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Selection of Green Recycling Suppliers for Shared Electric Bikes: A Multi-Criteria Group Decision-Making Method Based on the Basic Uncertain Information Generalized Power Weighted Average Operator and Basic Uncertain Information-Based Best–Middle–Worst TOPSIS Model

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Abstract: This study introduces a novel multi-criteria group evaluation approach grounded in the theory of basic uncertain information (BUI) to facilitate the selection of green recycling suppliers for shared electric bikes. Firstly, a comprehensive index system of green recycling suppliers is established, encompassing recycling capacity, environmental sustainability, financial strength, maintenance capabilities, and policy support, to provide a solid foundation for the scientific selection process. Secondly, the basic uncertain information generalized power weighted average (BUIGPWA) operator is proposed to aggregate group evaluation information with BUI pairs, and some related properties are investigated. Furthermore, the basic uncertain information-based best–middle–worst TOPSIS (BUI-BMW-TOPSIS) model incorporating the best, middle, and worst reference points to enhance decision-making accuracy is proposed. Ultimately, by integrating the BUIGPWA operator for group information aggregation with the BUI-BMW-TOPSIS model to handle multi-criteria decision information, a novel multi-criteria group decision-making (MCGDM) method is constructed to evaluate green recycling suppliers for shared electric bikes. Case analyses and comparative analyses demonstrate that compared with the BUIWA operator, the BUIGPWA operator yields more reliable results because of its consideration of the degree of support among decision-makers. Furthermore, in contrast to the traditional TOPSIS method, the BUI-BMW-TOPSIS model incorporates the credibility of information provided by decision-makers, leading to more trustworthy outcomes. Notably, variations in attribute weights significantly impact the decision-making results. In summary, our methods excel in handling uncertain information and complex multi-criteria group decisions, boosting scientific rigor and reliability, and supporting optimization and sustainability of shared electric bike green recycling suppliers.

Keywords: green recycling suppliers; shared electric bikes; BUIGPWA operator; BUI-BMW-TOPSIS model; MCGDM method

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Citation: Liu, L.; Shao, F.; He, C. Selection of Green Recycling Suppliers for Shared Electric Bikes: A Multi-Criteria Group Decision-Making Method Based on the Basic Uncertain Information Generalized Power Weighted Average Operator and Basic Uncertain Information-Based Best–Middle–Worst TOPSIS Model. *Sustainability* **2024**, *16*, 8647. https://doi.org/10.3390/su16198647

Academic Editor: Giada La Scalia

Received: 9 September 2024
Revised: 29 September 2024
Accepted: 5 October 2024
Published: 6 October 2024

1. Introduction

As the bike-sharing market continues to mature, society's acceptance of green travel continues to rise, yet the limitations of bike-sharing in short-to-medium distance travel have gradually surfaced. Electric bikes, as a rapidly developing alternative to the fuel-based mode of transportation, have garnered significant attention because of their effectiveness in addressing travel needs spanning 3 to 10 km [1]. In recent years, fueled by the application of the internet of things (IoT) and big data technologies, the deployment of shared electric bikes in cities has surged. Specifically, China's fleet of shared e-bikes surpassed 5 million units in 2023, and it is anticipated to expand to 8 to 10 million by 2025 [2,3]. However, issues stemming from high utilization rates and inadequate maintenance have sparked concerns over their sustainability, particularly the recycling of power batteries, which has emerged as a pivotal challenge facing the industry [3]. Failure to recycle these batteries properly poses a severe risk of heavy metal pollution, thereby threatening ecological environments and human health [4]. Consequently, research into the recycling of shared electric bikes holds significant practical importance.

In order to address the aforementioned challenges effectively, it is important to establish a comprehensive evaluation index system for green recycling suppliers of shared e-bikes. Firstly, it can assist decision-makers in selecting suppliers with outstanding performance in the green recycling process, thereby optimizing resource allocation, enhancing recycling efficiency, and mitigating environmental pollution risks. Secondly, the refinement of the evaluation index system will motivate suppliers to attach greater importance to environmental responsibility and social impact during their operations, thus driving the green transformation and sustainable development of the entire industry. Furthermore, a systematic index system can also provide a scientific basis for the formulation and implementation of relevant policies, further enhancing the effectiveness of policy support. Against this background, the existing research on the establishment of an evaluation index system for green recycling suppliers of shared e-bikes has encompassed crucial assessment indicators such as green image, financial capability, recycling capability, recycling costs, pollution and emissions, search capability, government cooperation, and battery recycling capability, as well as public awareness and education [5–8]. While the current evaluation index system is relatively comprehensive, there are still deficiencies, particularly in the assessment of suppliers' capabilities in waste reutilization capability and the management of the transition between old and new bike models. Consequently, this paper proposes to add the following critical indicators to the existing index system: waste reutilization capability and new–old transition handling capability. By incorporating these two new indicators, we can more comprehensively reflect the suppliers' capabilities and potential risks in the green recycling process, ensuring the scientific rigor and rationality of the selection process.

