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A Linear Metric Reconstruction by Complex Eigen-Decomposition *

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Abstract

This paper proposes a linear algorithm for metric reconstruction from projective reconstruction. Metric reconstruction problem is equivalent to estimating the projective transformation matrix that converts projective reconstruction to Euclidean reconstruction. We build a quadratic form from dual absolute conic projection equation with respect to the elements of the transformation matrix. The matrix of quadratic form of rank 2 is then eigen-decomposed to produce a linear estimate. The comparison of results of our linear algorithm to results of bundle adjustment, applied to sets of synthetic image data having Gaussian image noise, shows reasonable error ranges.

1 Introduction

Recent theoretical developments in projective geometric structure from motion algorithms showed that a minimal constraint like zero skew or known aspect ratio provides sufficient information to upgrade the result of projective reconstruction to Euclidean one up to an unknown global scale [15, 8, 5] in the case that the motion of the camera is sufficiently general so that the configuration of the viewing geometry does not belong to special degenerate cases like pure translation or pure rotation [14, 2, 12, 1, 13, 10].

In contrast to the theoretical advances, approaches of practical estimation have relied on linear approximations or non-linear iterative methods, and Euclidean bundle adjustment is eventually needed to minimize a geometrically meaningful error to get an optimal Euclidean reconstruction [8]. Hartley's algorithm of infinity searching provided another practical method of metric reconstruction [4]. Mendoça and Cipolla utilized the constraint of equal singular values of the essential matrix to compute the internal camera parameters [7]. Seo and Heyden proposed a linear iterative formulation from the orthogonality constraint [11].

We propose here a linear algorithm with the assumption of varying internal parameters with zero skew camera. The dual absolute conic projection equation [15] is re-formulated to make a quadratic equation for each of the projective camera matrices and the matrix of the quadratic equation is eigen-decomposed to build a linear equation to compute the projective-to-Euclidean transformation matrix.

2 Background on Auto-Calibration

The camera is modeled by the well known camera equation

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \gamma f & sf & u \\ 0 & f & v \\ 0 & 0 & 1 \end{bmatrix} [\, \boldsymbol{R} \mid -\boldsymbol{R}\boldsymbol{t} \,] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \Leftrightarrow$$
$$\lambda \boldsymbol{x} = \boldsymbol{K}[\, \boldsymbol{R} \mid -\boldsymbol{R}\boldsymbol{t} \,]\boldsymbol{X} = \boldsymbol{P}\boldsymbol{X} \ . \tag{1}$$

Let us assume that $\boldsymbol{P}_i$ and $\boldsymbol{X}$ are the obtained projective reconstruction. Then there exists a projective transformation $\boldsymbol{T}$ such that $\boldsymbol{P}_j\boldsymbol{T} \sim \boldsymbol{P}_j^{\mathcal{E}} = \boldsymbol{K}_j\boldsymbol{R}_j[\boldsymbol{I} | -\boldsymbol{t}_j]$, where $\boldsymbol{R}_j$ is rotation, $\sim$ denote equality up to scale and $\boldsymbol{K}_j$ is the appropriate intrinsic parameters. Now, let $\tilde{\boldsymbol{T}}$ denote the 4×3 matrix consisting of the first three columns of $\boldsymbol{T}$, giving $\boldsymbol{P}_j\tilde{\boldsymbol{T}} \sim \boldsymbol{K}_j\boldsymbol{R}_j$. Multiplying this with its transpose gives $\boldsymbol{P}_j\tilde{\boldsymbol{T}}\tilde{\boldsymbol{T}}^{\mathrm{T}}\boldsymbol{P}_j^{\mathrm{T}} \sim \boldsymbol{K}_j\boldsymbol{R}_j\boldsymbol{R}_j^{\mathrm{T}}\boldsymbol{K}_j^{\mathrm{T}} = \boldsymbol{K}_j\boldsymbol{K}_j^{\mathrm{T}}$. Define $\boldsymbol{\Omega} = \tilde{\boldsymbol{T}}\tilde{\boldsymbol{T}}^{\mathrm{T}}$, i.e. the dual to the absolute conic, and $\boldsymbol{\omega}_j = \boldsymbol{K}_j\boldsymbol{K}_j^{\mathrm{T}}$, i.e. the dual to the image of the absolute conic. Then the image of the dual absolute conic (absolute quadric) can be written as

