THE WIDTH OF THE Δ-ISOBAR IN CHIRAL SOLITON MODELS ☆

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The strong decay width $\Delta\to\pi N$ is estimated using a linear meson–nucleon coupling derived from a constrained quantization of the ω-stabilized version of the Skyrme model. Due to the inherent gauge conditions, this coupling is subleading in N_c and differs from the one proposed by Adkins, Nappi and Witten in the context of soliton models.

In the semiclassical approximation to the Skyrme model and its vector variants, the Δ-isobar is like the nucleon, a stable rotational bound state. In nature, however, the isobar is a bound state in the continuum with a width of about 120 MeV. Therefore, the strong decay process of the isobar ($\Delta\to\pi N$) is mediated by soft pions, absent at the semiclassical level. To go beyond the semiclassical description requires introducing and quantizing soft-pion fluctuations.

The skyrmion as a classical vacuum state breaks spin, isospin as well as translational invariance. As a result there are zero modes in the small amplitude oscillation spectrum that cause the conventional quantization scheme to break down. These modes are present even when vector mesons are introduced. They are spurious, and should be removed from the meson spectrum. In the canonical framework, this can be achieved using either the collective coordinate method (Dirac scheme) [1] or a variant of the Gupta–Bleuler procedure (subsidiary condition theorem) [2].

In this letter, we will restrict our discussion to the ω-stabilized [3] version of the Skyrme model for simplicity, and use the collective coordinate method to handle the zero-mode problem. In brief, the quantum fields in the soliton sector can be described as follows [4–6]:

$$\Phi^a(x)=R^{ab}[\phi^b(\boldsymbol{x}-\boldsymbol{X})+\xi^b(\boldsymbol{x}-\boldsymbol{X},t)], \tag{1}$$

$$\omega_i(x)=\omega_i(\boldsymbol{x}-\boldsymbol{X},t), \qquad (1\text{ cont'd})$$

where $\boldsymbol{\phi}(\boldsymbol{x})=f_\pi\hat{\boldsymbol{x}}F(x)$ and $\mathbf{0}$ are, respectively, the classical expectation values of $\boldsymbol{\Phi}$ and $\boldsymbol{\omega}$ in the vacuum, R a global isospin rotation, $\boldsymbol{X}$ the spatial position of the static skyrmion and ξ the intrinsic pion fluctuation. To avoid double counting the isospin degrees of freedom we impose

$$\int d^3x\,\xi\,g(\phi)T^a\phi=0, \tag{2}$$

where g is the O(3) metric of the conventional nonlinear σ-model and T^a is the generator of isospin rotations in the adjoint representation. This ensures that the spin–isospin zero-modes ($T^a\phi$) are orthogonal to the rest of the meson spectrum, hence removed. Similar constraints are required to remove the translational zero-modes ($\nabla\phi$), namely

$$\int d^3x\,\xi\,g(\phi)\nabla^a\phi=0. \tag{3}$$

In this case, the quantization procedure is that of constrained fields, and Dirac's [7] method has to be substituted to the canonical one. As a result, the πNN-, πNΔ- and πΔΔ-coupling are of order $N_c^{-3/2}$. They are mediated by (ignoring momentum dependent terms) [5]

$$\mathscr{H}_{\xi N}=-(J^a/2\Lambda_\pi)[\tfrac{1}{2}T^a\phi g(\phi)'\cdot\xi T^b\phi + T^a\phi g(\phi)T^b\xi]J^b/\Lambda_\pi+\text{h.c.}, \tag{4}$$

where J is the intrinsic isospin of the baryons, and Λ_π the classical moment of inertia,

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$$\Lambda_\pi=\tfrac{1}{3}\int \mathrm{d}^3x\,(T^a\phi)g(\phi)(T^a\phi). \tag{5}$$

The latter is just the norm of the spin–isospin zero-modes. Due to the constraint quantization conditions (2), (3), this coupling is subleading in N_c and differs from the one discussed originally by Adkins, Nappi and Witten [8] in the context of the original Skyrme model. This result is general, and does not depend on the dynamical details of the model. It should hold true for any version of the Skyrme model. Since the ω-field has zero vacuum expectation value, it is just the ordinary perturbative vector meson-field, and does not contribute to the moment of inertia (weak coupling). In the strong coupling case, the ω-field can be integrated into the baryon spectrum [5]. As a result $\Lambda_\pi\to\Lambda=\Lambda_\pi+\Lambda_\omega$ in agreement with the semiclassical result [3].

