Estimation of Black Globe Temperature for Calculation of the WBGT Index

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Abstract—The wet bulb globe temperature (WBGT) index is used in industry, sports and other areas to indicate the heat stress level for humans and animals. One of the values needed to calculate the WBGT index is the black globe temperature. The black globe temperature is measured using a WBGT instrument which includes a black globe with a thermometer inserted in the center. However, the WBGT instrument can be costly and many of these instruments may be needed to get measurements in many locations. The authors have derived a formula to estimate the black globe temperature using readily available data collected by the National Weather Service (NWS). The formula was derived from a formula suggested by Kuehn (1970), which was based on heat transfer theory. The resulting equation was a fourth degree polynomial in terms of the black globe temperature. It was determined that the fourth degree polynomial in terms of the black globe temperature can be very accurately approximated using a linear expression in terms of black globe temperature. Some preliminary tests indicate accuracy within 0.5˚F.

Introduction

One of the government regulations instituted by OSHA is heat stress management (OSHA, 2008). The manual states in Section III: Chapter 4 the second paragraph of the introduction:

> Outdoor operations conducted in hot weather, such as construction, refining, asbestos removal, and hazardous waste site activities, especially those that require workers to wear semipermeable or impermeable protective clothing, are also likely to cause heat stress among exposed workers.

A rating is calculated which indicates the safe amount of time a person can work outside on a hot day. This quantity is called the Wet Bulb Globe Temperature (WBGT). In the past, WBGT data has been collected manually using a portable instrument. The OSHA manual includes the following formulas for the WBGT:

1. For indoor and outdoor conditions with no solar load, WBGT is calculated as:

$$WBGT = 0.7NWB + 0.3GT$$

2. For outdoors with a solar load, WBGT is calculated as

$$WBGT = 0.7NWB + 0.2GT + 0.1DB$$

where:

WBGT	=	*Wet Bulb Globe Temperature Index*
NWB	=	*Nature Wet-Bulb Temperature*
DB	=	*Dry-Bulb Temperature*
GT	=	*Globe Temperature*

However, recently the National Weather Service (NWS) was asked to provide the WBGT using only data that is routinely collected by the NWS. The main problem with this is that one of the variables in the equation to calculate WBGT is the "globe temperature." This temperature is found by using a copper globe painted in black matte paint with a thermometer inserted so that the bulb is in the center of the globe. This temperature is not routinely collected by the NWS.

Turco, et. al. (2008) derived equations to estimate the black globe temperature based on meteorological data. However, their model was a statistical model, not a physical model. The equations derived were regression equations computed from meteorological data. Although the equations were extremely accurate, a more accurate model may be derived from the heat equations for the black globe. According to the authors:

> The models developed resulted in great performance to predict the black globe temperature, allowing the estimation of bioclimatic indices to assess the conditions of the environment, to accomplish regional studies, and to indicate best house designs for animals.

This paper shows how the globe temperature can be approximated using only data routinely collected by the NWS. A fourth degree polynomial equation is derived for globe temperature with the coefficients dependent on readily available data. Then, it is shown that the fourth degree polynomial can be reasonably approximated by a linear equation, thus making computation less costly and time-consuming. Finally, some experiments were done to verify the accuracy of the estimate using the linear expression in terms of temperature.

Derivation

The following heat equation was taken from a paper by Hunter and Minyard (1999), with the exception of the constant in the second term on the right:

$$(1-\alpha_{sps})S\left(f_{db}s_{sp}+(1+\alpha_{es})f_{dif}\right)+\left(1-\alpha_{spl}\right)\sigma\varepsilon_a T_a^4=\varepsilon\sigma T_g^4+0.115u^{0.58}(T_g-T_a) \quad (1)$$

The coefficient in the second term on the right side of equation (0.115) is from the convective heat flow coefficient. It was determined during testing that setting this coefficient equal to 0.437 gives a more accurate estimation of the globe temperature. This value may need to be adjusted for different spheres.

