The early attempts to use Yang–Mills theories to describe the weak interactions followed immediately the IVB hypothesis. Schwinger assumed the existence of a triplet of intermediate bosons, which he called $Z^{\pm,0}$, the two charged ones mediating the weak interactions and the neutral one being the photon. A year later, in 1958, S. A. Bludman[32] built the first $SU(2)$ Yang–Mills theory for weak interactions in which all three gauge bosons were coupled to $V - A$ currents. No connection with electromagnetism was assumed.

The most important contribution from this period dates from 1961 and it is due to Sheldon Lee Glashow.[33] He uses an $SU(2) \times U(1)$ gauge group, thus having two neutral gauge bosons. He is the first to propose a unified description for weak and electromagnetic interactions and introduces the idea of a mixing between the two neutral bosons. The photon field is a linear combination of the fields associated with $U(1)$ and the third generator of $SU(2)$ with an angle which he called θ (today it is called θ_W).

In the same year we have another important paper by Gell-Mann and Glashow.[34] This paper extends the Yang–Mills construction, which was originally done for $SU(2)$, to arbitrary Lie algebras. The well-known result of associating a single coupling constant with every simple factor in the algebra appeared for the first time in this paper. They even introduced the idea of a grand unified theory, as we shall explain later.

4.5. *A model for leptons*

The synthesis.

Gauge invariance requires the conservation of the corresponding currents and a zero masse for the Yang–Mills vector bosons. None of these properties seemed to be satisfied for the weak interactions. People were aware of the difficulty,[g] but had no means to bypass it. The mechanism of spontaneous symmetry breaking was invented a few years later,[35] in 1964. It is presented in a different Chapter in this Book, so here we shall continue the story after it.

The synthesis of Glashow's 1961 model with the mechanism of spontaneous symmetry breaking was made in 1967 by Steven Weinberg, followed a year later by Abdus Salam.[36,37] It is the work which gave rise to the Standard Model. The group is $U(1) \times SU(2)$ and has four gauge bosons, two charged ones and two neutral. At that time people did not yet know how to avoid the appearance of strangeness changing neutral currents, or $\Delta S = 2$ transitions, so the model applied only to leptons. The mechanism which allowed the extension to hadrons was found in 1970 and will be presented in the next section.

Many novel ideas have been introduced in this paper, mostly connected with the use of the spontaneous symmetry breaking which became the central point of the theory. They include:

- Its use for the weak interactions. We remind that the initial motivation was the breaking of flavour $SU(3)$.
- The fact that the same mechanism is the origin of the weak–electromagnetic mixing which had been postulated by Glashow.
- It is also the mechanism which gives masses to the fermions.

We shall present the general form of the model in a subsequent section.

[g] Glashow talks about *partially gauge invariant theories.*

5. Fighting the Infinities

5.1. *The phenomenology front*

A two-front battle.

A cut-off as low as 2–3 GeV was clearly unacceptable, it meant that, at least for some processes, Fermi's theory should be corrected already at low energies. The fact that the Fermi theory was non-renormalisable was known since the early years, but I believe it is fair to say that it was the work of Ioffe and Shabalin which showed that the problem was not only mathematical but also physical. A long and painful struggle against the infinities started. Although it was fought by few people,[h] it has been an epic battle given in two fronts: The first, the phenomenology front, aimed at finding the necessary modifications to the theory in order to eliminate the disastrous leading and next-to-leading divergences. The second, the field theory front,[38] tried to find the conditions under which a quantum field theory involving massive, charged, vector bosons is renormalisable. It took the success in both fronts to solve the problem. In this Chapter we shall describe the efforts in the first front.

5.2. *Early attempts*

Can we determine the Cabibbo angle? Are we ready to sacrifice elegance?

In the early attempts the effort was not focused on a particular physical problem, but aimed instead at eliminating the divergences, at least from physically measurable quantities. Some were very ingenious, but lack of space does not allow us to present them in any detail. A very incomplete list contains:

- The *physical* Hilbert space contains states with negative metric.[39] The introduction of negative metric states is considered unacceptable because it implies violation of the unitarity condition. However, Tsung Dao Lee and Gian Carlo Wick observed that, if the corresponding "particles", in this case the weak vector bosons, are very short lived, the resulting unitarity violations could be confined into very short times and be undetectable.
- The V-A form of the Fermi theory is an illusion and, in reality, the intermediate bosons mediating weak interactions are scalars.[40] By a Fierz transformation, the effective Lagrangian could look like a vector theory for some processes. This way the theory is renormalisable, but at the price of loosing all insight into the fundamental role of the weak currents.
- The theory (16) is an approximation and the real theory contains a large number of intermediaries with couplings arranged to cancel the most dangerous divergences.[41] The idea was simple: divergences arise in perturbation theory

[h] Most people doubted about the physical significance of the problem because of widespread mistrust towards field theory in general and higher order diagrams in particular. Since we had no theory, why bother about its higher order effects?

because a massive vector boson has a propagator which behaves like a constant at large momenta. This behaviour cannot be improved without violating unitarity. However, for a matrix valued field, we can obtain cancelations for some matrix elements. With a clever arrangement of the couplings, we can hide all bad divergences from the physically relevant quantities. A simple idea whose implementation turned out to be very complicated.[i]