$$\boldsymbol{\omega}_j \sim \boldsymbol{P}_j\boldsymbol{\Omega}\boldsymbol{P}_j^{\mathrm{T}} \ . \tag{2}$$

3 A Linear Estimation Background

Now let us consider the following:

Problem 1 *Find the solution vector $\boldsymbol{x}$ from N equations of the form*

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{M}_j\boldsymbol{x} = 0, \quad j = 1, ..., N, \tag{3}$$

provided that $\boldsymbol{M}_j$ is of rank 2, not positive nor negative definite, asymmetric, normal, *and defined up to a nonzero scale.*

If $\boldsymbol{M}$ is normal, then there exists a unitary matrix $\boldsymbol{U} \in \mathbb{C}^{n\times n}$ such that

$$\boldsymbol{U}^{\mathrm{H}}\boldsymbol{M}\boldsymbol{U} = \boldsymbol{D} = \mathrm{diag}(\lambda_1, \ldots, \lambda_n) \tag{4}$$

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where U^{H} is the complex conjugate transpose of U [3]. Let u_n be the column representation of the n-th row vector of U. Decomposition of M_j gives a simplified form:

$$\begin{aligned} 0 &= x^{\mathrm{T}} U_j \mathrm{diag}(\lambda_{j1}, \lambda_{j2}, 0, ..., 0) U_j^{\mathrm{H}} x \qquad (5)\\ &= \lambda_{j1} y_1^2 + \lambda_{j2} y_2^2, \qquad (6)\end{aligned}$$

where y_1 and y_2 are the elements of $y = U_j^{\mathrm{H}} x = [u_{j1}^{\mathrm{T}} x, u_{j2}^{\mathrm{T}} x, ..., u_{jn}^{\mathrm{T}} x]^{\mathrm{T}}$. From this equation, we have

$$z_j^{\mathrm{T}} x \equiv \left(u_{j1}^{\mathrm{T}} \pm \sqrt{\frac{\lambda_{j2}}{\lambda_{j1}}} u_{j2}^{\mathrm{T}} \right) x = 0. \qquad (7)$$

This gives us $2N$ complex linear equations. In matrix form it can be written as follows

$$\mathbf{0} = Zx = [z_1, z_2, ..., z_N]^{\mathrm{T}} x = (A + iB)x, \quad i = \sqrt{-1}, \qquad (8)$$

where A and B are respectively real and imaginary part of Z. Note that all the scalars belong to complex field $\mathbb{C}$. Finally, SVD of $W = [A^{\mathrm{T}}, B^{\mathrm{T}}]^{\mathrm{T}}$ will provide a linear solution of x.

4 DACP for Zero-Skew Camera

The dual absolute conic projection (DACP) equation is given by [15]

$$\omega_j = \rho K_j K_j^{\mathrm{T}} = P_j \Omega P_j^{\mathrm{T}}, \quad j = 1, \ldots, N. \qquad (9)$$

If we assume that the skew of the camera is zero, the image $K_j K_j^{\mathrm{T}}$ of the dual absolute conic are of the forms:

$$K_j K_j^{\mathrm{T}} = \begin{bmatrix} (\gamma f_j)^2 + u_j^2 & u_j v_j & u_j \\ u_j v_j & f_j^2 + v_j^2 & v_j \\ u_j & v_j & 1 \end{bmatrix}. \qquad (10)$$

