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Using Risk Dominance Strategy in Poker

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Abstract. *Risk dominance strategy is a complementary part of game theory decision strategy besides payoff dominance. It is widely used in decision of economic behavior and other game conditions with risk characters. In research of imperfect information games, the rationality of risk dominance strategy has been proved while it is also wildly adopted by human players. In this paper, a decision model guided by risk dominance is introduced. The novel model provides poker agents of rational strategies which is relative but not equals simply decision of "bluffing or no". Neural networks and specified probability tables are applied to improve its performance. In our experiments, agent with the novel model shows an improved performance when playing against our former version of poker agent named* HITSZ_CS_13 *which participated Annual Computer Poker Competition of 2013. The macroscopical results and analysis of details all cofirms the effectiveness of works provided in this paper.*
Keywords: Poker, Imperfect Information, Risk Dominance. .

1. **Introduction.** Games can be classified as perfect or imperfect information conditions, which are based on whether or not players have whole information of the game [1]. In imperfect information games, certain relevant details are withheld from players. Poker is an interesting test-bed for artificial intelligence research on imperfect information games where multiple competing agents must deal with risk management, agent modeling, unreliable information and deception, much like decisionmaking applications in the real world [2].

The concept of risk dominance was formulated by John Harsanyi and Reinhard Selten in 1990 [3] as a complementary part of game theory decision strategy. Straub compared the use of risk dominance and payoff dominance as equilibrium selection criteria [4]. The research of Cooper consists that risk dominance supposes correct approach of dealing with imperfect information conditions [5]. David discussed the role of risk dominance in coordination games [6]. Heinemann provided further research under imperfect information condition [7]. In recent years, more researches are raising about risk dominance models in imperfect information games [8].

The rationality of risk dominance strategy is based on that the "optimal strategies" in imperfect information conditions is actually randomized strategies [9]. And also, the opponents in the game are certainly not "optimal player", having idiosyncratic weaknesses that can be exploited to obtain higher payoffs than Nash value of the game that follows just payoff dominance [10]. In Texas Hold'em, "bluffing" is a typical strategy with risk characters which conflicts from payoff dominance criterion but practiced by all experienced human players. Thus, the motivation of this paper is to build a risk dominance strategy decision model which provides more gently and rational solutions on strategy selection.

1.1. **Related works.** Represented by University of Alberta research group, many researchers study for building a world class poker player. The basic structure of Texas Hold'em system is as Figure 1 shows.

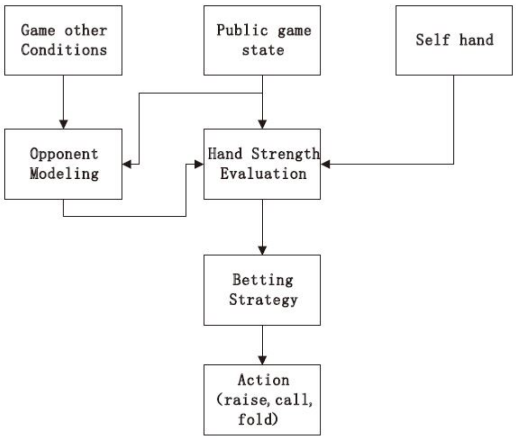


Figure 1. The basic structure of Texas Hold'em system

Generally speaking, there are several key components that decide the power of a computer poker player as: hand strength evaluation, hand potential, opponent modeling betting strategy, and bluffing strategy [2].