- The weak interaction divergences and the value of the Cabibbo angle.[42,43] The idea was to compute the coefficient of the divergent term, for example in a loop expansion, for both the weak and the electromagnetic contributions. Setting this coefficient equal to zero gives an equation for the Cabibbo angle. The work by itself has today only a historical interest, but, as by-products, two interesting results emerged, summarised in the following two relations:

$$\tan\theta = \sqrt{\frac{m_d}{m_s}} \;\; ; \;\; \frac{|m_d - m_u|}{m_d + m_u} \sim \mathcal{O}(1) \tag{17}$$

where the masses are those of the three quarks. The first is in good agreement with experiment and relates the Cabibbo angle with the medium strong interactions which break $SU(3)$. The second, obtained by Cabibbo and Maiani, is more subtle: The prevailing philosophy was that isospin is an exact symmetry for strong interactions broken only by electromagnetic effects. In this case one would expect the mass difference in a doublet to be much smaller than the masses themselves. The second relation of (17) shows instead that isospin is badly broken in the quark masses and the approximate isospin symmetry in hadron physics is accidental, due to the very small values, in the hadronic mass scale, of m_u and m_d.

5.3. *The leading divergences*

The breaking of $SU(3) \times SU(3)$.

The leading divergences in the series (11) raised the spectrum of strangeness and parity violation in strong interactions. The first step was to find the conditions under which this disaster could be avoided.[44] The argument is based on the following observation: at the limit of exact $SU(3)\times SU(3)$ one can perform independent right- and left-handed rotations in flavour space and diagonalise whichever matrix would multiply the leading divergent term. As a result, any net effect should depend on the part of the interaction which breaks $SU(3)\times SU(3)$. In particular, one can prove that, under the assumption that the chiral $SU(3)\times SU(3)$ symmetry breaking term transforms as a member of the $(3, \bar{3})\oplus(\bar{3}, 3)$ representation, the matrix multiplying the leading divergent term is diagonal in flavour space, i.e. it does not connect states with different quantum numbers, strangeness and/or parity. Therefore, all its effects could be absorbed in a redefinition of the parameters of the strong interactions and

[i] S. L. Glashow's remark: "Few would concede so much sacrifice of elegance to expediency".

no strangeness or parity violation would be induced. This was first found for the one-loop diagrams and then extended to all orders. This particular form of the symmetry breaking term has a simple interpretation in the formalism of the quark model: it corresponds to an explicit quark mass term and it was the favourite one to most theorists, so it was considered a welcome result.

5.4. *The next-to-leading divergences*

Lepton–hadron symmetry — Charm.

The solution of the leading divergence problem was found in the framework of the commonly accepted theory at that time. On the contrary, the next-to-leading divergences required a drastic modification, although, in retrospect, it is a quite natural one.[45]

Let us first state the problem. A firmly established experimental fact is that flavour changing weak processes obey certain selection rules: One of them, known as the $\Delta S = 1$ rule, states that the flavour number, in this case strangeness S, changes by at most one unit. A second rule is that the allowed ΔFlavour$=1$ processes involve only charged currents. It follows that $\Delta S = 2$ transitions, as well as Flavour Changing Neutral Current processes (FCNC), must be severely suppressed. The best experimental evidence for the first is the measured $K_L - K_S$ mass difference which equals 3.48×10^{-12} MeV and, for the second, the branching ratio $B_{\mu^+\mu^-} = \Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to all)$ which equals 6.87×10^{-9}. It was this kind of tiny effects which led to the small value of the cut-off we mentioned earlier. In fact, this problem can be addressed at two levels. They are both easier to visualise in the framework of the quark model. At the limit of exact flavour symmetry, quark quantum numbers, such as strangeness, are not well defined. Any basis in quark space is as good as any other. By breaking this symmetry the medium strong interactions choose a particular basis, which becomes the privileged one. Weak interactions, however, define a different direction, which forms an angle θ with respect to the first one. Having only three quarks to play with, one can form only one charged current of the form postulated by Cabibbo:

$$J_\mu(x) = \bar{u}(x)\gamma_\mu(1+\gamma_5)[\cos\theta\ d(x) + \sin\theta\ s(x)]. \tag{18}$$

The expression (18) can be interpreted as saying that the u quark is coupled to a certain linear combination of the d and s quarks, $d_C = \cos\theta\ d + \sin\theta\ s$. The orthogonal combination, namely $s_C = -\sin\theta\ d + \cos\theta\ s$ remains uncoupled. Notice the difference with the leptonic current. We have four leptons, two neutrals, the $\nu_{(e)}$ and the $\nu_{(\mu)}$ and two negatively charged ones, the electron and the muon. They are all coupled and the weak current (5) has two pieces.

The first level of the problem is to consider a theory satisfying a current algebra. The neutral component of the current will be related to the commutator of J_μ and $J_\mu^\dagger$ and will contain terms like $\bar{d}_C d_C$, thus having flavour changing pieces. Notice again that this does not happen with the leptonic current. The commutator of the current (5) with its hermitian adjoint has no terms violating the two lepton flavour numbers. Phrased this way, the solution is almost obvious: we must use the s_C combination, but, in order to do so, we must have a second up-type quark. If we call it c, for *charm* the charged weak current (18) will have a second piece:

$$J_\mu(x) = \bar{u}(x)\gamma_\mu(1+\gamma_5)d_C(x) + \bar{c}(x)\gamma_\mu(1+\gamma_5)s_C(x) \tag{19}$$

or, in a matrix notation,

$$J_\mu(x) = \bar{U}(x)\gamma_\mu(1+\gamma_5)CD(x) \tag{20}$$

with

$$U = \begin{pmatrix} u \\ c \end{pmatrix}; \quad D = \begin{pmatrix} d \\ s \end{pmatrix}; \quad C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{21}$$

The important point is that, now, a current J_3, given by the commutator of J and $J^\dagger$, is diagonal in flavour space.