Now, putting all T_g terms on the left of the equation, replacing 0.115 with 0.315 and dividing by $\varepsilon\sigma$ we get:

$$T_g^4+\frac{0.315u^{0.58}}{\varepsilon\sigma}T_g=\frac{(1-\alpha_{sps})S\left(f_{db}s_{sp}+(1+\alpha_{es})f_{dif}\right)+(1-\alpha_{spl})\sigma\varepsilon_a T_a^4}{\varepsilon\sigma}+\frac{0.315u^{0.58}}{\varepsilon\sigma}T_a \quad (2)$$

The values of all variables except T_g are either given or can be calculated from available data from the NWS. The following values are provided.

Globe albedo for short and long wave radiation: $\alpha_{sps}=\alpha_{spl}=0.05$ so $1-\alpha_{sps}=1-\alpha_{spl}=0.95$.

Black globe emissivity: $\varepsilon=0.95$

Stephan-Boltzman constant: $\sigma=5.67\text{x}10^{-8}$ is used.

Albedo for grassy surfaces: $\alpha_{es}=0.2$.

When these values are entered into equation (2) we get:

$$T_g^4+\frac{0.315u^{0.58}}{0.95(5.67\times10^{-8})}T_g=\frac{0.95S\left(f_{db}s_{sp}+(1.2)f_{dif}\right)+0.95(\varepsilon_a)\sigma T_a^4}{0.95(5.67\times10^{-8})}+\frac{0.315u^{0.58}}{0.95(5.67\times10^{-8})}T_a \quad (3)$$

Hunter and Minyard, in their paper, show that $s_{sp}=\frac{1}{4\cos(z)}$, where z is the solar angle to zenith. Putting this into (3), we get

$$T_g^4 + \frac{0.315u^{0.58}}{0.95(5.67x10^{-8})}T_g = \frac{S\left(\frac{f_{db}}{4\cos(z)}+(1.2)f_{dif}\right)+(\varepsilon_a)\sigma T_a^4}{(5.67x10^{-8})} + \frac{0.315u^{0.58}}{0.95(5.67x10^{-8})}T_a \quad (4)$$

Where S is solar irradiance, f_{db} is the direct beam radiation from the sun and f_{dif} is the diffuse radiation from the sun. Finally, the ambient temperature is represented by T_a and the wind speed by u in meters per hour. All of these are given or may be calculated directly from given data by the NWS.

The last parameter on which the globe temperature depends is the thermal emissivityε_a. According to Hunter and Minyard, thermal emissivity can be calculated using

$$\varepsilon_a = 0.575e_a^{(1/7)} \quad (5)$$

Where e_a is atmospheric vapor pressure, which may be calculated by

$$e_a = \exp\left(\frac{17.67(T_d - T_a)}{T_d + 243.5}\right) \times (1.0007 + 0.00000346P) \times 6.112\exp\left(\frac{17.502T_a}{240.97+T_a}\right), \quad (6)$$

where P is the barometric pressure and T_d is the dew point temperature.

When we take into consideration the fact that all parameter values in equation (4) are constants that can be entered at constant time intervals, we can reduce the equation to

$$T_g^4 + CT_g = B + CT_a \ , \quad (7)$$

where

$$C = \frac{0.315u^{0.58}}{(5.3865\times10^{-8})} \qquad \text{and} \qquad B = S\left(\frac{f_{db}}{2.268\times10^{-8}\cos(z)} + \left(\frac{1.2}{5.67\times10^{-8}}\right)f_{dif}\right) + (\varepsilon_a)T_a^4.$$

By doing this, we can treat (7) as a fourth degree polynomial in terms of T_g. The values of T_g in which we are interested are in the interval [20, 60], since values below 20˚C are too cold to cause heat stress and values above 60˚C, in general, do not occur. Figure 1 shows a graph of $y = t^4 + Ct$ and y1=Ct – 7,680,000 (the tangent line approximation for the function y at t=40) on the interval [20, 60] (C was calculated for a wind speed of about 15 mph).