When addressing the issue of selecting recycling suppliers for shared e-bikes, the existing research has predominantly adopted multi-criteria group decision-making (MCGDM) methods [5,6,8–10] based on the recycling studies of mobile batteries and other similar areas [7,11]. Given the exceptional capability of interval type-2 fuzzy sets in accurately quantifying and representing complex semantic evaluation information, scholars have proposed a VIKOR decision model based on this theory, aiming to apply it to the comprehensive evaluation of shared bike suppliers to achieve more precise and comprehensive evaluations [5]. Furthermore, recognizing that interval-valued Pythagorean fuzzy preference relations (IVPFPRs) can capture and integrate decision-makers' complex preference information multidimensionally (including degrees of membership, non-membership, and hesitation, etc.), scholars have subsequently constructed a decision model for shared bike suppliers based on IVPFPR to enhance the scientificity and effectiveness of the decision-making process [6]. Additionally, acknowledging the superior degradation ability of q-rung orthopair fuzzy sets, which not only maintain the original advantageous characteristics of Pythagorean fuzzy sets but also expand the boundaries of information representation, scholars have innovatively proposed a two-stage interval-valued q-rung orthopair fuzzy decision model specifically targeting the evaluation of shared bike suppliers, aiming to achieve a higher level of information fusion and precise decision-making in complex decision-making environments [9].

The aforementioned studies primarily focused on constructing group multi-criteria decision-making models for the selection of shared bike recycling suppliers based on uncertain information theories such as interval type-2 fuzzy sets, Pythagorean fuzzy sets, and q-rung orthopair fuzzy sets. However, they paid less attention to the support among group decision-makers and did not consider factors such as the credibility of evaluation information sources. Building upon the existing research, this study emphasizes the mutual support during the aggregation of group decision-making information and the credibility of decision-makers' information. It introduces the generalized power average (GPA) operator [12–16] and basic uncertain information (BUI) theory [17–23] to develop a novel decision-making model for shared bike recycling suppliers.

As mentioned earlier, with the rapid development of the shared e-bike market, the subsequent issues of recycling and reuse have become increasingly important. Therefore, the core research question of this paper is how to select green recycling suppliers for shared e-bikes scientifically. At the theoretical level, this study constructs a multi-criteria evaluation system for green recycling suppliers of shared e-bikes. To process the evaluation information from the assessment group within this system, the BUIGPWA operator based on the generalized power average (GPA) operator [12–16] and basic uncertain information (BUI) theory [17–23] is developed to aggregate group information, and the basic uncertain information-based best–middle–worst TOPSIS (BUI-BMW-TOPSIS) method is developed incorporating the best, middle, and worst reference points to support supplier selection. At the practical level, the evaluation system serves as a basis for supplier selection, while the decision-making method, which accounts for information credibility, provides enterprises with methodological guidance. This helps optimize resource allocation, enhance recycling efficiency, reduce environmental pollution risks, and promote green transformation within the industry. Specifically, the research path of this paper is as follows: Firstly, this study develops a selection index system for shared e-bike recycling suppliers. This system addresses the disposal and replacement of discarded e-bikes, offering comprehensive evaluation guidelines for supplier assessment. Secondly, this study proposes a selection and evaluation method for green recycling suppliers of shared e-bikes based on the BUIGPWA operator and the BUI-BMW-TOPSIS model, offering enterprises a scientific and reasonable supplier selection scheme.

In summary, this paper not only deepens the theoretical exploration in the field of shared e-bike recycling but also provides powerful solutions to practical problems in the real world, thereby possessing significant theoretical and practical implications.

This paper is structured as follows: Section 2 reviews the operators and methods for handling MCGDM problems. Section 3 briefly introduces the concepts of BUI and the GPA operator. In Section 4, an evaluation index system for green recycling suppliers of shared e-bikes is constructed. Section 5 presents the selection and evaluation methodology for green recycling suppliers of shared e-bikes based on the BUIGPWA operator and the BUI-BMW-TOPSIS model. Section 6 shows the case study and comparative analysis. Section 7 provides a summary and discusses future research directions.