This solves the first level of the problem, but it is not enough to explain the observed rates. For example, the $K_L \to \mu^+\mu^-$ decay can be generated by the box diagram of Fig. 1 which, although of higher order in the weak interactions, it is quadratically divergent and contributes a term proportional to $G_F g_{\text{eff}}$.

Here comes the second ingredient of the mechanism. With a fourth quark, there is a second diagram, with c replacing u, Fig. 2.

In the limit of exact flavour symmetry the two diagrams cancel. The breaking of flavour symmetry induces a mass difference between the quarks, so the sum of the two diagrams is of order $g^4(m_c^2 - m_u^2)/m_W^2 \sim G_F(G_F m_c^2)$. Therefore, Ioffe and Shabalin's estimations can be translated into a limit for the new quark mass and

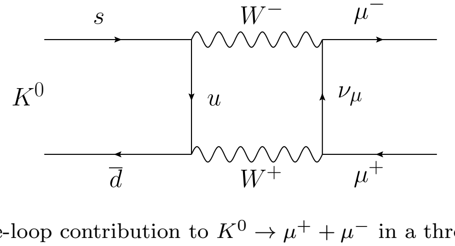

Fig. 1. The one-loop contribution to $K^0 \to \mu^+ + \mu^-$ in a three quark theory.

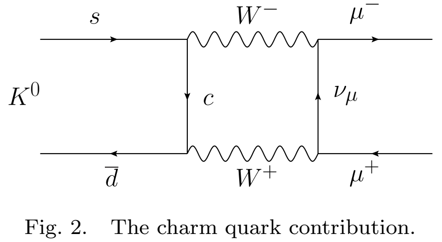

Fig. 2. The charm quark contribution.

yield an upper bound of a few GeV for the masses of the new hadrons. This fact is very important. A prediction for the existence of new particles is interesting only if they cannot be arbitrarily heavy.

In the early days of the Fermi theory there was a kind of symmetry between hadrons and leptons: proton–neutron *vs* neutrino–electron. The first discovery of heavy flavours appeared to break this symmetry: we had two new leptons, the muon and its associated neutrino, but only one new hadron, the strange quark. The introduction of the charmed quark restored this symmetry. By doing so, the mechanism obtained two important results: (i) It solved the technical problem of the low value of the Ioffe and Shabalin cut-off by replacing it with the masses of new hadrons. (ii) It opened the way to a formulation of the theory in terms of current operators which satisfy algebraic properties. It is this second result which allowed the use of Yang–Mills theories for the entire weak interactions, leptonic as well as hadronic.

6. The Standard Model

With the work done in the field theory front, and especially that of Gerard 't Hooft and Martinus Veltman, which is presented in a special Chapter in this Book, it became clear that weak *and* electromagnetic interactions are described by a gauge theory. The ball now was again in the phenomenology camp to decide which one.

6.1. *Which model?*

Do-it-yourself guide for gauge models.

We want to apply all the powerful machinery of gauge theories to the real world. The essential steps are the following:

- Choose a gauge group G.
- Choose the fields of the "elementary" particles whose interactions you want to describe and assign them to representations of G. Include scalar fields to allow for the Brout–Enblert–Higgs (BEH) mechanism.
- Write the most general renormalisable Lagrangian invariant under G. At this stage gauge invariance is still exact and all gauge vector bosons are massless.
- Choose the parameters of the BEH potential so that spontaneous symmetry breaking occurs. In practice, this often means to choose a negative value for a parameter μ^2.
- Translate the scalars and rewrite the Lagrangian in terms of the translated fields. Choose a suitable gauge and quantise the theory.

A remark: Gauge theories provide only the general framework, not a detailed model. The latter will depend on the particular choices made in steps 1) and 2).

6.1.1. *No neutral currents*

Today all experimental evidence points unmistakably to a single model, but this was not the case in the early days. In particular there was no evidence for the existence of weak neutral currents, so some models tried to avoid them. We mention two among them:

- The $SO(3)$ model.[46] The photon is the only neutral gauge boson, so the spontaneous symmetry breaking is $SO(3) \to U(1)$. The leptons belong to a triplet of $O(3)$, so we need heavy, positively charged partners of the electron and the muon. The hadronic sector is much more complicated. Even after the discovery of the weak neutral currents, this model has survived as a toy model because it has one interesting feature: the photon is a gauge boson of a simple Lie algebra. In this sense it is an elementary version of what we shall call *grand-unified theories* in the last section.
- A model without neutrino induced neutral currents.[47] This model had a short lifetime and it was proposed as a possible theoretical answer to a confusion regarding the existence of neutral currents in the early neutrino experiments.

6.1.2. *The $U(1) \times SU(2)$ model*

This is *the Standard Model*. It has four gauge bosons $W^{\pm}$, Z^0 and the photon. Following the notation which was inspired by the hadronic physics, we call T_i, $i = 1, 2, 3$ the three generators of $SU(2)$ and Y that of $U(1)$. Then, the electric charge operator Q will be a linear combination of T_3 and Y. By convention, we write: $Q = T_3 + \frac{1}{2}Y$. The coefficient in front of Y is arbitrary and only fixes the normalisation of the $U(1)$ generator relatively to those of $SU(2)$.

