

## FUZZY RELATIONAL MODEL WITH 3-D STRUCTURAL INFORMATION OF OBJECTS

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### ABSTRACT

An approach is proposed that gets a fuzzy relational model of objects by taking 3-D geometrical information of objects obtained from stereo vision as inputs. To solve the scale problem occurred in quantitative representation of 3-D structural information of objects and lower the matching cost, a fuzzy relational description language (FRDL) is defined in this paper that represents the 3-D geometrical relations of surface pairs of objects qualitatively. By FRDL, the quantitative values of 3-D information of objects could be smoothly transformed into qualitative values without losing the 3-D information of objects largely. This FRDL also serves as the intermediate description between the quantitative geometric values obtained from the vision process and the abstract symbolic description utilized in the machine learning process. With concept learning method, such fuzzy relational model can be generalized and a fuzzy-based 3-D generic model of objects can be learned.

Some examples of object houses show the effectiveness of proposed methods.

### 1. INTRODUCTION

3-D object recognition is one of the major challenge in computer vision<sup>[1,2]</sup>. This task is often carried out through the model-based recognition method that matches the features extracted from object images to the 3-D geometric model of objects. However, it is impossible in practice, to put all kinds of the geometric models of objects into the database. Moreover, as the number of objects increases, the cost of matching time is also growing greatly. In order to avoid using exact metric models in a 3-D object recognition system, many attempts have been made at obtaining a generic model that can represent the objects of the same class abstractly. A canonical example of such work is ACRONYM system<sup>[1]</sup> that uses volumetric primitives (known as GEONS) as the representation of 3-D object models. However, not all kinds of objects can be represented in such way.

On the other hand, learning concept from examples has been one of the most widely studied problem in artificial intelligence. These learning algorithms, however, assumed highly abstracted symbolic description as inputs and often dealt with simple objects like arch in the blocks world<sup>[3]</sup>.

In order to fill up the gap between model acquiring from image data and concept learning with symbolic description. We have proposed a integrated system that obtains generic model of objects by using concept learning method with image data from vision as input<sup>[4]</sup>. In such system, we offered a concept description language(CDL),

which represents the geometrical features of surfaces and relations between surfaces of the objects qualitatively to solve scale problem of the quantitative values. However, this method just divided the quantitative values of features and relations into several qualitative intervals in a very simple way. This made the qualitative representation be too coarse and resulted in the loss of the useful information.

To solve these problems, we make the improvement on the qualitative concept description language(CDL) by introducing the fuzzy relational description language (FRDL) to the system. With this FRDL, the quantitative values of 3-D information of objects could be smoothly transformed into qualitative values without losing the 3-D information of objects largely. In this paper, we make the following original contributions:

- A fuzzy relational description language (FRDL) that represents 3-D relations of surface pairs of objects qualitatively. It can solve the scale problem of quantitative values obtained from stereo vision and also serves as the intermediate description between the quantitative geometric values obtained from the vision process and the abstract symbolic description utilized in the machine learning process;
- A fuzzy-based 3-D generic model of objects that can be learned with such FRDL representation, by taking 3-D structural information of objects obtained from stereo vision as input.

Some examples of object houses show the effectiveness of proposed methods.

### 2. SYSTEM CONFIGURATION

The system configuration is shown in Fig.1. Trinocular vision instead of binocular vision is used to obtain 3-D information of objects and solve the correspondence problem occurred in the binocular vision<sup>[5]</sup>. Fig.3 shows the 3-D information of objects obtained from the input image shown in Fig.2. With image understanding process based on geometrical rules<sup>[6]</sup>, the 3-D hierarchical structure of objects is obtained (see Fig.4). These 3-D structural information of objects are represented by fuzzy relational description language (FRDL) qualitatively and used as inputs for learning process of fuzzy-based generic model.

In the learning part of the system, the quantitative values of 3-D structural information of an example object are first transformed into qualitative description with FRDL. By this, the initial fuzzy model of objects is generated. In our paper, we take the house objects as example objects. In this case, the basic parts of houses such as roof, wall are taught interactively and the initial fuzzy relational model of an object is generated in the form of semantic network which has fuzzy confidence for arc parts and FRDL representation for node parts of such network. Then, the initial fuzzy model is refined by matching the examples of

objects to it fuzzily, and the final fuzzy-based generic model(structural concept) is obtained as the output of the system as shown in Fig.5.