Let p_j^k is the k-th row vector of $P_j = [p_j^{1\mathrm{T}}, p_j^{2\mathrm{T}}, p_j^{3\mathrm{T}}]^{\mathrm{T}}$. Then the DACP equation becomes

$$\omega_j = P_j \Omega P_j^{\mathrm{T}} = \left[p_j^m \Omega p_j^{n\mathrm{T}} \right]_{3\times 3} \quad \text{for } m, n = 1, 2, 3. \qquad (11)$$

Now the (m, n)-th component of ω_j is $p_j^m \Omega p_j^{n\mathrm{T}}$ which can be re-written as a vector inner-product of two 10-D vectors $b_{m,n}$ and $\underline{\Omega}$:

$$p_j^m \Omega p_j^{n\mathrm{T}} = b_{m,n}^{\mathrm{T}} \underline{\Omega}. \qquad (12)$$

From the equations (9)–(11), we know that

$$u_j = \frac{b_{1,3}^{\mathrm{T}} \underline{\Omega}}{b_{3,3}^{\mathrm{T}} \underline{\Omega}}, \quad v_j = \frac{b_{2,3}^{\mathrm{T}} \underline{\Omega}}{b_{3,3}^{\mathrm{T}} \underline{\Omega}}, \quad \text{and} \quad u_j v_j = \frac{b_{1,2}^{\mathrm{T}} \underline{\Omega}}{b_{3,3}^{\mathrm{T}} \underline{\Omega}}. \qquad (13)$$

Removing u_j and v_j from the last equation using the first two, we have the final form

$$\underline{\Omega}^{\mathrm{T}} \left[b_{3,3} b_{1,2}^{\mathrm{T}} - b_{1,3} b_{2,3}^{\mathrm{T}} \right] \underline{\Omega} = \underline{\Omega}^{\mathrm{T}} M_j \underline{\Omega} = 0, \qquad (14)$$

where $M_j = b_{3,3} b_{1,2}^{\mathrm{T}} - b_{1,3} b_{2,3}^{\mathrm{T}}$. Note that the matrix $M_j' = b_{3,3} b_{1,2}^{\mathrm{T}} - b_{2,3} b_{1,3}^{\mathrm{T}}$ also satisfies Equation (14). From a symbolic computation and algebraic reasoning, we can find that *the matrix M_j is of rank 2; $M_j + M_j^{\mathrm{T}}$* has rank 4.

5 How to compute Ω.

In practice, M_j is not normal; however, even though M_j is not normal actually, we may compute an eigen-decomposition $M_j U = U \mathrm{diag}(\lambda_1, \lambda_2, 0, ..., 0)$, which can be applied for the computation of Ω, instead. Here is our practical algorithm:

1. Given image matches, compute projective camera matrices P_j and structure X_k using the iterative factorization of [6] and projective bundle adjustment.
2. For every projective camera matrix P_j, build four 10×10 matrices M_j, M_j', M_j^{T}, $M_j'^{\mathrm{T}}$(Equation (14)).
3. Construct eight 10-D row vectors z from eigen-decomposition for each of the four matrices. The conjugate transpose U^{H} of U in Equation (5) is now replaced by U^{-1}.
4. Construct the matrix $Z \in \mathbb{C}^{8N \times 10}$ using z's. A linear solution of $\underline{\Omega}$ is given from SVD of $[A^{\mathrm{T}}, B^{\mathrm{T}}]^{\mathrm{T}}$ where A and B are real and imaginary part of Z.
5. Build 4×4 symmetric matrix Ω from the vector $\underline{\Omega}$. Then, compute T

   $$T = V \mathrm{diag}(\sqrt{\sigma}_1, \sqrt{\sigma}_2, \sqrt{\sigma}_3, 0) \qquad (15)$$

   from SVD of Ω:

   $$\Omega = V \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3, \sigma_4) V^{\mathrm{T}}, \quad \sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \sigma_4, \qquad (16)$$