This ends our discussion of the first step. Regarding the choice of the matter fields, the model assumes a modular structure with the basic unit being a *family* of spin-1/2 chiral fermions. We start with the leptons for which the assignment in $SU(2)$ representations is as follows (the index i denotes the three families, the electron, the muon and the tau):

$$\Psi^i_L(x) = \frac{1}{2}(1+\gamma_5)\begin{pmatrix} \nu_i(x) \\ \ell^-_i(x) \end{pmatrix}; \quad i = 1, 2, 3. \tag{22}$$

The right-handed components are assigned to singlets of $SU(2)$:

$$\nu_{iR}(x) = \frac{1}{2}(1-\gamma_5)\nu_i(x) \quad (?) \quad ; \quad \ell^-_{iR}(x) = \frac{1}{2}(1-\gamma_5)\ell^-_i(x). \tag{23}$$

The question mark next to the right-handed neutrinos means that the presence of these fields is not confirmed by the data. We shall drop them in this Chapter, but there is a special one devoted to the neutrino masses. Notice that, with this assignment and in the absence of ν_R, the neutrinos will be massless and individual lepton numbers will be separately conserved.

For the hadrons, the model is written in terms of elementary quark fields. In order to explore the lepton–hadron universality property, it uses also doublets and singlets, but with some novel features, as compared to leptons:

- All quarks appear to have non-vanishing Dirac masses, so we must introduce both right-handed singlets for each family.
- Quark numbers are not individually conserved, so the formalism must allow for mixings among them, but not in the neutral current sector.
- We know that each quark appears in three species, called *colours*, so we have three times as many fields for each family.

Since left and right fields belong to different representations of $SU(2)$, all fermions are massless

$$Q_L^i(x) = \frac{1}{2}(1+\gamma_5)\begin{pmatrix} U^i(x) \\ D^i(x) \end{pmatrix}; \quad U_R^i(x); \quad D_R^i(x) \tag{24}$$

with the index i running over the three families as $U^i = u, c, t$ and $D^i = d, s, b$ for $i = 1, 2, 3$, respectively. An additional index a, running also through 1, 2 and 3 and denoting the colour, is understood.

We still have to introduce scalar fields for the mechanism of spontaneous symmetry breaking and the simplest choice is to have a doublet Φ containing a ϕ^+ and a ϕ^0 with the conjugate fields ϕ^- and ϕ^{0*} forming $\Phi^\dagger$.

These choices fully determine the model. What follows is straightforward algebra. We write the most general, renormalisable, Lagrangian, invariant under gauge transformations of $SU(2) \times U(1)$. The requirement of renormalisability implies that all terms in the Lagrangian are monomials in the fields and their derivatives and their canonical dimension is smaller or equal to four. The presence of the scalar fields generate Yukawa interactions with the fermions. The physical consequences of the model are obtained after spontaneous symmetry breaking and translation of the scalar field. We give a list of the most important ones:

- *Gauge boson mass terms.* The breaking is $U(1) \times SU(2) \to U(1)_{\rm em}$ with the generator of the electromagnetic group Q obtained as a superposition of T_3 and Y. As a result three gauge bosons become massive and the fourth one, the photon, remains massless. Let us call $\vec{W}$ and B the gauge fields associated to $SU(2)$ and $U(1)$ respectively and g and g' the corresponding coupling constants. After the breaking we obtain two charged bosons $W^\pm$ and two neutral ones, Z^0 and A, orthogonal combinations of W^3 and B:

$$Z_\mu = \sin\theta_W B_\mu - \cos\theta_W W_\mu^3; \quad A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3 \tag{25}$$

with $\tan\theta_W = g'/g$. They correspond to the mass eigenvalues

$$m_W = \frac{vg}{2}; \quad m_Z = \frac{v(g^2+g'^2)^{1/2}}{2} = \frac{m_W}{\cos\theta_W}; \quad m_A = 0 \tag{26}$$

where v is the vacuum expectation value of the scalar field.

• *Fermion masses.* They come from the Yukawa couplings between the fermions and the scalar field. They are given by:

$$\begin{aligned}\mathcal{L} = \sum_{i=1}^{3}\left[-G_i(\bar{\Psi}^i_L R_i\Phi + \text{h.c.}) + G^i_u(\bar{Q}^i_L U^i_R\tilde{\Phi} + \text{h.c.})\right]\\ + \sum_{i,j=1}^{3}\left[(\bar{Q}^i_L G^{ij}_d D^j_R\Phi + \text{h.c.})\right]\end{aligned} \tag{27}$$

where the summation runs over the three families. A further summation for the three quark colours is understood. $\tilde{\Phi}$ is the doublet made out of ϕ^{0*} and ϕ^-. It has the same transformation properties under $SU(2)$ as Φ, but the opposite Y charge.

Two remarks on this expression: (i) We have assumed the absence of right-handed neutrinos and this explains the fact that we have only one term for leptons. (ii) In the two terms for the quarks we have chosen to diagonalise the one referring to the up-quarks. This is a convention. As a result, the coupling of the second term involving the down-quarks is non-diagonal in flavour space.

After translation of the scalar field we obtain masses for all fermions with the exception of the neutrinos. The model does not predict their values, but allows to fit them. They are all proportional to the corresponding Yukawa coupling constant G. In addition, after diagonalisation of the down-quark masses, we obtain the Cabibbo–Kobayashi–Maskawa matrix[48] for the weak couplings:

$$\text{CKM} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_3 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta}\end{pmatrix} \tag{28}$$

with the notation $c_k = \cos\theta_k$ and $s_k = \sin\theta_k$, $k = 1, 2, 3$. The novel feature is the possibility of introducing the phase δ. This means that a six-quark model has a natural source of CP, or T, violation, while a four-quark model does not.