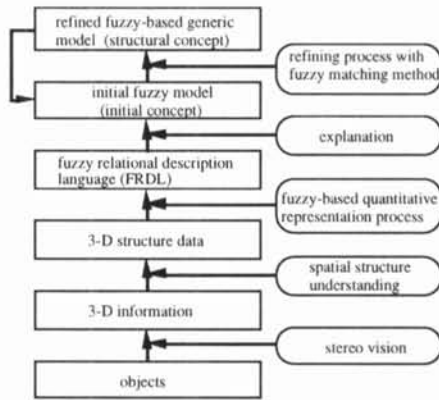


Fig.1 The System Configuration

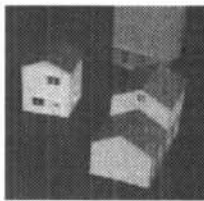


Fig.2 The Image of Objects

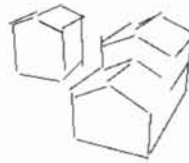


Fig.3. 3-D information of Objects

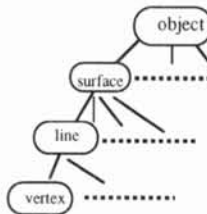


Fig.4 3-D Hierarchical Structure of Objects

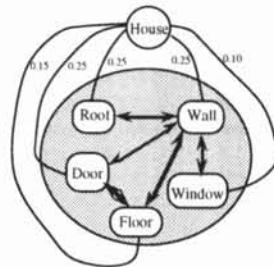


Fig.5 Fuzzy-based Generic Model of Houses

### 3. FUZZY RELATIONAL DESCRIPTION LANGUAGE(FRDL)

Using the quantitative values of 3-D structural information of objects as the concept description will cause scale problem and make it difficult to obtain a generic model because quantitative values are too detailed description for symbolic machine learning. Therefore, we transform the quantitative values of 3-D structural information to their qualitative description. Here, we take the surfaces of objects as the fundamental elements and define the fuzzy relational description language (FRDL) on the surface relations.

The elementary formulas used in our FRDL is written as  $[L=R]$  where  $L$  represents the 3-D relations between surface pairs and  $R$  represents the qualitative values corresponding to  $L$ . The FRDL representation defined in our paper is shown in Table 1.

In Table1, the domain values of CoEdge and CoSurface relations are defined respectively as those shown in Fig.6 and Fig.7. From Fig.6 and Fig.7, we know that:

(I) for the same qualitative domain value, for example, the domain value Contain of CoEdge relation shown in (a) of Fig.6, the degree of its qualitative property is still different like those shown in (a1), (a2) and (a3) of Fig.6.  
(II) for different domain values, for example, the domain values Contain and Equal of CoEdge relation shown in (a) and (c) of Fig.6, there still exist some cases in which the domain values of them are similar, or approximately same like those of (a3) and (c1) shown in Fig.6.

The similar results could also be found in Fig.7. Such difference and similar properties on domain values have been considered in this paper to solve the problem of being too coarse in qualitative representation with CDL[4].

Table 1 Fuzzy Relational Description Language(FRDL)

classification	Function Symbol	Domain Value
Connected Relation	CoEdge(X1,X2)	{Contain,Cross,Equal}
	Angle(X1,X2)	{Parallel, Acute, Perpendicular, Obtuse}
Coplanar Relation	CoSurface(X1,X2)	{Contain,Cross,Independence,Equal}
Spatial Relation	Spatial R(X1, X2)	{ Upper,Same, Lower }

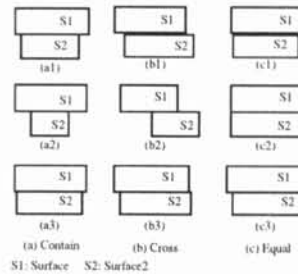


Fig.6 Domain value of CoEdge Relation between Two Surfaces

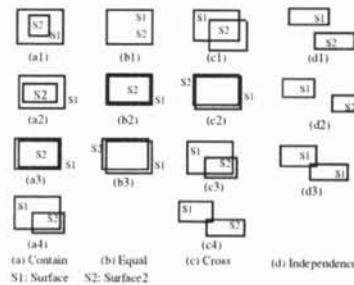


Fig.7 Domain value of CoSurface Relation between Two Surfaces

### 4. FUZZY MEMBERSHIP FUNCTIONS FOR DOMAIN VALUES OF FRDL

In this section, we give the definitions of fuzzy membership functions for each qualitative domain values of surface relations in FRDL, and then present the formulas to acquire such fuzzy values.

(a) Fuzzy domain value of Angle relation

The fuzzy membership function for domain value of Angle(X1,X2) is defined as Fig.8. Here, the domain values Perpendicular and Parallel are not just for the angle  $90^\circ$ , the angle  $0^\circ$  or  $180^\circ$ , they are allowed to have some bias around these degrees. Such kind of fuzzy domain values are flexible for qualitative description.