In poker, bluffing is to lie about one's hand strength. Given a weak hand, bluffing signals the opposite to opponents. Many researches consist that game should be arises not only from mathematical but also psychological problem. As Slansky theorized [11], bluffing should be attempted whenever the pot odds one gets by raising is favorable in relation to the probability that all other opponents fold. However, Salim insists that too much bluffing will results in a rapid loss of credibility [12]. In general poker agent framework, too much bluffing will obviously decrease your raising threshold in your opponents' prediction, which leads much more probability that opponent calling your bluff. Thus, bluffing should be done judiciously. Andrea presents a simple adaptive learning model of a poker-like game and shows how a bluffing strategy emerges very naturally and can also be rational and evolutionarily stable [13]. Magdalen discusses a solution of hands bluff using mix strategies as a sequential equilibrium based on von Neumann and Morgenstern's simple model of poker [14]. Salim provides his bluff strategy systematically [12]. A decision whether to bluff or not, provided the cost of "raising" is judged to be advantageous relative to the probability estimate that all other opponents will fold, is made by generating a random number and comparing it to the tan-h function output.

1.2. **Study motivation.** While contributes the theoretical and practical basement, most researches tread bluffing as an independent strategy. While studies on the rationality and mathematical method on bluffing, the final results is still a decision about "bluff or not". That is because when based on decision models guided by payoff dominance, bluffing strategy is tremendously ignored. For this motivation, the decision model of risk dominance strategy is discussed in this paper, which concerns "bluffing" as common strategies and raises them naturally and rationally.

This paper is organized as following. In section 2, decision model guided by risk dominance is introduced including the decision criterion suggested in this paper. Section 3 focuses on opponents' prediction that influences the effectiveness of this model. Neural network is adopted to predict income of risk dominance strategies and special probability tables are built to improve the accuracy further. Section 4 shows the experiments results in practice. And finally, Section 5 gives the conclusions.

2. **Decision model guided by risk dominance.**

2.1. **Introduction of risk dominance in game theory.** Risk dominance and payoff dominance are two related refinements of the Nash equilibrium (NE) solution concept in game theory, defined by John Harsanyi and Reinhard Selten. With the condition of imperfect information which means the low level of trust, the risk dominance comes to an important factor in the decision strategy that we should consider. Table 1 shows a typical example of risk dominance and payoff dominance. In Game 1 and 2 shown in table, row player (marked with player 1) has the payoff of low left corner in each cell and the line player (marked with player 2) has the payoff of up right corner. It is easily to judge that both the strategy pair (A, A) and (B, B) is strict Nash equilibrium. Because both the payoff of the two players in (A, A) is higher than in (B, B), the strategy A for both of the two players has payoff dominance.

Table 1. Payoff and risk dominance in two games

game 1		
player1,player2	A	B
A	8 , 8	4 , 3
B	3 , 4	6 , 6
game 2		
A	8 , 8	0 , 4
B	4 , 0	6 , 6

In the condition of game 2, (A, A) is still payoff dominant, but (B, B) is now risk dominant. That is because strategy B provides a more gently expectation for both players with the absence of credible of the other.

Harsanyi and Selten's risk dominance is based on what they refer to as a tracing procedure, the details of which are beyond the scope of this paper. As an alternative, Selten proposes a measure of risk dominance that is easy to calculate for our games [15], and is indicative of the outcome of the tracing procedure. Let u1 (X, X) be the payoff of player 1 with strategy pair (X, X), average log is used to weight the measure of risk dominance of the equilibrium (A, A) over (B, B).

$$R = \log \frac{u_1(A, A) - u_1(B, A)}{u_2(B, B) - u_2(A, B)} \tag{1}$$

If R is positive, Harsanyi and Selten's tracing procedure selects (A, A) as risk dominant. If R is zero, the mixed strategy Nash equilibrium is risk dominant. If R is negative, (B, B) is risk dominant. Notice that any affine transformation of the payoffs in the game would not change either the sign or the magnitude of R.

With this criterion we can explain the experimental results of these games. In game 1, for player 1 we can calculate by (1) that R of player 1 is Log (2.5) , so (A, A) is both payoff and risk dominance. In game 2, R of player 1 changes to Log(0.67) which is a negative value, so the strategy B has risk dominance.

Generally speaking, low credibility of opponents can strengthen the influence of risk dominance. In uncoordinated games, the credibility comes from the accuracy of opponents' actions' predictions, which is markedly higher in perfect information games than in imperfect information games. That is the reason that Cooper [5] consists that risk dominance supposes correct approach of dealing with imperfect information conditions.

