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Link Importance Evaluation Based on Gray Relational Analysis for Communication Networks

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Abstract. *One of important research branches of communication networks is to evaluate their link importance, since the network vulnerability is of great importance in the presence of unexpected disruptive events or adversarial attacks targeting on critical links. As is well known, most of existing methods only consider one factor, but not the integration of multiple factors in evaluating critical links. In this paper, we first present three criteria to evaluate the link importance, and then adopt the gray relational analysis technique to combine these criteria to give an overall index to describe the link importance. Simulation results based on several test networks demonstrate the effectiveness of our method.*
Keywords: Communication networks, Critical links, Link importance, Multi-criteria, Gray relational analysis.

1. **Introduction.** To evaluate the importance of communication networks' links is one of important research topics of communication network reliability[1]. Due to factitious and natural factors, the links in communication networks are prone to failure, which affects the reliability of communication networks[2]. When multiple links fail simultaneously, we need to consider the order of repairs in order to make the suffered loss small. On the other hand, in the design of the network, we need to focus on certain links in the network maintenance and reduce their failures in order to improve the reliability of the entire communication network. Existing evaluation criteria of link importance can be classified into five categories, i.e., the shortest path based methods[4], the minimal cut sets and minimal path sets based methods[5], the reliability polynomial based method[6], the minimum spanning tree weight method, and the number of spanning trees based method. The shortest path based methods select the source and sink nodes stationarily and thus can only evaluate the importance of the links in the selected shortest path, but cannot evaluate the link importance in the sense of the entire network. Scholars have demonstrated that the minimal path sets - cut sets based methods are less accurate than the reliability polynomial based methods. But the reliability polynomial based

methods require the comparison of the link importance between each pair of links to produce the sorted link importance of the whole network, and sometimes we cannot get the importance relationship between certain links. The minimum spanning tree weight method presupposes the weights of the links, and therefore the link importance is closely related to the link weight.

At present, many researchers have proposed link importance evaluation methods based on the following evaluation criterion[7, 8], i.e., the most important link can be found if its deletion will cause the maximum reduction of network performance, such as the length of the shortest path or the number of spanning trees. However, the biggest problem of the deletion based evaluation methods is that when the deletion of the link makes the network unconnected or in the evaluation of series links, the importance of these links are identical, but from the intuitive judgment, there is difference between their importance. This paper presents a link importance evaluation method by combining three indices using gray relational analysis. We consider three indices from two aspects, one is based on link deletion, the other is based on edge betweenness[9]. With regard to link deletion, we considers the reduction ratio of the number of spanning trees and the increase ratio of the average shortest path[10].

The remainder of the paper is organized as follows. Section 2 introduces three indices for link importance evaluation. Section 3 introduces the gray relational analysis technique to combine three indices into a single overall index. Section 4 provides simulation results based on several test networks and compare the proposed method with the manual-weighted method. Section 5 concludes the whole paper.

2. Proposed Three Indices for Link Importance Evaluation.

2.1. Reduction rate of the number of spanning trees. For a give network $\mathtt{G} = (\mathtt{V}, \mathtt{E})$, where $\mathtt{V}$ is the set of vertices(nodes) and $\mathtt{E}$ is the set of links(edges), after deleting a link $\mathtt{e} \subseteq \mathtt{E}$, the number of spanning trees $\tau(\mathtt{G})$ will be definitely reduced. Thus, we can use the reduction rate of $\tau(\mathtt{G})$ to evaluate the importance of the deleted link. With regard to the number of spanning trees, based on the Laplacian matrix $\mathtt{L}$, we have the following theorem[11]:

Theorem 2.1. *Delete the information of any node from the $N \times N$ Laplace matrix $\mathtt{L}$, through finding the determinant of the rest $(N-1) \times (N-1)$ matrix $\mathtt{L}^*$, we can get the number of spanning trees of the given N nodes connected undirected graph $\mathtt{G}$. That is*

$$\tau(G) = \mathtt{det}(\mathtt{L}^*) \tag{1}$$

Thus, based on the above theorem, the time complexity required to compute the number of spanning trees of an undirected connected graph is $O(N^3)$. For an undirected connected graph $\mathtt{G}$, its Laplacian matrix $\mathtt{L}$ has the following relationship with its adjacency matrix $\mathtt{A}$:

$$\mathtt{L} = \mathtt{D} - \mathtt{A} \tag{2}$$

where $\mathtt{D}$ is a diagonal matrix, whose diagonal elements correspond to the degree values of nodes. Thus, the diagonal elements L_{ii} of the Laplacian matrix $\mathtt{L}$ are just nodes' degree values, while the remaining values $L_{ij} (i \neq j)$ is defined as: if Node i and Node j are adjacent, then $L_{ij} = -1$; if Node i and Node j are not adjacent, then $L_{ij} = 0$.