(b) Fuzzy domain value of CoEdge and CoSurface relations

The fuzzy membership function for domain values of CoEdge(X1,X2) and CoSurface(X1,X2) are shown in

Fig.9 and Fig.10. The fuzzy domain values are calculated with formulas (1-1), (1-2), (1-3) and (2-1), (2-2), (2-3) respectively. The meaning of symbols in these formulas can be seen in Appendix of this paper.

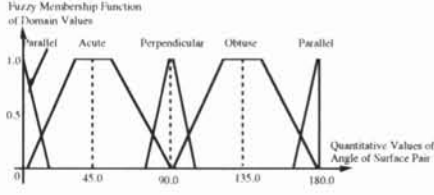


Fig.8 Fuzzy Domain Values of Quantitative Values of Angle Relation

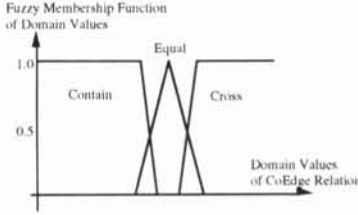


Fig.9 Fuzzy Domain Values of Quantitative Values of CoEdge Relation

$$F(\text{Contain}) = \begin{cases} 1 & \frac{(L_{MAX} - L_{MIN})}{L_{\gamma}} \geq 0.2 \\ \frac{5(L_{MAX} - L_{MIN})}{L_{\gamma}} & \frac{(L_{MAX} - L_{MIN})}{L_{\gamma}} < 0.2 \end{cases} \quad (1-1)$$

$$F(\text{Equal}) = \begin{cases} 1 & L_{\gamma} = L_{\gamma} = L_{MIN} = L_{MAX} \\ \frac{L_{\gamma}}{L_{\gamma}} & L_{\gamma} > L_{MAX} \\ \frac{L_{\gamma}}{L_{MAX}} & L_{\gamma} = L_{MAX}, L_{\gamma} = L_{MIN} \\ & L_{MAX} > L_{MIN} \end{cases} \quad (1-2)$$

$$F(\text{Cross}) = \begin{cases} 1 & L_{\gamma} \leq L_{MIN}^{1/2} \leq 2L_{\gamma} \\ \frac{2L_{MIN}}{L_{\gamma}} & 2L_{MIN}^{1/2} < L_{\gamma} \\ \frac{2L_{\gamma}}{L_{MIN}^{1/2}} & L_{MIN}^{1/2} > 2L_{\gamma} \end{cases} \quad (1-3)$$

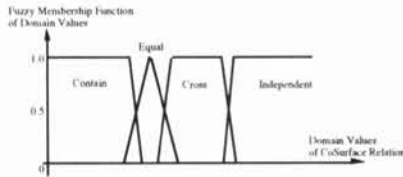


Fig.10 Fuzzy Domain Values of Quantitative Values of CoSurface Relation

$$F(\text{Contain}) = \begin{cases} 1 & \frac{(S_{MAX} - S_{MIN})}{S_{\gamma}} \geq 0.2 \\ 1 - \frac{S_{\gamma} - S_{MAX}}{S_{\gamma}} & S_{\gamma} > S_{MAX} \\ \frac{S_{MAX} - S_{MIN}}{S_{\gamma}} & \frac{(S_{MAX} - S_{MIN})}{S_{\gamma}} < 0.2 \end{cases} \quad (2-1)$$

$$F(\text{Equal}) = \begin{cases} 1 & S_{\gamma} = S_{\gamma} = S_{MIN} = S_{MAX} \\ \frac{S_{\gamma}}{S_{\gamma}} & S_{\gamma} > S_{MAX} \text{ or } \\ \frac{S_{\gamma}}{S_{\gamma}} & S_{\gamma} = S_{MAX}, S_{\gamma} = S_{MIN}, S_{\gamma} \neq S_{\gamma} \end{cases} \quad (2-2)$$

$$F(\text{Cross}) = \begin{cases} 1 & S_{\gamma} \leq S_{MIN}^{1/2} \leq 2S_{\gamma} \text{ or } S_{\gamma}^{1/2} \leq S_{\gamma} \leq 2S_{MIN}^{1/2} \\ \frac{2S_{MIN}^{1/2}}{S_{\gamma}} & 2S_{MIN}^{1/2} < S_{\gamma} \\ \frac{2S_{\gamma}}{S_{MIN}^{1/2}} & S_{MIN}^{1/2} > 2S_{\gamma} \end{cases} \quad (2-3)$$

$$F(\text{Independence}) = \begin{cases} 1 & S_{\gamma} = S_1 + S_2 \text{ or } S_{\gamma} = \emptyset \\ 1 - \frac{S_{\gamma}}{S_{MIN}^{1/2}} & S_{\gamma} < S_1 + S_2, \emptyset < S_{\gamma} < S_{MIN}^{1/2} \end{cases} \quad (2-4)$$