2.2. **Risk dominance decision model in Poker.** In imperfect information games, many researches are focused on the prediction of unknown information. Take Texas Hold'em for example, the prediction methods of opponent's card strength and future actions have been well studied. Comparing with this, decision model of betting strategy are comparatively consistent in researches. The "raising" ratio is proportional with the comparison between the hand strength of us and opponents' in prediction. One of the distinctions is using pure strategy with methods of mathematical statistics or mix strategy with random or sampling method. The following formula simplifies but not conflicts with most betting strategy decision model.

$$Raiseratio = \frac{\tanh(EHS_{self} - EHS_{opponent}) * scaleFactorX}{scaleFactorY} + delZ \tag{2}$$

In formula 2, hyperbolic tangent function is used to describe the relationship between "raising" ratio and the players' hand strength. EHS, defined by Billings [2], denotes the card strength of both players. scaleFactorX, scaleFactorY and delZ are control factors to compress and shifts the tanh curve to fit within 0 and 1. Based on decision models guided by formula 2 or similar modes, the probability of "raising" should be maximum when hand strength is high, and decreases as hand strength decreases down to a minimum.

On the other side, researchers who appreciate "bluffing" strategy consider that having bigger hand strength is not the only way to win the game. The same purpose can also be attached by leading your opponent to "fold" which is no relationship with what cards you hold. The following figure shows a further description.

Figure 2 shows a common situation in Texas Holdem. We suppose the money previously put into pot is y. Each time of "raising" will cost Player A or B of x. E_A and E_B donate the hand strength of the two players. The bottom of the decision tree provides the payoffs in different conditions. Thus, suppose the probability of $E_A > E_B$ is p, and B means other influence factors excludes opponent's previous action. Let G_{Ar} , G_{Ac} and G_{Af} donates the expected gains of player A when he adopts strategies of raise, call or fold. denotes the condition probability of player B's strategy. For example, means the probability of player B "raising" under the condition of player A's "raising" this turn and other environment factors donated by B. Thus, for player A, the expected payoffs of the three strategies can be calculated as following:

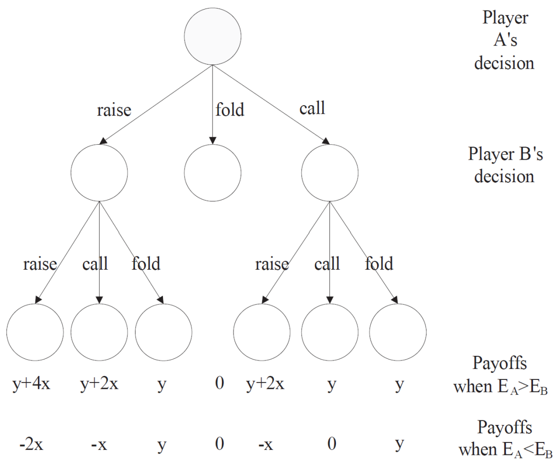


FIGURE 2. Decision tree of a common condition in Texas Hold'em

$$G_{Ar} = [p(y+4x)+(1-p)(-2x)]*P(r_B|r_A,B)+[p(y+2x)+(1-p)(-x)] \\ *P(c_B|r_A,B)+y*P(f_B|r_A,B) \tag{3}$$

$$G_{Ac} = [p(y+2x)+(1-p)(-x)]*P(r_B|c_A,B)+(p*y)*P(c_B|c_A,B)+y*P(f_B|c_A,B) \tag{4}$$

$$G_{Af} = 0 \tag{5}$$

Formula 3 5 show the expected gains for different strategies of player A. The conditions can be also described in table 2.