• $W^\pm$*-fermion couplings.* The charged W's couple to the usual $V-A$ weak current. The hadronic part involves the CKM matrix. The model was designed for that.
• *Photon-fermion couplings.* Again, we obtain the usual electromagnetic current.
• Z^0*-fermion couplings.* This is a new feature of the model. The weak neutral current is a particular mixture of right- and left-components given by the same parameter θ_W which enters in the gauge boson masses. For example, the Z^0-lepton

coupling is given by:

$$-\frac{e}{\sin(2\theta_W)}\left[\bar{\nu}_L\gamma^\mu\nu_L - \cos(2\theta_W)\bar{e}_L\gamma^\mu e_L + 2\sin^2\theta_W\bar{e}_R\gamma^\mu e_R\right]Z_\mu. \tag{29}$$

- *Vector boson self-couplings.* They are characteristic of the Yang–Mills structure of the theory. In particular, the coupling of the photon with the charged W's involve a single coupling constant e and gives very specific predictions concerning the electromagnetic parameters of the W's. The gyromagnetic ratio equals two and the quadrupole moment equals $-em_W^{-2}$.
- *Scalar field-fermion couplings.* They come from the Yukawa term (27) and give the most important signature of the model: the strength of the coupling is proportional to the fermion mass.
- *Scalar self-coupling.* Another prediction of the model is that the value of the mass of the physical scalar particle determines the strength of its self-coupling. With the value of 126 GeV we find $\lambda \sim 1/6$, appreciable, but according to our experience, still in the perturbative regime.

The agreement of the theory with experiment has been spectacular.

6.2. *A problem of anomalies*

An obscure higher-order effect determines the structure of the world.

Gauge invariant quantum field theories present a special feature as compared to other field theories, to wit gauge invariance. In order to define the theory one needs to *fix the gauge*, i.e. to impose some condition to eliminate redundant degrees of freedom. A change of gauge produces a completely new theory. All these theories which look, and in many respects are, very different, must give the same answer for physical quantities. This is achieved because they are linked together through a set of relations called *Ward identities.* They are the results, at the level of the Green functions, of the conservation equations for the symmetry currents of the theory. But this raises a new problem:

Let us consider the example of quantum electrodynamics and, for simplicity, neglect the electron mass. At the classical level the Lagrangian is invariant under separate phase transformations of the right- and left-components of the electron field. This $U(1)\times U(1)$ symmetry yields two currents, a vector and an axial:

$$j_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x); \quad j^5_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma^5\psi(x) \tag{30}$$

and, using the classical equations of motion, we see that they are both conserved. The problem arrives at the quantum level where we can prove that the two currents cannot be simultaneously conserved. It is the famous Adler–Bell–Jackiw[49] triangle anomaly[j] which tells us that, if we enforce the conservation of the vector current $J_\mu(x)$, the equation for the axial one becomes:

$$\frac{\partial}{\partial x_\mu} j^5_\mu(x) = \frac{e^2}{8\pi^2} \epsilon_{\nu\rho\sigma\tau} F^{\nu\rho}(x) F^{\sigma\tau}(x) \tag{31}$$

where e is the charge of the electron, $\epsilon_{\nu\rho\sigma\tau}$ is the completely anti-symmetric four index tensor which equals 1 if the indices form an even permutation of (0,1,2,3) and $F^{\nu\rho}$ is the electromagnetic field strength given by $F^{\nu\rho}(x) = \frac{\partial A^\rho(x)}{\partial x_\nu} - \frac{\partial A^\nu(x)}{\partial x_\rho}$ with $A^\nu(x)$ the electromagnetic vector potential. The r.h.s. of Eq. (31) is called *the axial anomaly*, which is a fancy way to say that the axial current of massless quantum electrodynamics is not conserved, contrary to what the classical equations of motion indicate.

This result has important physical consequences in particle physics but here we shall present only its implications for the electroweak theory. For quantum electrodynamics the non-conservation of the axial current can be considered as a curiosity because this current does not play any direct physical role. However, in the electroweak theory both vector and axial currents are important and in deriving the Ward identities we need the conservation of both. The axial anomaly breaks this conservation and the entire program collapses. As a result, the purely leptonic model, the one which was first constructed, is mathematically inconsistent.

The solution was first found in 1972.[50] The important observation is that the anomaly is independent of the fermion mass. Every fermion of the theory, light or heavy, contributes the same amount and we must add all contributions in order to get the right answer. For the electroweak theory this means that we need both the leptons and the quarks. A simple calculation shows that the total anomaly produced by the fermions of each family will be proportional to $\mathcal{A}$ given by:

$$\mathcal{A} = \sum_i Q_i \tag{32}$$

where the sum extends over all fermions in a given family and Q_i is the electric charge of the ith fermion. Since $\mathcal{A} = 0$ is a necessary condition for the mathematical consistency of the theory, we conclude that each family must contain the right amount of leptons and quarks to make the anomaly vanish. This condition is satisfied by the three colour model with charges 2/3 and $-1/3$, but also by other models such as the old Han–Nambu model which assumes three quark doublets with integer charges given by (1,0), (1,0) and (0,-1). In fact, the anomaly cancellation condition (32) has a wider application. The Standard Model could have been invented after the Yang–Mills theory was written, much before the discovery of the quarks. At that

[j] The term is slightly misleading, as it may give the impression that something contrary to common sense has happened. The real reason is that going from the classical equations to the quantum theory involves a series of steps which often include a limiting procedure, for example the limit of some parameter, the cut-off, going to infinity. This limit, although well defined, may not respect some of the symmetries of the classical equations.

time the "elementary" particles were thought to be the electron and its neutrino, the proton and the neutron, so we would have used one lepton and one hadron doublet. The condition (32) is satisfied. When quarks were discovered we changed from nucleons to quarks. The condition is again satisfied. If tomorrow we find that our known leptons and/or quarks are composite, the new building blocks will be required to satisfy this condition again. Since the contribution of a chiral fermion to the anomaly is independent of its mass, it must be the same no matter which mass scale we are using to compute it.