2. Literature Review

The selection of shared e-bike recycling suppliers represents a typical MCGDM problem, encompassing multiple evaluation criteria and opinions from various decision-makers. This process necessitates the adoption of suitable aggregation operators to integrate group decision information, as well as appropriate multi-criteria decision-making methods to process this information. In terms of aggregating group decision information, the generalized power average (GPA) operator exhibits notable advantages by considering the degree of support among decision-makers. Additionally, the TOPSIS method has garnered significant attention for its efficiency in handling multi-criteria decision-making problems, providing clear rankings and evaluation results, and is particularly suitable for complex supplier selection scenarios. Therefore, this paper reviews the GPA operator and TOPSIS method and, based on this foundation, proposes a novel comprehensive decision-making model to enhance the accuracy and reliability of decision-making in uncertain environments.

2.1. Generalized Power Average Operator for Aggregating Group Information in MCGDM

From the perspective of the generalized power average (GPA) operator used for aggregating group decision information, the GPA operator demonstrates significant advantages in processing information aggregation because of its consideration of the degree of support among decision-makers [12]. On this foundation, further research has introduced refined

GPA methodologies [13–16], whose effectiveness in uncertain environments has been validated. For instance, integrating the GPA operator with the probabilistic dual hesitant fuzzy approach enables more effective navigation of uncertainties in complex decision-making settings [13]. Additionally, the dual hesitant fuzzy linguistic power averaging operator based on Archimedean t-conorm and t-norm offers a novel path for information aggregation [14], further expanding the GPA operator's potential in group decision-making within fuzzy environments [15,16]. These refined methods exhibit unique advantages in tackling uncertainty and ambiguity, significantly enhancing the scientific rigor and fairness in MCDM. In this context, applying the GPA operator in basic uncertain information (BUI) environments demonstrates particular merits. BUI environments often involve incomplete or imprecise information, which is crucial to the decision-making process [17–23]. Through its flexible weighting mechanism, the GPA operator can effectively manage the uncertainty in such information, thus reducing deviations and errors in the decision-making process. For instance, research leveraging I-subgroup-based weighted generalized interval t-norm has significantly improved information aggregation in BUI environments [19]. Furthermore, employing the ordered weighted geometric averaging (OWGA) operator for BUI information fusion not only boosts decision accuracy but also enhances robustness and reliability [21]. Based on these insights, this paper introduces a novel support degree calculation approach within the BUI context. This method captures the difference between individual and group information. Additionally, it proposes a relevance weight calculation method that takes into account both the subjective and objective weights of decision-makers, providing a more holistic perspective. Subsequently, we construct the BUIGPWA operator to address the aggregation of expert information for qualitative attributes. This approach aims to optimize decision-making processes in complex and uncertain environments.

2.2. The TOPSIS Method for Processing Multi-Criteria Information in MCGDM

In the realm of MCGDM methods, supplier selection has consistently emerged as a critical research topic. Over the years, numerous researchers have endeavored to refine MCGDM approaches to better address the intricacies of supplier selection problems [24–27]. For instance, Wu et al. [24] enhanced the accuracy of prediction models by integrating grey relational analysis with Choquet fuzzy integral; Huang et al. [25] leveraged DEMATEL and ANP models to identify key indicators within sustainable emergency material reserve systems; and Liu et al. [26] constructed an intellectual capital evaluation model based on hybrid MCDM techniques. These research approaches have provided robust support for complex decision-making issues such as supplier selection. Concurrently, the technique for order preference by similarity to an ideal solution (TOPSIS) has garnered significant attention because of its effectiveness in handling multi-criteria evaluations [28–33]. To confront the environmental sustainability challenges in modern supply chains, hierarchical methods based on the best–worst method (BWM) integrated with TOPSIS have been developed to support the selection of environmentally sustainable suppliers [28]. Furthermore, in the context of green supplier selection in the food industry, the Pythagorean fuzzy TOPSIS approach has proven effective in managing uncertainty and fuzziness [29]. With the advent of industry 4.0, supplier selection has become increasingly complex, and the combination of Pythagorean fuzzy AHP and fuzzy TOPSIS has demonstrated unique advantages in tackling this complexity [30]. Similarly, the q-Rung orthopair fuzzy TOPSIS method has been applied to green supplier selection, showcasing its ability to provide stable results in uncertain environments [31]. An improved BWM-TOPSIS method, using scenario-based z-numbers and the reverse pagerank algorithm, has enhanced the accuracy and robustness of supplier selection [32]. Additionally, studies have proposed TOPSIS methods based on complex fuzzy rough information, particularly suited for energy supplier selection, exhibiting innovation in handling multi-attribute decision-making [33]. Building upon the BWM-TOPSIS framework, Wang et al. [34] introduced the BMW-TOPSIS model. This model enhances traditional TOPSIS by incorporating best, middle, and worst reference points, leading to more accurate and effective multi-criteria decision-making. These studies highlight that TOPSIS is popular because it is simple, effective, and robust in solving decision problems with multiple criteria. It is often used together with other tools like fuzzy logic and AHP to make better decisions [30–32]. However, in fundamentally uncertain environments, TOPSIS's reliance on deterministic data for evaluation criteria limits its ability to manage uncertainty and fuzziness effectively. To address this limitation and enhance the handling of fuzziness and uncertainty in the decision-making process, this paper proposes the BUI-BMW-TOPSIS integrated decision-making model. By integrating BUI theory into the BMW-TOPSIS model, this approach elevates the quality and reliability of evaluation information, enabling it to better navigate complex decision problems in uncertain and fuzzy environments. Consequently, the BUI-BMW-TOPSIS model enhances the accuracy and reliability of selecting recycling suppliers for shared electric bikes.