TABLE 2. Payoff matrix of Texas Hold′em poker

player A and B	raise	call	fold
raise	$[p(y+4x)+(1-p)(-2x)]$ $*P(r_B\|r_A,B)$	$[p(y+2x)+(1-p)(-x)]$ $*P(c_B\|r_A,B)$	$y*P(f_B\|r_A,B)$
call	$[p(y+4x)+(1-p)(-2x)]$ $*P(r_B\|r_A,B)$	$[p(y+2x)+(1-p)(-x)]$ $*P(c_B\|r_A,B)$	$y*P(f_B\|r_A,B)$
fold	0	0	0

Table 2 shows expected payoff of player A as payoff matrix mode so that classic game theory methods can be used and risk dominance strategy can be explored as section 2.1 introduced.

$$R = log\left[\frac{Gsi}{Gsj}\right]\begin{cases} s_i \quad dominances_j \quad if \quad R>0 \\ s_j \quad dominances_i \quad if \quad R<0 \\ s_i \quad equalss_j \quad if \quad R=0 \end{cases} \tag{6}$$

If $s_i \in S$, $\forall sj \in S$ s_i dominance s_j, then s_i is the optimal strategy of the risk dominance decision model recommended in this paper. Otherwise, if more than single optimal strategy exits, mixed strategy is sampled by this decision model.

2.3. **Summary.** In summary, payoff dominance provides a gently approach of strategy selection. It guides the model recommended in this paper to a balance between ignorance and irrationally abuse of "bluffing" characteristic strategies. In this model, none of the selected strategies are classified as "bluffing" or not but all of them are rational under risk dominance.

3. **Prediction of opponent actions.** Accurate prediction of game conditions is the crucial factors of the previous decision model's performance. Using methods of opponent modeling has been proved to make two distributions effectively. One is to predict how likely the opponent is to take a particular action given the current state. Formula description is $P(a_n|a_1a_{n-1}, B)$ in which ai donates the ith players' action and B means other influence factors in current condition. It can be specified as $P(r_B|r_A, B)$ and similar terms used in formula 3 5. The other distribution is the estimation of the strength of opponent, which can be described as $P(E_B|a_1a_{n-1}, B)$. By comparing it with self strength E_A, probability p used in formula 3 5 can be estimated.

3.1. **Neural network for predicting opponents' actions and hand strength.** As a popular and sophisticated approach, neural networks are adopted to build opponent models in our system. The structure of network for predicting opponent is shown in figure 3. In our network, 14 input nodes are used to describe current conditions and history data of the game. The input nodes can be classified as three groups. First is about the actions frequency token by players in this game. Second is about the current conditions of the game which contains the stage of game processes, the conditions of public cards and so on. The third group is about the history data statistics in our database.

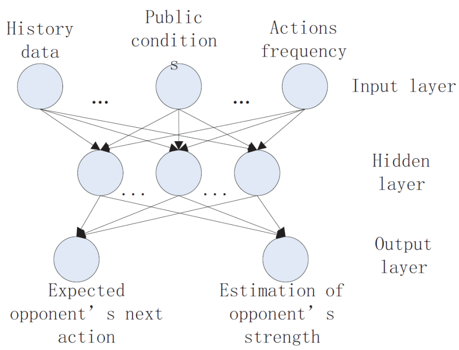


FIGURE 3. Neural network for predicting opponent

For training the network, game data are collected from former matches of ACPC (Annual Computer Poker Competition). About 800,000 data of game are used as the training set. The created network contains 12 nodes in hidden layer and shows a train correct ratio of 81.66 in our test data set.

The neural network is the black-box nature and the output node of expected opponent's action from the neural network represented the most likely action from the probability triple, not the probability triple itself. For building probability table of action frequencies, an ergodic process of all possible cards arrangements is used as the input of the neural network. The statistical result of the output is recorded and partial of the table are listed in table 3.

Table 3 shows the action prediction model of one opponent. Based on these works, the decision model can be supported to prediction of opponents as a general probability

TABLE 3. Action sequence statistical times and probability table of "marv", who is the top project in ACPC-13

Action times		Action probability	
opponent call	2410512	$P(r\|r)$	0.1466089
opponent raise	5110788	$P(c\|r)$	0.6951249
raise after raise	749287	$P(f\|r)$	0.1582662
call after raise	3552636	$P(r\|c)$	0.7277591
fold after raise	808865	$P(c\|c)$	0.2722409
raise after call	1754272	$P(f\|c)$	0
call after call	656240		
fold after call	0		

table. For improve the accuracy further, the probability table is statistical integrated as a specified manner classified by opponents and conditions with high relative characters.