The moral of the story is that families must be complete.[k] Thus, the discovery of a new lepton, the tau, implied the existence of two new quarks, the b and the t, prediction which was again verified experimentally.

The above discussion was confined to the $SU(2) \times U(1)$ gauge theory but the principle of anomaly cancellation should be imposed in any gauge theory in order to ensure mathematical consistency. This includes models of strong interactions and grand-unified theories. H. Georgi and S.L. Glashow[51] found the generalisation of the anomaly equation (32) for a gauge theory based on any Lie algebra. It takes a surprisingly simple form:

$$\mathcal{A}_{abc} = \mathrm{Tr}\left(\gamma^5 \{\Gamma_a, \Gamma_b\} \Gamma_c\right) \tag{33}$$

where Γ_a denotes the Hermitian matrix which determines the coupling of the gauge field W_a^μ to the fermions through the interaction $\bar{\Psi}\gamma_\mu \Gamma_a \Psi W_a^\mu$. As we see, Γ_a may include a γ^5. Georgi and Glashow showed that the anomaly is always a positive multiplet of $\mathcal{A}_{abc}$, so this quantity should vanish identically for all values of the Lie algebra indices a, b and c.

Since gauge theories are believed to describe all fundamental interactions, the anomaly cancellation condition plays an important role not only in the framework of the Standard Model, but also in all modern attempts to go beyond, from grand unified theories to superstrings. It is remarkable that this seemingly obscure higher order effect dictates, to a certain extent, the structure of the world.

6.3. *The Standard Model becomes the Standard Theory*

The Standard Model wins all the battles.

The detailed comparison of the Standard Model with experiment will be shown in several Chapters of this Book. Obviously, in computing the theoretical predictions, one should include also the strong interactions, so the model is really the gauge theory of the group $U(1) \times SU(2) \times SU(3)$. Here we shall present only a list of the most spectacular successes in the electroweak sector.

[k]The title of the GIM paper was *Weak interactions with lepton–hadron symmetry.* With this work we showed that the title was indeed correct.

- The discovery of weak neutral currents by Gargamelle in 1972. Both, their strength and their properties were predicted by the Model.
- The discovery of charmed particles at SLAC in 1974–1976. Their characteristic property is to decay predominantly in strange particles.
- A necessary condition for the consistency of the Model is that $\sum_i Q_i = 0$ inside each family. When the τ lepton was discovered the b and t quarks were predicted with the right electric charges.
- The discovery of the W and Z bosons at CERN in 1983 was a remarkable achievement of experimental physics and accelerator technology. The characteristic relation of the Standard Model with an isodoublet scalar $m_Z = m_W / \cos\theta_W$ is checked with very high accuracy (including radiative corrections).
- The t-quark was *seen* at LEP through its effects in radiative corrections before its actual discovery at Fermilab.
- An impressive series of experiments have tested the Model at a level such that the weak interaction radiative corrections are important.
- The final touch: The recent discovery of the Brout–Englert–Higgs scalar.

All these successes give us full confidence that we have THE STANDARD THEORY of strong, electromagnetic and weak interactions of elementary particles.

7. Beyond the Standard Model

7.1. *Why and how*

We know why — we do not know how.

In spite of its enormous success, there are several reasons to suspect that the gauge theory of the Standard Model cannot be considered as a truly fundamental theory. Let us only mention some of its shortcomings.

- The family problem: why do we observe three, apparently similar, families of elementary fermions?
- The problem of masses. It is hard to believe that all these widely spread mass values are arbitrary parameters in a fundamental theory. This problem existed already with mass values such as m_e and m_t. It is accentuated with the values of the neutrino masses. We expect in a fundamental theory to be able to compute mass ratios.
- $U(1) \times SU(2) \times SU(3)$ is not a unified theory at all. Each group factor comes with its own coupling strength. Even worse is the presence of $U(1)$ because it allows for any number of coupling constants. We have already explained that in a non-Abelian group the coupling constant is fixed, but for $U(1)$ this is not so. In other words, the present theory does not explain why electric charge appears to be quantised and we do not see particles with charge πe. For the standard

model the observed very precise equality (up to one part in 10^{20}) of the electric charges of the proton and the positron, seems to be accidental.