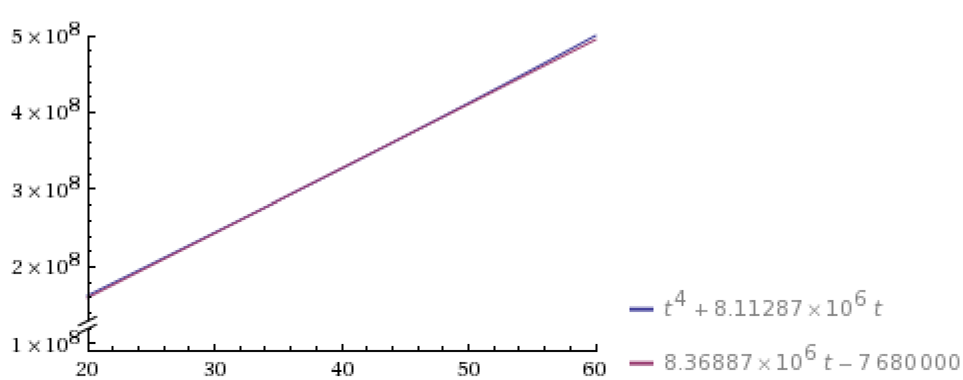

Figure 1. The graph of $y = t^4 + Ct$ and y1=Ct – 7,680,000 for C≈8,389,000 and t between 20 and 60.

Notice that the curve appears to be very close to the linear graph. We can compute the curvature for y to see how close to a linear function y is. The curvature of $y = t^4 + Ct$ is given by

$$k = \frac{12t^2}{(3t^3+C)^{(3/2)}} \tag{8}$$

In order to get an understanding of the magnitude of the curvature, consider the graph (Figure 2) of the function $k(t,u) = \frac{12t^2}{(3t^3+C(u))^{(3/2)}}$ for u between 1 mph and 40 mph (1690 meters per hour to about 65,000 meters per hour) and t between 20˚C and 60˚C.

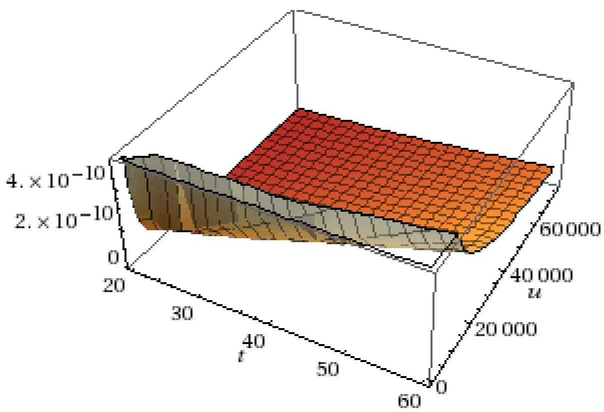

Figure 2. Pictured above is the curvature of y for 20<t≤60 and 1≤u≤76,000.

Notice that the curvature is on the order of 4 x 10^{-10} or less on the domain of interest. This confirms the assumption that y is nearly linear for values of t and u that make sense for this context. It is therefore reasonable to use a linear approximation for y to solve for t (=T_g). In other words we may use a linear approximation on the left side of equation 7 to estimate the value of the globe temperature.

Using differential calculus to find the equation of the tangent to the curve at t=40 (the midpoint of the interval [20, 60]), we find that the left side of equation 7 may be substituted by

$$y_{est} = CT_g + 256000T_g - 7680000. \tag{9}$$

Putting this in place of the left side of equation7 and solving for T_g, we get

$$T_g = \frac{B+CT_a+7680000}{C+256000} \tag{10}$$

with B, C and T_a as defined previously. Now we have an estimate of Tg dependent only on values which are either readily available from the NWS or may easily be calculated from data available from the NWS. Also, the equation is linear making for easier computation than what was necessary to solve the original fourth degree polynomial.