3.2. **Building special probability tables.** Based on the built neural network, relationship between environment conditions and player's actions can be systematically analyzed. For example, probability of opponent's "fold" after my "raise" is interested by our risk strategy decision model. Making same input of opponent's hand cards, game stages and player's actions frequency ended by "raise", the relationship between opponent's willing of "fold" and the conditions of public cards can be observed by a traverse of all possible arrangement of current public cards. Although all inputs of neural network influence the final results, three factors, which are described in following figures, are chosen as the classify factors of special tables.

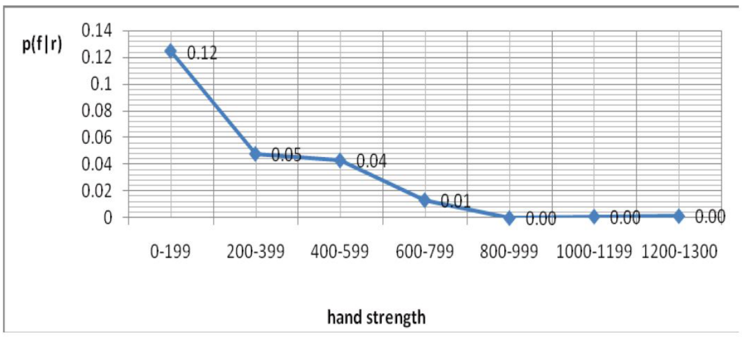


FIGURE 4. Changing tendency of $p(f|r)$ with the value of EHS_{self}

The results of these experiments reveal some accordant regulations of common cognition. Firstly, comparison of participants' hand strength is always key point of betting strategy decision. The more likely player estimates he has a bigger strength than his opponents, the less likely he will take "fold", and the reverse is also true. Secondly, the player's aggressiveness shows obvious reciprocal ratio with potential strength of public cards. For example, player will certainly take "raise" when he hold a double "A" in most cases. However, the ratio of "call" risen when public cards show like "3, 4, 7" style. That is because the possible of opponent's holding "5, 6" makes player more cautious. Thirdly, players are more likely to "fold" when chips in pot are little. Based on these factors, the general model can be classified to special as figure 7 shows.

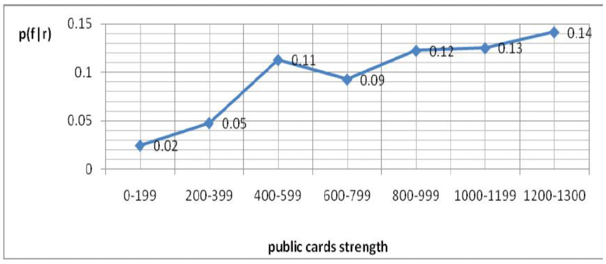


Figure 5. Changing tendency of $p(f|r)$ with potential strength of public cards

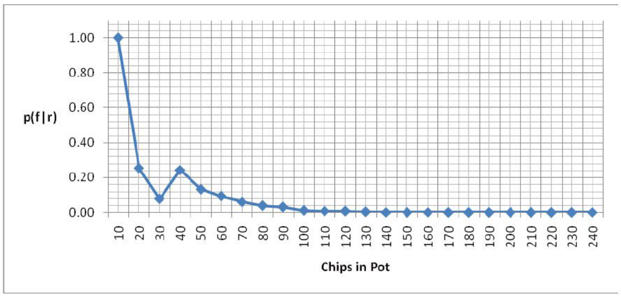
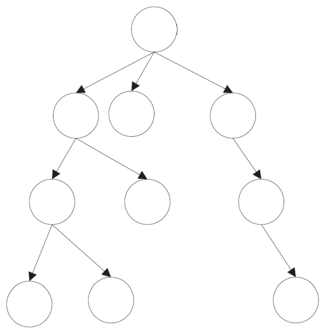