In summary, this paper employs the BUIGPWA operator and the BUI-BMW-TOPSIS method to address the multi-criteria group decision-making problem of selecting shared e-bike recycling suppliers.

3. Preliminaries

3.1. The Basic Uncertain Information

This section briefly summarizes the concepts of basic uncertain information, the power average operator, and the generalized power average operator, providing a theoretical foundation for subsequent discussions.

Definition 1 ([21]). *Given a binary pair $\langle x; c\rangle$, where $x \in [0,1]$ represents the input value (evaluation value) and $c \in [0,1]$ denotes the confidence level of the evaluation. As the binary pair $\langle x; c\rangle$ encompasses both the evaluation value and its reliability, it is denoted as basic uncertain information (BUI).*

Definition 2 ([21]). *Given two BUI pairs, $\alpha_i = \langle x_i; c_i\rangle (i = 1,2)$, the distance between α_1 and α_2 is defined as follows:*

$$d(\alpha_1, \alpha_2) = \frac{1}{2}(|x_1 - x_2| + |c_1 - c_2|) \tag{1}$$

and fulfills the following criteria:

(1) $d(\alpha_i, \alpha_j) \in [0,1]$;
(2) $d(\alpha_i, \alpha_j) = d(\alpha_j, \alpha_i)$;
(3) $d(\alpha_i, \alpha_j) = 0$ if and only if $\alpha_i = \alpha_j$.

Definition 3 ([21]). *Consider $\delta_i = \langle x_i; c_i\rangle (i = 1,2)$ as two BUI pairs, and the associated sorting method is outlined as follows:*

(1) If $x_1 \cdot c_1 < x_2 \cdot c_2$, then $\delta_1 \prec \delta_2$, that is, δ_1 is preferable to δ_2.
(2) If $x_1 \cdot c_1 = x_2 \cdot c_2$, then:
(a) If $x_1 < x_2$, then $\delta_1 \prec \delta_2$;
(b) If $x_1 = x_2$, then:
(i) If $c_1 \prec c_2$, then $\delta_1 \prec \delta_2$;
(ii) If $c_1 = c_2$, then $\delta_1 = \delta_2$.

3.2. The Power Average Operator

The literature has introduced a power average (PA) operator that considers the variations among individuals within an information group, and we now elucidate its conceptual framework and properties.

Definition 4 ([15]). *The power average operator is defined as a mapping $PA : \Omega^n \to \Omega$ that satisfies the following conditions:*

$$PA(\alpha_1, \ldots, \alpha_n) = \sum_{i=1}^{n} \frac{(1 + T(\alpha_i))\alpha_i}{\sum_{j=1}^{n}(1 + T(\alpha_j))} \tag{2}$$

where $T(\alpha_i) = \sum_{j\neq i}^{n} Supp(\alpha_i, \alpha_j)$, *where* $Supp(\alpha_i, \alpha_j)$ *denotes the support of* α_i *and* α_j, *and it fulfills the following criteria:*

(1) $Supp(\alpha_i, \alpha_j) \in [0, 1]$;
(2) $Supp(\alpha_i, \alpha_j) = Supp(\alpha_j, \alpha_i)$;
(3) If $|\alpha_i - \alpha_j| \leq |\alpha_s - \alpha_t|$, *then* $Supp(\alpha_i, \alpha_j) \geq Supp(\alpha_s, \alpha_t)$.