The N and Δ states follow by diagonalizing

$$H=M_s+J^2/2\Lambda_\pi+P^2/2M_s, \tag{6}$$

where M_s is the classical mass of the skyrmion and P its total momentum. Their wavefunctions are given by

$$\Psi_{JMM',P}(x,\theta)=\sqrt{(2J+1)/2\pi^2}\,\exp(-\mathrm{i}P\cdot x)\times(-1)^{(J+M)}D^J_{-MM'}(\theta). \tag{7}$$

The meson states are obtained by diagonalizing the order-one hamiltonian discussed in ref. [6]. In the lab-frame, the pion field is given by (plane wave normalization)

$$\pi^a(\boldsymbol{x},t)=\int\frac{\mathrm{d}^3k}{(2\pi)^3 2E(\boldsymbol{k})}\{\xi_k(\boldsymbol{x},t)A^a_{\boldsymbol{k}}[\hat{\boldsymbol{X}},\hat{\boldsymbol{\theta}}]+\text{h.c.}\}. \tag{8}$$

If the $c^\dagger$'s are the meson creation operators associated to the quantum field ξ in the intrinsic frame, satisfying the usual canonical commutation relations, then [#1]

$$A^\dagger_{\boldsymbol{k},a}[\hat{\boldsymbol{X}},\hat{\boldsymbol{\theta}}]=\exp(\mathrm{i}\boldsymbol{k}\cdot\hat{\boldsymbol{X}})R_{ab}[\hat{\boldsymbol{\theta}}]c^\dagger_{\boldsymbol{k},b}. \tag{9}$$

Since $RR^{\mathrm{T}}=1$, the A's satisfy also canonical commutation relations,

$$[A_{\boldsymbol{k},a},A^\dagger_{\boldsymbol{k}',b}]=(2\pi)^3 2E(\boldsymbol{k})\delta(\boldsymbol{k}-\boldsymbol{k}'). \tag{10}$$

A one-pion-baryon state can be described as follows:

$$|\pi^a(\boldsymbol{q});JMM',P\rangle=A^\dagger_{\boldsymbol{q},a}[\hat{\boldsymbol{X}},\hat{\boldsymbol{\theta}}]\,|JMM',P\rangle. \tag{11}$$

Similar relations can be defined for the states with ω-mesons.

To lowest order, the decay process $\Delta^{++}\to\pi^+\mathrm{p}$ can be described by the following S-matrix element:

$$S_{\Delta^{++}\to\pi^+\mathrm{p}}=\langle\pi^+(\boldsymbol{q});\tfrac12\tfrac12\tfrac12,P|-\mathrm{i}\int\mathrm{d}^4x\,\mathcal{H}_{\xi\mathrm{N}}\,|\tfrac32\tfrac32\tfrac32,P'\rangle. \tag{12}$$

Using the linear character of the meson–nucleon vertex (4) together with (9)–(11) yields

$$S_{\Delta^{++}\to\pi^+\mathrm{p}}=\mathrm{i}(2\pi)^4\delta^4(Q+P-P')\int\mathrm{d}^3x\,U_{+a}\xi_q(\boldsymbol{x})\times\langle\tfrac12\tfrac12\tfrac12,P|R_{ab}\sum_i\mathcal{A}^{(i)}_b(\boldsymbol{x})|\tfrac32\tfrac32\tfrac32,P'\rangle, \tag{13}$$

where U is the matrix transformation from cartesian to spherical coordinates

$$U=\frac{1}{\sqrt2}\begin{pmatrix}-1&\mathrm{i}&0\\0&0&\sqrt2\\1&\mathrm{i}&0\end{pmatrix},$$

and the $\mathcal{A}$'s are short for

$$\mathcal{A}^{(1)}_a=-\frac{f_\pi}{2\Lambda^2_\pi}\boldsymbol{J}^2\sin2\phi\,\hat\phi_a, \tag{14}$$

$$\mathcal{A}^{(2)}_a=\frac{f_\pi}{2\Lambda^2_\pi}\frac{\sin^2\phi}{\phi}\delta_{ai}\hat\phi_j[J_i,J_j]_+, \tag{15}$$

$$\mathcal{A}^{(3)}_a=\frac{f_\pi}{2\Lambda^2_\pi}\left(\tfrac12\sin2\phi-\frac{\sin^2\phi}{\phi}\right)\hat\phi_a\hat\phi_i\hat\phi_j[J_i,J_j]_+. \tag{16}$$