(c) Fuzzy domain value of SpatialR

The fuzzy membership function for domain value of SpatialR(X1,X2) is given as Fig.11. Here, the horizontal axis of Fig.11 represents the relative distance of their heights measured by their geometric centers. The ground (floor) surface is assumed to be at lowest position.

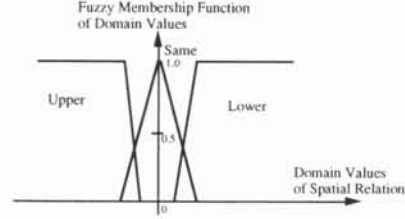


Fig.11 Fuzzy Domain Values of Quantitative Values of Spatial Relation

Next, we take 3-D geometric model of a house with its 3-D hierarchical structure information shown in Fig.12, as an example to describe the process of transforming quantitative values of the structural information to the FRDL representation.

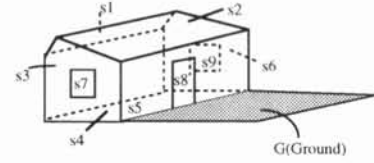


Fig.12 3-D Geometric Model of House1

Here, we just give the calculated results for those surface pairs that have connected relations with s1 in Sce. The quantitative values of angles and their qualitative values with fuzzy membership are shown in Table2. Sce is the surface pair set Sce which has the connected relations in Fig.12. The symbols Pa, Ac, Pe and Ob are for Parallel, Acute, Perpendicular and Obtuse respectively.

The fuzzy domain values of the other kinds of relations in FRDL could also be calculated in the similar way.

Table2

Sce	(s1, s2)	(s1, s3)
Angle	96.8	160.6
Fuzzy Values	$\{^{0.0}/_{Pa}, ^{0.0}/_{Ac}, ^{0.7}/_{Pe}, ^{0.7}/_{Ob}\}$	$\{^{0.0}/_{Pa}, ^{0.0}/_{Ac}, ^{1.0}/_{Pe}, ^{0.0}/_{Ob}\}$

Table2 (Continued)

Sce	(s1, s4)	(s1, s6)
Angle	91.5	90.0
Fuzzy Values	$\{^{0.0}/_{Pa}, ^{0.0}/_{Ac}, ^{0.7}/_{Pe}, ^{0.7}/_{Ob}\}$	$\{^{0.0}/_{Pa}, ^{0.0}/_{Ac}, ^{1.0}/_{Pe}, ^{0.0}/_{Ob}\}$

With the processing described above, the qualitative representation of House1 with FRDL is obtained. Here, we give the part of such representation that has connected relation with surface s1 and those having CoSurface relation. Fig.13 shows the FRDL representation of those parts. The other part of FRDL representation of House1 can also be obtained with the similar way.

## 5. FUZZY RELATIONAL MODEL

The FRDL representation of 3-D structural information of House1 obtained show in Fig.13 is just the qualitative description set for surface relations. There are no structural information that a house is composed from

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[Angle(s1, s2)={0.0/Pa, 0.0/Ac, 0.3/Pe, 0.2/Ob}]
[Angle(s1, s3)={0.0/Pa, 0.0/Ac, 0.0/Pe, 0.6/Ob}]
[Angle(s1, s4)={0.0/Pa, 0.0/Ac, 0.9/Pe, 0.1/Ob}]
[Angle(s1, s6)={0.0/Pa, 0.0/Ac, 1.0/Pe, 0.0/Ob}]
[SpatialR(s1, s2)={0.0/Up, 1.0/Sa, 0.0/Lo}]
[SpatialR(s1, s3)={1.0/Up, 0.0/Sa, 0.0/Lo}]
[SpatialR(s1, s4)={1.0/Up, 0.0/Sa, 0.0/Lo}]
[SpatialR(s1, s6)={1.0/Up, 0.0/Sa, 0.0/Lo}]
[CoEdge(s1, s2)={0.0/Co, 0.0/Cr, 1.0/Eq}]
[CoEdge(s1, s3)={0.0/Co, 0.0/Cr, 1.0/Eq}]
[CoEdge(s1, s4)={0.0/Co, 0.0/Cr, 1.0/Eq}]
[CoEdge(s1, s6)={0.0/Co, 0.0/Cr, 1.0/Eq}]
[CoSurface(s4, s7)={1.0/Co, 0.0/Cr, 0.0/In, 0.0/Eq}]
[CoSurface(s5, s8)={1.0/Co, 0.0/Cr, 0.0/In, 0.0/Eq}]
[CoSurface(s6, s9)={1.0/Co, 0.0/Cr, 0.0/In, 0.0/Eq}]