3.3. The Generalized Power Average Operator

Definition 5 ([15]). *A generalized power average (GPA) operator of dimension n is a mapping GPA,* $R^n \to R$, *defined by a parameter* $\lambda \in (-\infty, +\infty)$ *and* $\lambda \neq 0$, *according to the following formula:*

$$GPA(a_1, a_2, \ldots, a_n) = \left(\frac{\sum_{i=1}^{n} (1 + T(a_i)) a_i^{\lambda}}{\sum_{i=1}^{n} (1 + T(a_i))} \right)^{1/\lambda}, \tag{3}$$

where $T(\alpha_i) = \sum_{j\neq i}^{n} Supp(\alpha_i, \alpha_j)$ *and* $a_j (j = 1, 2, \ldots, n)$ *are a collection of arguments.*

If $\lambda = 1$, *the GPA operator degenerates into the PA operator*

$$GPA(a_1, \ldots, a_n) = PA(a_1, \ldots, a_n) = \sum_{i=1}^{n} \frac{1 + T(a_i)}{\sum_{j=1}^{n} (1 + T(a_j))} a_i \tag{4}$$

If $\lambda \to \infty$, *the GPA operator degenerates into the PG operator*

$$GPA(a_1, \ldots, a_n) = PG(a_1, \ldots, a_n) = \prod_{i=1}^{n} a_i^{\frac{1 + T(a_i)}{\sum_{j=1}^{n} (1 + T(a_j))}} \tag{5}$$

where $T(\alpha_i) = \frac{1}{n} \sum_{j=1; j\neq i}^{n} Supp(\alpha_i, \alpha_j)$.

4. Selection of Green Recycling Suppliers for Shared Electric Bikes and Its Indicator System

4.1. Problem Description

To evaluate the green recycling suppliers of shared electric bikes, a set of m recycling suppliers are selected as the evaluation objects $A = \{A_1, A_2, \ldots, A_m\}$. A group of g experts is organized to form the expert evaluation group $E = \{e_1, e_2, \ldots e_g\}$, with the weight vector of the experts denoted as $v = (v_1, \cdots, v_g)^{\mathrm{T}}, v_k \in [0, 1], \sum_{k=1}^{g} v_k = 1$. Each expert evaluates the green recycling suppliers of shared electric bikes based on n aspects, forming the attribute set $S = \{s_1, s_2, \ldots s_n\}$, which includes n_1 qualitative indicators and n_2 quantitative indicators, where $n_1 + n_2 = n$. The weight vector of the indicators is denoted as $\omega_j = (\omega_1, \cdots, \omega_n), \omega_j \in [0, 1], \sum_{j=1}^{n} \omega_j = 1$. The decision matrix for qualitative information is $U_1^k = \left(\alpha_{ij}^k\right)_{m \times n_1}$, where $\alpha_{ij}^k = \left\langle x_{ij}^k ; c_{ij}^k \right\rangle$ represents the basic uncertain information provided by expert $e_k \in E$ for the alternative $A_i \in A$ on attribute $S_j \in S$. The quantitative information matrix is converted into $U_2 = (\alpha_{ij})_{m \times n_2}$.

4.2. Indicator System for Green Recycling Suppliers of Shared Electric Bikes

Shared electric bikes extend the range of travel compared with standard bike-sharing services, yet they offer a more environmentally friendly and sustainable alternative to personal automobiles. As a result, they have become an ideal choice for fulfilling the diverse demands of users. However, the rapid growth of the shared electric bicycle market has resulted in a significant number of abandoned and idle bicycles, prompting an urgent need for operators to seek recycling suppliers for recycling and disposal tasks. In the field of shared electric bicycle recycling, enterprises must carefully consider various factors when selecting suppliers, including their financial capacity and vehicle search capabilities.

Based on the existing related recycling suppliers research [5–8], we identified the following eight evaluation indicators and their corresponding assessment criteria:

(1) Weight of recycled bikes s_1: The total weight of bikes that the recycling supplier can handle, with a particular emphasis on the effective recovery of incomplete or faulty bikes [5,6].