An Algorithm

In this section, an algorithm is created for the calculation of globe temperature estimates. First, we will consider the values readily available from the NWS. These will be input values to be entered at the beginning of the program.

1. The values to be entered are wind speed (u in meters per hour), ambient temperature (T_a in degrees Celsius), dew point temperature (T_d in degrees Celsius), solar irradiance (S in Watts per meter squared), direct beam radiation from the sun (f_{db}) and diffuse radiation from the sun (f_{dif}).
2. The zenith (z) angle may be entered or calculated. (In Excel, this must be in radians.)
3. The thermal emissivity must be calculated next. Using the following two equations.

   a. $e_a = \exp\left(\frac{17.67(T_d - T_a)}{T_d + 243.5}\right) \times (1.0007 + 0.00000346P) \times 6.112\exp\left(\frac{17.502T_a}{240.97 + T_a}\right)$

   b. $\varepsilon_a = 0.575e_a^{(1/7)}$

4. Now B and C can be calculated using the following equations.

   a. $B = S\left(\frac{f_{db}}{4\sigma\cos(z)} + \left(\frac{1.2}{\sigma}\right)f_{dif}\right) + (\varepsilon_a)T_a^4$, where $\sigma=5.67\times10^{-8}$

   b. $C = \frac{hu^{0.58}}{(5.3865\times10^{-8})}$, where h=0.315

5. Finally the estimate for globe temperature is calculated using equation (10).

$$T_g = \frac{B + CT_a + 7680000}{C + 256000}$$

Testing

In September, 2010, three preliminary tests were conducted to test the accuracy of the globe temperature estimate. A WBGT measuring unit was created by an employee of the NWS.

A picture of the unit is included in Figure 3. This unit was used to get some preliminary reading to check for accuracy of the equation.

The first test was done on September 9, 2010 in front of the NOAA offices in Tulsa, Oklahoma. The weather conditions were hazy that day with air temperature 86˚F, and dew point temperature 69˚F. The barometric pressure was 30.08 in. of Hg (about 993 mb for pressure not adjusted for sea level) and the solar irradiance was 336 W/m^2. The wind speed averaged around 5 to 6 mph during the measurement. The globe temperature was measured to be 91˚F using a black globe as described earlier in this paper. Using Excel, a spread sheet was created to use the derived equation to estimate globe temperature. The equation estimated the globe temperature to be about 91.434 ˚F.

Another test was performed on September 10, 2010. The conditions were sunny with air temperature 93˚F and dew point temperature 76˚F. The barometric pressure was 29.75 in. of Hg (about 982 mb for pressure not adjusted for sea level) and the solar irradiance was 754 W/m^2. The wind speed was measured at about 7 mph during the measurement. The globe temperature was measured to be 103˚F using a black globe. The equation estimated the globe temperature to be about 102.757˚F.

The third test was performed on September 17, 2010. The conditions were similar to the conditions on September 10. The air temperature was 94˚F and dew point temperature 76˚F. The barometric pressure was 30.05 in. of Hg (about 992 mb for pressure not adjusted for sea level) and the solar irradiance was 579 W/m^2. The wind speed was measured at about 3.7 mph during the measurement. The globe temperature was measured to be 105˚F using a black globe. The equation estimated the globe temperature to be about 105.175˚F.

The three preliminary tests indicate that the formula used to estimate the globe temperature is very accurate. If the estimate is within 1˚F, it is sufficient to estimate the WBGT index. As can be seen from the preliminary tests, the estimates are within about 0.5˚F. The main problem with our tests is that we had to estimate the wind speed and the formula is very sensitive to the value of the wind speed. However, the estimates for wind speed should be within about 0.5-1 mph.

Bibliography

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Wet Bulb Temperature = $(-5.806 + 0.672 * T_a - 0.006 * T_a^{2} + (0.061 + 0.004 * T_a + 99*10\text{-}6 * T_a^{2}) * RH + (-33*10\text{-}6 - 5*10\text{-}6 * T_a - 1*10\text{-}7 * T_a^{2}) * RH^2)$