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Fig.13 Part of FRDL Representation of House1

several basic parts such as roof, walls, windows, door and floor like those shown in Fig.5. In order to get such structure representation, we explain the fact to computer that from which surfaces each basic part is composed. For example, the fact that the roof part of House1 in Fig.12 is composed from surface s1 and s2. And then, the FRDL subsets that represent each subpart of House1 and the subsets that represent the relations between these subparts can be obtained from FRDL set of House1. Through this method, we can obtain the fuzzy relational model of House1 in the form of sematic network shown in Fig.14.

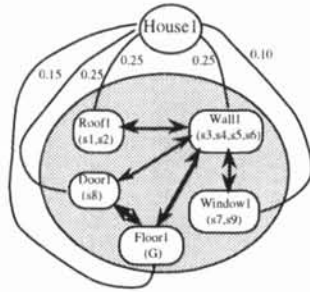


Fig.14 Fuzzy Relational Model of House1

As we know, different basic parts of a house have different importance in the composition of this house. For example, a house without windows could still be looked as a house approximately, but a house without walls could not be looked as a house obviously. Therefore, we add the weighing values to each basic part of House1 as shown in Fig.14.

In the processing of section 4, we assumed that every kind of the relational description in FRDL has the same importance. In practice, the importance for different relational description should be dealt in different degree. For example, the CoSurface relation between two surfaces has more important function than that of Angle relation since one surface usually has several Angle relations. So we add the weighing value to every kind of relational description in FRDL.

In section 4, we also dealt with each kind of domain value in a relational description formula in FRDL with the same importance. In fact, there do exist difference between different domain values of a FRDL formula. For example, the domain value Perpendicular of Angle relation has more important function than that of Acute of this relation since this domain value are mapped with a limited angle region (almost equal to 90° degree) than that of Acute and also plays a decisive role in some cases such as whether a surface is a wall or not. Therefore, we add the weighing value to each domain values of a FRDL description formula.

## 6. CONCLUSION

A system is presented in this paper that gets a fuzzy relational model of objects by taking 3-D geometrical information of objects obtained from stereo vision as inputs. By the fuzzy relational description language(FRDL) given in this paper, the scale problem and matching cost problem occurred in quantitative representation has been solved. And, the qualitative representation of the quantitative values become more flexible than CDL[4].

The further work should include that learning fuzzy-based generic model of objects from fuzzy relational model of example objects and the 3-D object recognition based on fuzzy relational model.

## APPENDIX

For two lines  $L_1$  and  $L_2$  that are located on the same line as shown in Fig.A, and two surfaces  $S_1$  and  $S_2$  that are located on the same plane as shown in Fig.B, the below geometric relations are defined.

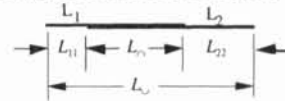


Fig.A Geometric Relations of Two Lines

$$\begin{aligned}
L_{MAX} &= MAX(L_1, L_2), & L_{MIN} &= MIN(L_1, L_2) \\
L_{\cap} &= L_1 \cap L_2, & L_{\cup} &= L_1 \cup L_2 \\
L_{11} &= L_1 - L_{\cap}, & L_{22} &= L_2 - L_{\cap} \\
L_{MIN}^{L_{11}L_{22}} &= MIN(L_{11}, L_{22})
\end{aligned}$$

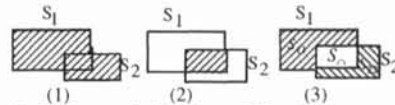


Fig.B Geometric Relations of Two Surfaces

$$\begin{aligned}
S_{MAX} &= MAX(S_1, S_2), & S_{MIN} &= MIN(S_1, S_2) \\
S_{\cap} &= S_1 \cap S_2, & S_{\cup} &= S_1 \cup S_2 \\
S_{11} &= S_1 - S_{\cap}, & S_{22} &= S_2 - S_{\cap} \\
S_{MIN}^{S_{11}S_{22}} &= MIN(S_{11}, S_{22})
\end{aligned}$$

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