- With the discovery of the scalar boson the Standard Model is complete. We can compute any quantity at any given order in the perturbation expansion. Following K. Wilson, we can fix a scale Λ and imagine that we integrate over all degrees of freedom with energies above Λ. We thus obtain an effective field theory describing the light, meaning lighter than Λ, degrees of freedom. Even without computations, we can guess the form of this effective theory by dimensional analysis. Integrating over the heavy degrees of freedom does not break any symmetry, so the effective theory will be a sum over all operators built out of the light fields consistent with the symmetries of the Standard Model. We can distinguish three classes of operators: (i) Operators whose dimension d_i is larger than four. Their contribution decreases as a power of Λ, so they become irrelevant for large Λ. (ii) Operators with $d_i = 4$. They are precisely the operators appearing in the Standard Model Lagrangian. (iii) Operators with $d_i < 4$. Their coefficients grow like positive powers of Λ, so they become dominant at large scales. In the Standard Model there are only two such operators: the unit operator $\mathbf{1}$ with dimension equal to zero and the operator Φ^2, where Φ is the scalar field, with dimension equal to two. The first contributes only to the induced cosmological constant which, in the absence of gravitational interactions, is not observable. We conclude that *the only dominant operator of the Standard Model is the mass term of the scalar boson.* It receives corrections which grow quadratically with the energy scale. This problem is often referred to as *the hierarchy problem* and it is a genuine instability of all generic quantum field theories involving scalar fields. This argument allows us to introduce the concept of *naturalness.* The underlying idea is that all physical theories are effective theories valid up to a certain scale, because we can never assume that we know physics at all scales. A quantum field theory will be called natural if the values of its parameters depend only logarithmically on this large energy scale. According to this definition, the Standard Model is not natural. It must be replaced by a different theory above a certain scale Λ.
- Last, but not least, the Standard Model leaves out the gravitational forces. Although, at present energies, the latter are very weak, we expect a fundamental theory to describe *all* fundamental interactions.

7.2. *The most beautiful speculations*

A personal choice.

If there are plenty of answers to the question *Why*, we are still in the dark concerning the question *How.* An old theoretical prejudice states that a better theory is a more symmetric one, so it is not surprising that most theoretical speculations aim at increasing the symmetry of the Standard Model. We give a short selection of theories, each one of which tries to address some of the problems mentioned above. None solves them all, which probably means that none is *the* correct theory.

7.2.1. *Grand unified theories*

The seed for grand unification can be found in the 1961 paper of Gell-Mann and Glashow.[34] In a footnote they write: The remarkable universality of the electric charge would be better understood were the photon not merely a singlet, but a member of a family of vector mesons comprising a simple partially gauge invariant theory. The first "realistic" grand-unified theories were proposed in the early seventies, just after the Standard Model was complete.[52]

The basic hypothesis of grand unification states that $U(1) \times SU(2) \times SU(3)$ is the remnant of a larger, simple or semi-simple group G, which is spontaneously broken at very high energies. The scheme looks like:

$$G \xrightarrow{M} U(1) \times SU(2) \times SU(3) \xrightarrow{m_W} U(1)_{e.m.} \times SU(3) \tag{34}$$

where the breaking of G may be a multistage one and M is one (or several) characteristic mass scale(s). As Gell-Mann and Glashow had observed, in order to explain electric charge quantisation, the charge operator should be a generator of G. Therefore it must be represented by a traceless matrix, which implies that the sum of the charges of the particles belonging to an irreducible representation must be equal to zero. This property is not true for either the known leptons, or the known quarks considered separately. We conclude that, unless we assume the existence of unknown exotic particles, an irreducible representation of G must include both leptons *and* quarks. This means that there exist gauge bosons of G which can change a lepton into a quark, or vice versa. Therefore, a generic prediction of GUT's can be stated as an alternative: new exotic particles, and/or violation of the separate conservation of baryons and leptons. An intense experimental effort has been devoted to the detection of a possible proton decay. The amplitude for such a decay is given by the exchange of the corresponding gauge boson and therefore, it is of order M^{-2}, where M is the gauge boson's mass. The resulting proton life-time will be of order:

$$\tau_p \sim \frac{M^4}{{m_p}^5}. \tag{35}$$

Using the experimental limit (for particular decay modes), of a few times 10^{33} years, we can put a lower limit on M:

$$M \geq 10^{16}\,\text{GeV}. \tag{36}$$

Grand unification is not a low-energy phenomenon!

Another general feature of grand unification concerns the three coupling constants of the Standard Model. At present energies they have very different numerical values. We can use the renormalisation group equations and follow their evolution as a function of the energy scale.[53] For the grand unification idea to be correct, they must reach roughly the same value at a scale of M. This property can be used to test each particular model and the result is not completely satisfactory. We shall come back to this in the next section.

A last general remark is that grand unified theories predict the existence of magnetic monopoles,[54] although their masses are of order M.

Several groups have been used for grand unification and an incomplete list includes:

- $SU(5)$. It is the simplest possible choice. The group of the Standard Model $U(1) \times SU(2) \times SU(3)$ is of rank four and we can prove that the only simple, or semi-simple, group of rank four which could be used for grand unification is $SU(5)$. The leptons of each family fill a $(10 + \bar{5})$ reducible representation. If a right-handed neutrino exists, it should belong to a singlet. There are 24 gauge bosons, 12 belonging to the Standard Model group and 12 new ones which, since they may mediate proton decay, they must be superheavy with masses of order M. The two step spontaneous symmetry breaking requires at least two distinct scalar field representations, the simplest system consisting of dimensions 24 and 5. We can immediately see the hierarchy problem we mentioned above: Let H and h denote the scalar fields of the 24 and 5 dimensional representations, respectively. They both get non-zero vacuum expectation values, the first $V \sim 10^{16}\,\mathrm{GeV}$ and the second $v \sim 10^{2}\,\mathrm{GeV}$. For this, the scalar potential must have terms M^2H^2 and μ^2h^2, with $M \sim V$ and, presumably, $\mu \sim v$. But if these terms are present, the term λH^2h^2 must also be present. Translating the field H by V will generate an induced mass term for h equal to λV^2. So, unless the coupling constant λ is of order 10^{-28}, the "light" field h will be pushed to the high mass scale, in other words the system is not capable to sustain naturally this hierarchy of widely separated mass scales. This is a problem with practically all grand unified theories.
- $SO(10)$. The simplest $SU(5)$ model does not fit the data very well and people have looked for higher groups. An attractive choice is the rank five group $SO(10)$. It has a sixteen dimensional spinorial irreducible representation capable of accommodating all chiral spinors of a family, including a right-handed neutrino. $SO(10)$ contains $SU(5)$ as a subgroup and the 16-plet decomposes under $SU(5)$ into $16 = 10 \oplus \bar{5} \oplus 1$. The proton decay prediction is similar to that of $SU(5)$. The main experimental prediction of $SO(10)$, which differs substantially from that of $SU(5)$, concerns the neutrino mass, but this problem will be addressed in another Chapter.
- The exceptional groups E_6 and E_8 have also been used. They offer many theoretical advantages but they have quite large representations (for example, the adjoint representation of E_8 has 248 dimensions), which means that a large number of up to now unknown particles are predicted.