Figure 6. Changing tendency of $p(f|r)$ with amount chips in pot



Class 1	Class 2	…	Class m*n*k
P_1 (r|r)	P_2 (r|r)		P_{mnk} (r|r)
P_1 (c|r)	P_2 (c|r)		P_{mnk} (c|r)
P_1 (f|r)	P_2 (f|r)		P_{mnk} (f|r)
P_1 (r|c)	P_2 (r|c)		P_{mnk} (r|c)
P_1 (c|c)	P_2 (c|c)		P_{mnk}(c|c)
P_1 (f|c)	P_2 (f|c)		P_{mnk} (f|c)

Figure 7. Building classified probability model

We divided each factor as m, n and k intervals thus the general probability table can be divided into $m * n * k$ classes. The proper values of m, n and k can be adjusted by the running computer environment and response time request in practice

The classified probability tables specified different conditions to improve the precise of prediction which support our decision model. In practical competitions, real-time conditions are analyzed or predicted to map the probability model to a special one firstly.

And then, the decision model will work depending on it. Finally, the probability table is updated timely based on the results of the revealed conditions of past round.

4. **Experiments.** Annual Computer Poker Competition (ACPC) provides a very good test platform for poker researches [16]. We have participated ACPC 2013 with our program named HITSZ_CS_13 which got the fourth place in 3-player Limit Texas Holdem finally. Thus, HITSZ_CS_13 is chosen as the reference to examine the performance of agents guided by risk dominance strategy model.

Our experiment is set as a tournament mode of 3-player Limit Texas Hold'em. Each player's wealth is initialed as 0 and there is no limit of player's holding money. This means that the player's wealth can become a minus value and the tournament can be played to any set scale of rounds.

The participants in the experiment are set as following. Hitsz13 is the previous version of our poker system which participate ACPC 2013 and shows a good performance in 3-player Limit Texas Hold'em. In this version, classical methods are also used like neural networks, opponent modeling model and so on. Risk_g is guided by risk dominance strategy model with general probability table. Risk_g, which is guided by risk dominance strategy model with specified probability table, is tested as the recommended system of this paper.

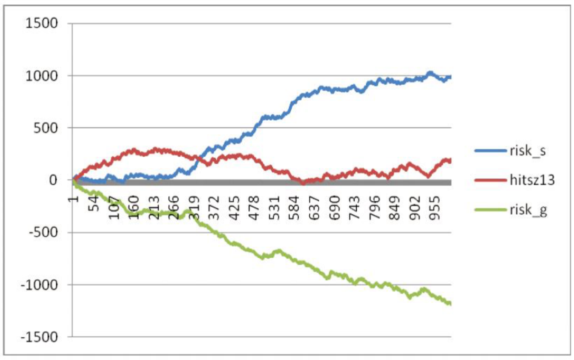


FIGURE 8. 1000 rounds' performance of different poker agents in 3-player Limit Texas Hold'em

Figure 8 shows the experiment's results of 1000 rounds game of the three tested poker agents. The Y-axis shows the changing bankroll of each agent along with game processes. In the beginning 300 rounds, Hitsz13 shows advantages over the two versions of risk strategy agents. However, Risk_s agent asserts its superiority over the hitsz13 by its growing accuracy of opponent's prediction. In another side, Risk_g agent is totally defeated by upper two versions of agents and loses its money rapidly.

Figure 9 shows the same set of experiments' results of 3000 rounds, which shows similar tendencies of the three agents.

The following tables analyzed the details of the agent's strategies' selection influenced by risk dominance, which reveals the exact performance of the novel decision model.