Thus, we can obtain the normalized link importance index as follows:

$$r(\mathtt{e}_{ij}) = 1 - \frac{\tau(\mathtt{G} - \mathtt{e}_{ij})}{\tau(\mathtt{G})} \tag{3}$$

where $r(\mathtt{e}_{ij})$ is the normalized importance of Link $\mathtt{e}_{ij}$, $\tau(\mathtt{G}-\mathtt{e}_{ij})$ is the number of spanning trees of $\mathtt{G}$ after deleting the edge $\mathtt{e}_{ij}$. The smaller the value $\tau(\mathtt{G}-\mathtt{e}_{ij})$ is, the greater the value $r(\mathtt{e}_{ij})$ is, the more influence of deleting the edge $\mathtt{e}_{ij}$ on the entire network is, and the more importance the link $\mathtt{e}_{ij}$ is. When the corresponding number of spanning trees $\tau(\mathtt{G}-\mathtt{e}_{ij})$ related to the link $\mathtt{e}_{ij}$ is zero, the graph after deleting the link is unconnected.

2.2. **Increase rate of the average distance.** To obtain the increase rate of the average distance, we first calculate the average distance of the network, denoted as $l(\mathtt{G})$. For an undirected graph, the average distance[12] is defined as the average value over all the distances between every two nodes. When an edge $\mathtt{e}_{ij}$ is deleted, we recalculate the average distance of the new network, denoted as $l_{\mathtt{e}_{ij}}(\mathtt{G})$. Thus, the increase rate of the average distance $D(\mathtt{e}_{ij})$ can be calculated as follows:

$$l(\mathtt{G}) = \frac{1}{N(N-1)} \sum_{m \neq n} d_{mn} \tag{4}$$

$$D(\mathtt{e}_{ij}) = \frac{l_{\mathtt{e}_{ij}}(\mathtt{G}) - l(\mathtt{G})}{l(\mathtt{G})} \tag{5}$$

Here, d_{mn} denotes the length of the shortest paths between Node $\mathtt{v}_m$ and Node $\mathtt{v}_n$, and N is the number of nodes. The larger the increase rate is, the more important the link is.

2.3. **Edge betweenness.** Edge betweenness[13] is a measure to quantify the ability of an edge in controlling the communication between nodes in a complex network. This index does not require deleting the link and can reflect the influence of the link on the transmission performance directly. Let p_{mn} denote the number of shortest paths between Node $\mathtt{v}_m$ and Node $\mathtt{v}_n$, $p_{mn}(\mathtt{e}_{ij})$ be the number of shortest paths between Node $\mathtt{v}_m$ and Node $\mathtt{v}_n$ which must pass through the edge $\mathtt{e}_{ij}$, and $B(\mathtt{e}_{ij})$ denote the edge betweenness of $\mathtt{e}_{ij}$. Then we have

$$B(\mathtt{e}_{ij}) = \sum_{m \neq n} \frac{p_{mn}(\mathtt{e}_{ij})}{p_{mn}} \qquad \mathtt{v}_m, \mathtt{v}_n \in \mathtt{V}, \mathtt{e}_{ij} \in \mathtt{E} \tag{6}$$

The larger the value $B(\mathtt{e}_{ij})$ is, the more important the edge $\mathtt{e}_{ij}$ is.

3. **Link Importance Evaluation Based on Gray Relational Analysis.** The gray relational analysis method[14] considers the difference or similarity among the trends of various factors and expects to adopt a special method to find the numerical relationship among them, which is an effective way to measure the degree of association among various factors. At the same time, this method can quantify the changes in the system, which is well suited to dynamic process analysis, and thus it is also consistent with the fact that complex networks are changing all the time. With regard to link importance evaluation with multiple indices, this method can be described as follows: Step 1. Suppose we use m (here, $m = 3$) indices to characterize the link importance and there are n links in the network. For the i-th link, its m indices are computed and stored in the following vector:

$$\boldsymbol{X}_i = (x_{i1}, x_{i2}, \ldots, x_{im}), 1 \leq i \leq n \tag{7}$$

Where $x_{ik}, 1 \leq k \leq m$ denotes the k-th index of the i-th link.