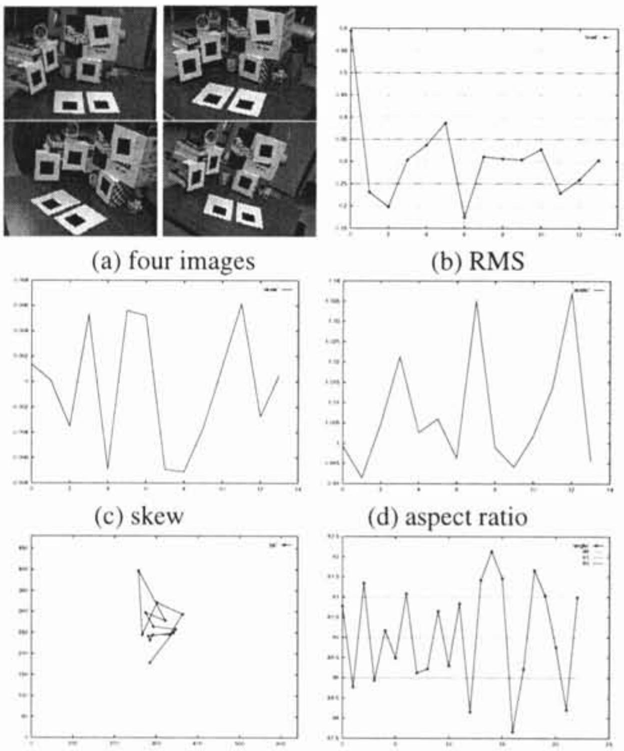
   and set the $(4, 4)$-th element of T to 1.
6. Upgrade projective quantities to Euclidean:

   $$X_k^{\mathcal{E}} = T^{-1} X_k \quad \text{and} \quad P_j^{\mathcal{E}} = P_j T. \qquad (17)$$

   Compute calibration matrix K_j and motion R_j, T_j through QR decomposition of the first 3×3 sub-matrix of $P_j^{\mathcal{E}}$.

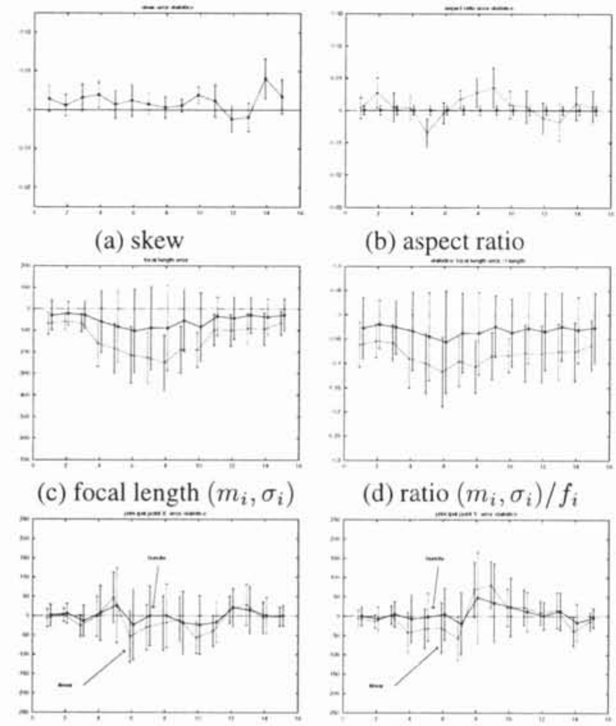
6 Real Experiment

Figure 1 shows the result of linear metric reconstruction. The first real experiment exploits image matches from 14 views. Image matches are obtained from the intersection points of the edges of black squares. Figure 1(b) is the plot of RMS re-projection errors for each image



(a) four images (b) RMS (c) skew (d) aspect ratio (e) principal points (f) angle measurements

Figure 1: (a) Four from 14 images. Image size is 640×480. (b) RMS errors for each view after projective bundle adjustment (24 point matches). (c) skew (d) aspect ratio (e) principal points (f) angle measurements of the orthogonal angles. The true angles are 90 degrees.