The overall δ-function in (13) reflects energy–momentum conservation as expected from restoring translational invariance into the problem [#2]. Using the fact that

$$\langle\tfrac12\tfrac12\tfrac12|R_{ak}[J_k,J_l]_+|\tfrac32\tfrac32\tfrac32\rangle\hat r_l=-\tfrac54(1/\sqrt2)(\delta_{a1}+\mathrm{i}\delta_{a2})\hat r_+ \tag{17}$$

[#1] Here we are using a linearized version of the general Bogoliubov transformation discussed in ref. [6]. This approximation is justified asymptotically where $\phi\sim0$ around the pion.

[#2] As already mentioned, we have omitted the momentum dependence in the linear coupling (4) and thus from (14)–(16).

and a plane wave approximation to the meson fluctuations ξ, we obtain the following contributions to the T-matrix:

$$T_{\mathrm{fi}}^{(1)}=\frac{\pi f_\pi}{\Lambda_\pi^2}\frac{15}{4}\hat{q}^+\int r^2\,\mathrm{d}r\sin 2F j_1(qr),\qquad(18)$$

$$T_{\mathrm{fi}}^{(2)}=\frac{\pi f_\pi}{\Lambda_\pi^2}\frac{5}{2}\hat{q}^+\int r^2\,\mathrm{d}r\frac{\sin^2F}{F}j_1(qr),\qquad(19)$$

$$T_{\mathrm{fi}}^{(3)}=\frac{\pi f_\pi}{\Lambda_\pi^2}\frac{1}{2}\hat{q}^+\int r^2\,\mathrm{d}r\left(\frac{\sin^2F}{F}-\tfrac{1}{2}\sin 2F\right)j_1(qr).\qquad(20)$$

This decomposition of the T-matrix is commensurate with (14)–(16). Note that the T-matrix involves a single Fourier transform since one of the Fourier transforms in the S-matrix yields overall energy–momentum conservation. Note also that the contribution of (16) to the T-matrix is small since it comes solely from the derivative of the metric. In terms of (18)–(20) the width of the Δ is

$$\Gamma_{\Delta^{++}\to p+\pi^+}=\frac{\pi f_\pi^2}{\Lambda_\pi^4}\frac{1}{12}\left|\int r^2\,\mathrm{d}r j_1(qr)\right.$$
$$\left.\times\left(7\sin 2F+\frac{6\sin^2F}{F}\right)\right|^2|q|.\qquad(21)$$

Here, $|q|$ is the modulus of the pion momentum defined through

$$q^2/2M+\sqrt{q^2+m_\pi^2}=M_\Delta-M_{\mathrm{N}},\qquad(22)$$

and turns out to be $q\sim 225$ MeV in the Δ rest frame. With $f_\pi=62.2$ MeV and $g_\omega=15.2$ we can fit both the N and Δ masses in the strong coupling case ($\Lambda_\pi\to\Lambda\sim 1$ fm). In this case the width Γ is about 550 MeV which is about a factor of four too large compared to the empirical value of about 120 MeV. This value, however, should be viewed as an upper bound because of our use of the plane wave approximation. In general, the ξs in (8) are distorted by the soliton background to order 1. Moreover, the momentum-dependent terms omitted from (4) might also be important. Our value of the width does not agree with the one recently discussed by Holzwarth, Hayashi and Schwesinger [9] in the canonical Skyrme model using different arguments. A similar conclusion has also been reached independently by Verschelde [10] using also the conventional Skyrme model.

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Note added. Very recently, Verschelde and Verbeke [11] pointed out that the gauge condition (2), which forces the fluctuations to be orthogonal to the zero modes in the *entire* space, should be modified to allow for a more appropriate treatment of dynamical effects. In the modified "non-rigid" gauge, the scattering modes are allowed to have a small zero mode contribution such that a time dependent rotation of the skyrmion does not violate locality (the fluctuations in the vicinity of the skyrmion are dragged along). For soft-pion processes, this translates into sizeable corrections, which in the "rigid" gauge are tedious to sum up. Using this approach Verschelde has obtained a decay width much closer to the experimental value.

In the light of these developments, it appears that our value for the width reflects the rigid-body limit of a more appropriate "non-rigid" treatment.

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