Step 2. After a comprehensive comparison of all links' indices, the following reference vector can be derived:

$$\boldsymbol{Y} = (y_1, y_2, \ldots, y_m) \tag{8}$$

Where $y_k, 1 \leq k \leq m$ denotes the best value among all links' k-th indices.

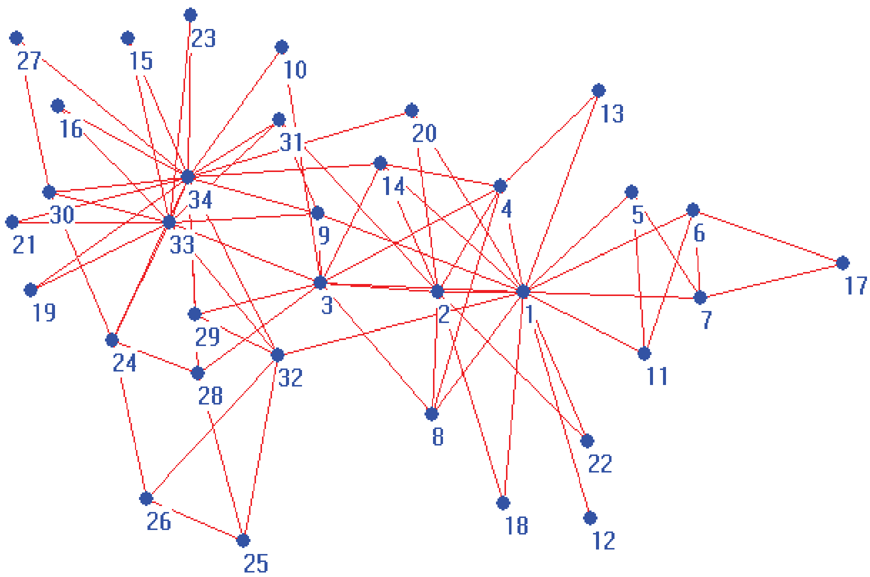

FIGURE 1. Zachary karate club network

Step 3. Since the dimensions of various indices are not necessarily the same and there may be significant differences among the magnitude orders of various indices, we should perform the non-dimensional operation on these indices. Here, we adopt the so-called "average method", that is, after the treatment, the i-th vector is output as follows:

$$\boldsymbol{X}_i^* = (\frac{x_{i1}}{ave_1}, \frac{x_{i2}}{ave_2}, \dots, \frac{x_{im}}{ave_m}), 1 \le i \le n \tag{9}$$

Similarly, the reference vector is processed as follows:

$$\boldsymbol{Y}^* = (\frac{y_1}{ave_1}, \frac{y_2}{ave_2}, \dots, \frac{y_m}{ave_m}) \tag{10}$$

Where ave_k represents the k-th$(1 \le k \le m)$ component of the mean vector $\overline{\boldsymbol{X}}$, calculated as follows:

$$\overline{\boldsymbol{X}} = (ave_1, ave_2, \dots, ave_m) = (\frac{1}{n}\sum_{j=1}^{n} x_{j1}, \frac{1}{n}\sum_{j=1}^{n} x_{j2}, \dots, \frac{1}{n}\sum_{j=1}^{n} x_{jm}) \tag{11}$$

Step 4. Calculate the difference matrix Δ as follows:

$$\Delta = \begin{pmatrix} |y_1^* - x_{11}^*| & |y_2^* - x_{12}^*| & |y_3^* - x_{13}^*| & \dots & |y_m^* - x_{1m}^*| \\ |y_1^* - x_{21}^*| & |y_2^* - x_{22}^*| & |y_3^* - x_{23}^*| & \dots & |y_m^* - x_{2m}^*| \\ |y_1^* - x_{31}^*| & |y_2^* - x_{32}^*| & |y_3^* - x_{33}^*| & \dots & |y_m^* - x_{3m}^*| \\ \vdots & \vdots & \vdots & \dots & \vdots \\ |y_1^* - x_{n1}^*| & |y_2^* - x_{n2}^*| & |y_3^* - x_{n3}^*| & \dots & |y_m^* - x_{nm}^*| \end{pmatrix} \tag{12}$$

And then find the maximum and minimum values in the matrix Δ, denoted as $\Delta_{\mathtt{max}}$ and $\Delta_{\mathtt{min}}$ respectively.

Step 5. Calculate the correlation coefficient as follows:

$$r_{ik} = \frac{\Delta_{\mathtt{min}} + \rho\Delta_{\mathtt{max}}}{|y_k^* - x_{ik}^*| + \rho\Delta_{\mathtt{max}}} \tag{13}$$

Where r_{ik} represents the correlation coefficient of the k-th $(k = 1, 2, 3, ..., m)$ index of the i-th $(i = 1, 2, 3, \dots, n)$ link, and ρ is the distinction coefficient between 0 and 1, usually we take $\rho = 0.5$.