7.2.2. *Supersymmetry*

Gauge theories contain three independent worlds. The world of radiation with the gauge bosons, the world of matter with the fermions and the world of BEH scalars. In the framework of gauge theories these worlds are essentially unrelated to each other. Given a group G the world of radiation is completely determined, but we have no way to know a priori which and how many fermion representations should be introduced; the world of matter is, to a great extent, arbitrary.

This arbitrariness is even more disturbing if one considers the world of BEH scalars. Not only their number and their representations are undetermined, but their mere presence introduces a large number of arbitrary parameters into the theory. Notice that this is independent of our computational ability, since these are parameters which appear in our fundamental Lagrangian. What makes things worse, is that these arbitrary parameters appear with a wild range of values. From the theoretical point of view, an attractive possibility would be to connect the three worlds with some sort of symmetry principle. Then the knowledge of the vector bosons will determine the fermions and the scalars and the absence of quadratically divergent counterterms in the fermion masses will forbid their appearance in the scalar masses. We shall call such transformations supersymmetry transformations and we see that a given irreducible representation will contain both fermions and bosons.[55] It is not *a priori* obvious that such supersymmetries can be implemented consistently, but in fact they can. The generators of the algebra contain operators Q which are fermionic with spin 1/2. The algebra closes using both commutators and anticommutators and, in its simplest version, takes the form:

$$[Q_\alpha, \bar{Q}_\beta]_+ = -2\gamma^\mu_{\alpha\beta}P_\mu \tag{37}$$

where P_μ are the generators of space–time translations.

There is a special Chapter in this Book devoted to supersymmetry,[56] so we will not go into any details here. We shall see there that supersymmetric field theories have remarkable renormalisation properties[57] which make them unique. In particular, they offer the only field theory solution of the hierarchy problem. Another attractive feature refers to grand unification. The presence of the supersymmetric particles modifies the renormalisation group equations and the effective coupling constants meet at high scales.

An interesting extension consists of considering gauge supersymmetry transformations, i.e. transformations whose infinitesimal parameters — which are anticommuting spinors — are also functions of the space–time point x. There are several reasons to go from global to local supersymmetry[58]:

- We have learned in the last years that all fundamental symmetries in nature are local (or gauge) symmetries.
- The supersymmetry algebra contains the translations. So local supersymmetry transformations imply local translations and we know that invariance under local

translations leads to general relativity which, at least at the classical level, gives a perfect description of the gravitational interactions.

- In the last section we saw that in a supersymmetric grand unified theory the unification scale approaches the Planck mass (10^{19} GeV) at which gravitational interactions can no more be neglected.
- The miraculous cancelation of divergences we find in supersymmetry theories raises the hope that the supersymmetric extension of general relativity will give a consistent quantum field theory. In fact local supersymmetry, or "supergravity", is the only field theoretic extension of the Standard Model which addresses the issue of quantum gravity.

Since the supersymmetry generators have spin 1/2, when applied to a state with spin projection s_z, they transform it into one with $s_z \pm 1/2$. A well-established theoretical prejudice is that in Nature there are no elementary particles with spin higher than 2. It follows that the maximum number of independent supersymmetry generators we can consider is $N = 8$.[59] The irreducible representation of one-particle states contains:

$$\begin{aligned}&1 \text{ spin-2 graviton}\\&8 \text{ spin-3/2 Majorana gravitini}\\&28 \text{ spin-1 vector bosons}\\&56 \text{ spin-1/2 Majorana fermions}\\&70 \text{ spin-0 scalars}\end{aligned} \tag{38}$$

$N = 8$ supergravity promised to give us a truly unified theory of all interactions, including gravitation and a description of the world in terms of a single fundamental multiplet. The main question is whether it defines a consistent field theory. At the moment we have no clear answer to this question, although it sounds rather unlikely. In some sense $N = 8$ supergravity can be viewed as the end of a road, the road of local quantum field theory. The usual response of physicists whenever faced with a new problem was to seek the solution in an increase of the symmetry. This quest for larger and larger symmetry led us to the standard model, to grand unified theories and then to supersymmetry, to supergravity and, finally, to the largest possible supergravity, that with $N = 8$. In the traditional framework we are working, that of local quantum field theory, there exists no known larger symmetry scheme. The next step had to be a very radical one. The very concept of point particle, which had successfully passed all previous tests, was abandoned. During the last decades the theoretical investigations have moved towards the theory of interacting extended objects.

This Chapter has touched so many subjects that a complete list of references is impossible. The selection is arbitrary and I have chosen either the original articles, or some which are not so well-known. I apologise for the numerous omissions.

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