(a) skew (b) aspect ratio (c) focal length (m_i, σ_i) (d) ratio $(m_i, \sigma_i)/f_i$ (e) x coord. of principal point (f) y coord. of principal point

Figure 2: Statistics of simulation results after 100 runs when input noise level is 0.5pixel for each of the image coordinates. Bold lines (—) are for bundle adjustment and dashed lines (- - -) for the linear algorithm. Vertical line segments denote one standard deviation ranges from the mean values.

after projective bundle adjustment. Total RMS becomes 0.32pixel. Initial projective reconstruction is obtained using the method of Heyden *et. al.* [6]. Figure 1(c) shows the computed skew values for each of the cameras. Its range is ± 0.008 which was assumed to be zero. Figure 1(d) gives aspect ratios whose range is $0.99 \sim 1.04$. The location of the image center is $(320, 240)$ and the principal points of Figure 1(e) are around the image center. Finally the result of angle estimation of the metric reconstructed 3D points is shown in Figure 1(f); their errors are distributed within $\pm 2^\circ$ from true angle 90°.

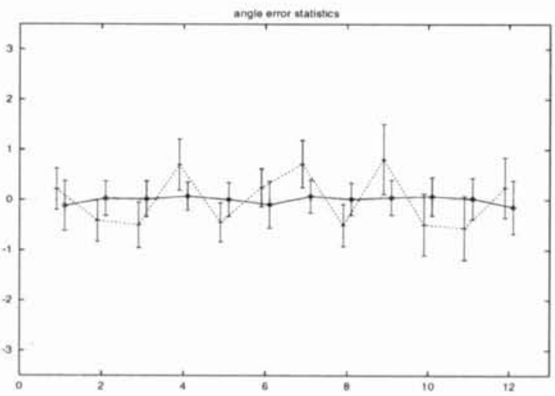
More experiments can be found in [9], and from these real experiment, we come to see that our linear algorithm yields a reasonable calibration parameters as well as Euclidean motion parameters.

7 Simulation with Synthetic Data

We generated a set of synthetic matching data of 15 views. We added Gaussian noise of standard deviation $\sigma_{\text{noise}} = 0.5$pixel to image coordinates of the synthetic data. By repeating noise addition to the original data set, we generated 100 contaminated data sets to which the linear algorithm and Euclidean bundle adjustment algorithm were respectively applied.

Figure 2 shows the statistics of the 100 computations; mean and standard deviation of each of the estimation errors – skew, aspect ratio, focal length, principal point – with respect to the true calibration parameter values are shown in each of the plots. For example, Figure 2(a) is the plot of the error of skew values. When we applied Euclidean bundle adjustment, all the skew values were zero; but the linear algorithm shows biases, maximally 0.008, from the truth (zero) and standard deviation of 0.005 at maximum. Figure 2(d) shows ratio of statistics of estimation error of focal length to its true value; this is to compensate for the change of focal lengths. Compared to the results of bundle adjustment, these error range graphs show the feasibility of the linear algorithm.

Figure 3 shows mean and standard deviations of twelve angle measurements whose true angle values are 90°. The bias of mean error is within $\pm 1^\circ$, which supports the feasibility of the linear algorithm.



$\sigma_{\text{noise}} = 0.5\text{pixel}$

Figure 3: Statistics for the measurement of angle errors after 100 runs. Vertical line segments denote one standard deviation ranges from the mean values.

8 Discussions and Conclusion

A novel linear metric reconstruction algorithm is proposed; the real and synthetic experiments provide sufficient support for the feasibility of the algorithm.

Our future research to make a full mathematical treatment of the algorithm; in spite of the reasonable experimental results, it is true that we are short of full rigorous mathematical analysis.

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