TABLE 1. Comparison of link importance ranking between two methods for the Zachary network.

Rank	Proposed GRA method	MW method	Questionnaire
1	2-20	2-20	2-20
2	1-32	1-32	1-32
3	3-33	3-33	3-33
4	1-9	1-9	1-9
5	20-34	20-34	20-34
6	1-7	1-12	1-7
7	1-6	3-28	1-6
8	1-12	14-34	1-12
9	14-34	9-34	14-34
10	1-3	1-7	1-3
11	27-34	1-6	27-34
12	1-11	26-32	1-11
13	3-28	27-34	3-28
14	9-34	1-11	9-34
15	26-32	25-32	26-32
16	25-32	1-3	25-32
17	1-13	2-31	1-13
18	21-34	21-34	21-34
19	23-34	23-34	23-34
20	2-31	1-13	2-31
21	32-34	30-33	32-34
22	1-20	28-34	1-20

Step 6. Calculate the correlation degree, i.e., the overall index for each link as follows:

$$R_i = \frac{1}{m} \sum_{k=1}^{m} r_{ik} \tag{14}$$

Where R_i represents the correlation degree of the i-th link ($i = 1, 2, 3, \dots, n$. For convenience, we also use $R(\mathsf{e}_{ij})$ to denote the correlation degree of the link e_{ij}.

Step 7. Sort the links according to their correlation degree values in the descending order. The more the correlation degree is, the more important the corresponding link is.

4. **Simulation Results.** In our experiments, the proposed gray relational analysis (GRA) based method is compared with the manual-weighted (MW) method in estimating the link importance for different test networks. Here, the MW method sets the weights of different indices subjectively, and the weights of different criteria can be changed according to different cases, and the sum of all the weights is 1. In our experiments, we set three weights for $r(\mathsf{e}_{ij})$, $D(\mathsf{e}_{ij})$ and $B(\mathsf{e}_{ij})$ as 0.33, 0.33 and 0.34 respectively.

Experiment 1: Firstly, we choose the Zachary karate club network (Zachary network), which is widely used as a research example in complex network analysis. There are 34 nodes and 78 links in this network. In order to get the accurate comparative result, we obtain data from different methods, including questionnaire, the MW method and the proposed GRA method. The data collected by questionnaire are subjective. The results are shown in Table 1. From Fig. 1, we can see that the Zachary network can be divided into two parts, Node 1 and Node 34 are the corresponding centers of the two parts. Obviously, the links surrounded with the two centers should be more important. From Table 1, we can see that the link importance is ranked differently with different methods.

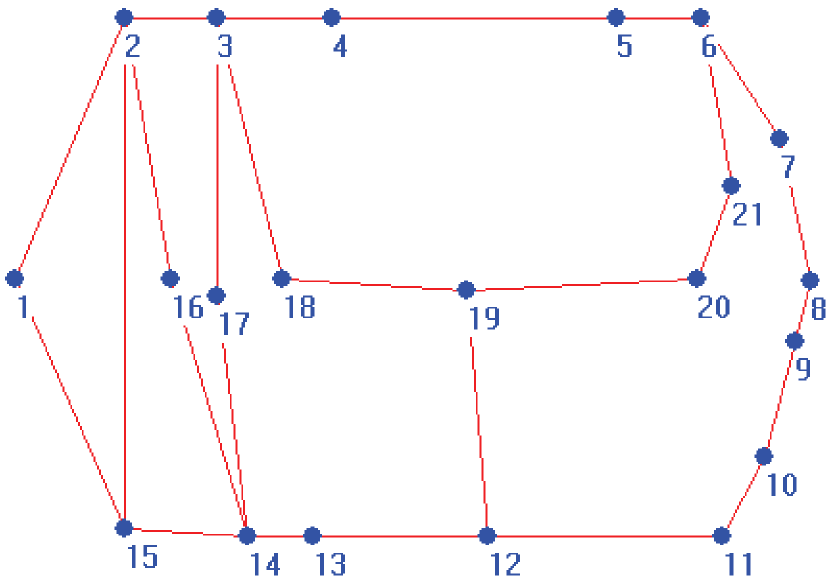

FIGURE 2. ARPA network

TABLE 2. Comparison link of importance ranking between two methods for the ARPA network.