TABLE 4. Decision details of agents guided by risk strategy model

agent	raise ratio	different ratio	"$f\|r$" ratio	success rate	Finial payoff
Hitsz13	0.273	0	0.0428	0.5130	398
Risk_s	0.235	0.0713	0.0991	0.8704	554
Risk_g	0.249	0.1125	0.1007	0.2698	-428

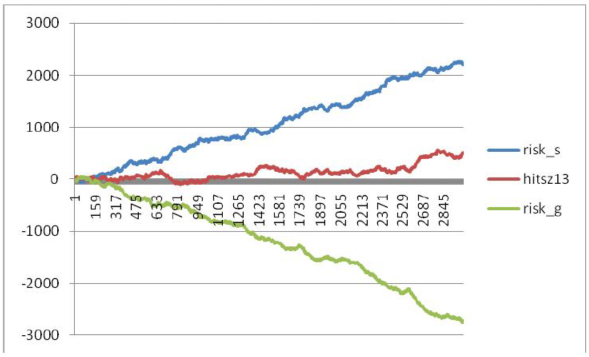


Figure 9. 3000 rounds' performance of different poker agents in 3-player Limit Texas Hold'em

For the purpose of better observing the difference decision guided by risk dominance strategy model, decision model of agent HITSZ_CS_13 is also applied in the new system as a reference. In our experiments, the two decision models are making strategies independently and recoded when they decides differently. Table 4 shows the statistical results of 3000 rounds experiments.

In table 4, Risk_s and Risk_g show 7.13 percent and 11.25 percent ratio that risk dominance strategy model makes different decisions from former version. Both of them get more than double times that lead their opponent to "fold" by self "raise". However, Risk_s performs a much higher success rate of its recorded risk strategies than Risk_g does (87.04 percent:26.98 percent). Thus, Risk_s gains extra 554 money from the different strategies but Risk_g loses 428 in total 3000 rounds games.

Another aspect deserves attention is besides the increase of "$f|r$" ratio, the total raise ratio of all strategies is not increased but a little reduces in Risk_s and Risk_g. This result reflects the contribution of the smooth influence of risk dominance strategy. While considering the strategies that influences on opponents, the additional loss cost by these strategies are also calculated in our decision model to prevent the abuse of risk character strategies.

In the end, the performance of risk dominance strategy model is greatly influenced by accuracy of opponents' prediction. Based on precise prediction of opponent, risk dominance can provide more keen strategies in conditions of divergence exits between risk and payoff dominance decision criterion. However, when the prediction is unreliable, the things will progress to the opposite. Table 5 and 6 show the "confusion matrix" that describe prediction accuracy of the two versions of agents. Higher prediction accuracy of 8 percent leads the better performance of Risk_s.

Table 5. Prediction accuracy of Risk_g Prediction

Actual	Fold	Call	Raise	%
Fold	12.25	0.16	0.41	12.82
Call	3.50	47.65	10.28	61.43
Raise	2.37	7.15	16.22	25.74
%	18.12	54.96	26.91	76.12

Table 6. Prediction accuracy of Risk_g Prediction

Actual	Fold	Call	Raise	%
Fold	19.66	0.25	0.20	20.11
Call	1.96	52.31	11.39	65.66
Raise	0.05	2.18	12.00	14.23
%	21.67	54.74	23.59	83.97

5. **Conclusions.** In this paper, a novel decision model guided by risk dominance is introduced. Based on the analysis and experiments on this model, at least two points can be concluded in this paper.

Firstly, decision model guided by risk dominance can provides rational strategies that well balance between ignorance and irrationally abuse of risk characteristic strategies like "bluffing". In our experiments, risk dominance strategy model shows 7 percent different decisions from payoff dominance model Hitsz13 and 87 percent of them gain better payoff. Secondly, the effectiveness of risk dominance strategy model is greatly reliable to the accuracy of opponents' prediction. Agent Risk_s agent asserts its superiority over the other version of risk dominance guided agent Risk_g for its higher accuracy.

The further work of our study will be mainly focused on improving accuracy of our prediction methods. The neural network will be further studied. Classification characters that used for building more specified opponent models are also important for our research.

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