Rank	Proposed GRA method	MW method
1	11-12	11-12
2	3-4	3-4
3	6-7	6-7
4	4-5	4-5
5	10-11	10-11
6	12-13	5-6
7	5-6	12-13
8	13-14	13-14
9	12-19	12-19
10	19-20	19-20
11	2-3	2-3
12	9-10	7-8
13	7-8	9-10
14	3-18	3-18
15	18-19	18-19
16	8-9	8-9
17	20-21	20-21
18	6-21	6-21
19	3-17	3-17
20	14-17	14-17
21	14-15	14-15
22	2-16	2-16

However, in the proposed GRA method, the important links overall prefer to the links around Node 1 and Node 34, such as Link 1-7, Link 1-6 and Link 1-12. Besides, in the proposed GRA method, if a link is connected with nodes of greater importance, the link will be ranked ahead. Therefore, for the Zachary network, from above ranking results, we can see that the proposed GRA method is more reasonable and effective than the MW method in most cases.

Experiment 2: In this experiment, we use the Advanced Research Projected Agency network (ARPA network) in Fig. 2, which is widely used as a test example in many researches. There are 26 links in this test network, the ranking results by two different methods are shown in Table 2. From Table 2, we can see that there are little difference between the proposed GRA method and the MW method. Because the ARPA network is an approximate symmetrical network, some links are very similar. We can see that, Link 12-13 is more close to the center of the network, and Node 12 is connected to Node 19, so there are more information flows through Link 12-13. That is to say, Link 12-13 is more important than Link 5-6. Thus, for the ARPA network, from above ranking results, we can see that the proposed GRA method is more reasonable and effective to evaluate link importance.

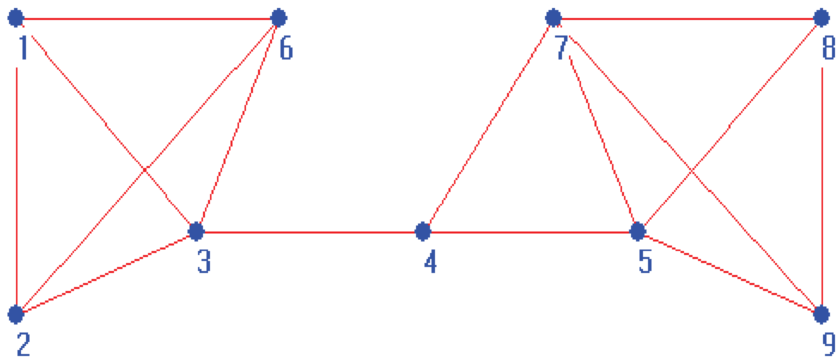

FIGURE 3. The nine nodes test network

TABLE 3. Comparison link of importance ranking between two methods for the nine nodes test network.

Proposed GRA method		**MW method**	
Link	**Overall index**	**Link**	**Overall index**
3-4	1.0000	3-4	1.0000
4-5	0.6613	4-5	0.3429
4-7	0.6613	4-7	0.3429
1-3	0.6312	1-3	0.2555
2-3	0.6312	2-3	0.2555
3-6	0.6312	3-6	0.2555
5-8	0.6146	7-9	0.1975
5-9	0.6146	5-8	0.1975
7-8	0.6146	7-8	0.1975
7-9	0.6146	5-9	0.1975
1-2	0.6057	1-6	0.1675
1-6	0.6057	1-2	0.1675
2-6	0.6057	8-9	0.1675
8-9	0.6057	2-6	0.1675
5-7	0.5981	5-7	0.1375

Experiment 3: In this experiment, we use a simple nine nodes network as shown in Fig. 3. There are 9 nodes and 15 links, and the network can be divided into two parts, and Node 4 connects the two parts. The results are in Table 3. We can see that Link 5-8, Link 5-9, Link 7-8, and Link 7-9 are ranked differently with different methods. From Fig. 3, we can see that Node 5 is more important than Node 7, and thus the links start with Node 5 will be more important than the links start with Node 7. That is to say, Link 5-8 and Link 5-9 should be ranked before Link 7-8 and Link 7-9, as obtained by the proposed GRA method.

5. **Conclusions.** This study mainly considers the multi-criteria based comprehensive ranking method for evaluating Link importance for communication networks using the gray relational analysis method. We characterize link importance based on three criteria. We do experiments to compare the gray relational method with the manual-weighted method using three different networks and get persuasive results. The gray relational analysis method gets a more reasonable ranking result, and it is an objective method which can avoid artificial deviation. However, further studies are still necessary to understand how to make reasonable weights in more complex and special networks.

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