(2) Financial capability s_2: An assessment of whether the supplier has sufficient financial resources to support long-term, stable recycling operations, including funding for personnel salaries, training, and management expenses; equipment procurement and maintenance; and site leasing and operational costs, as well as the feasibility and flexibility of short- and long-term financial planning [5,6].

(3) Search capability s_3: An evaluation of the supplier's technical means and efficiency, particularly how it efficiently and accurately locates and collects abandoned shared electric bicycles from different regions. Key considerations include the effectiveness of location technologies (e.g., GPS, GIS), data collection and analysis capabilities, and collaboration with other relevant entities (such as shared bike platforms) [5,6].

(4) Repair and redeployment capability s_4: An assessment of the supplier's ability to repair and redeploy recovered shared electric bicycles. This includes the availability of repair technologies and equipment, the professional skills of personnel, and inspection processes to ensure vehicles meet safety and performance standards [5,6].

(5) Distribution and organization capability s_5: An evaluation of the supplier's ability to sort and transport large volumes of recovered bicycles, especially in terms of scheduling when public spaces are occupied. This encompasses the number and types of transport vehicles, the efficiency and flexibility of the dispatching system, and rapid solutions for managing accumulated bicycles. This indicator is primarily based on the evaluation metrics for mobile phone recycling suppliers studied in reference [7].

(6) Government public relations capability s_6: An assessment of the supplier's ability to communicate and coordinate promptly with government regulatory departments, ensuring smooth operations under policies, regulations, and management requirements. Key factors include establishing and maintaining relationships with local governments, sensitivity and understanding of policies and regulations, and crisis management and problem-solving capabilities [6].

(7) Battery recycling capability s_7: An evaluation of the supplier's ability to recycle and dispose of used batteries effectively, including safe storage facilities and management systems; compliance and efficiency in industrial solid waste disposal; and exploration and implementation of secondary utilization opportunities [7,8].

(8) Helmet recycling program s_8: An assessment of the supplier's ability to provide effective helmet recycling solutions, including the setup and operation of collection channels, sorting and processing technologies and processes, and the feasibility and environmental friendliness of resource-based treatment plans [6].

Although the current evaluation index system is relatively comprehensive, there are still deficiencies, particularly in assessing the capabilities of shared e-bike recycling suppliers in waste resource recycling and the management of new and old model transitions. Therefore, based on the existing index system, this paper introduces the following new indicators: the waste reutilization capability and the new–old transition handling capability of shared e-bikes. The evaluation criteria for these two novel indicators are as follows:

(9) Waste reutilization capability s_9: An evaluation of the supplier's ability to maximize the residual value of abandoned shared electric bicycles, including the development of reusable products, innovative practices in resource recycling, and specific measures to reduce waste and environmental pollution.

(10) New–old transition handling capability s_{10}: An assessment of the supplier's ability to respond to market and policy changes swiftly, promptly retiring old models and introducing new ones to ensure continuous business operations and management efficiency. This includes the efficiency of old model retirement processes, the speed and quality control

of new model introductions, and the flexibility and adaptability of business operations and management.

The specific indicator system is outlined in Table 1.

Table 1. The specific indicator system for green recycling suppliers of shared electric bikes.

Indicators	Meanings	Reference	Characteristics
Weight of recycled bikes s_1	Maximum total weight of electric bikes recycled daily, etc.	[5,6]	Quantitative
Financial capability s_2	Financial capability includes the ability to manage finances, conduct financial activities, maintain financial relationships, and achieve financial performance.	[5,6]	Qualitative
Search capability s_3	The capability to search for shared electric bikes that need to be recycled.	[5,6]	
Repair and redeployment capability s_4	The capability to repair damaged vehicles so that they meet the standards for redeployment.	[5,6]	
Distribution and organization capability s_5	The capability to distribute and organize shared electric bikes according to the distribution of pedestrian and vehicle flows, as well as traffic control requirements.	[7]	
Government public relations capability s_6	The ability to maintain long-term, close relationships and cooperation with regulatory bodies such as traffic management departments, to be familiar with government operations of shared bikes, and to meet governmental requirements related to traffic management.	[6]	
Battery recycling capability s_7	The capability to recycle and process used batteries from shared electric bikes, such as providing energy storage to the power grid with used batteries, and managing industrial solid waste after battery processing.	[7,8]	
Helmet recycling program s_8	Refers to a centralized helmet recycling program, such as collecting helmets for disassembly and classification, and sending recyclable materials to specialized institutions for processing.	[6]	
Waste reutilization capability s_9	The capability to recycle and reutilize severely damaged or old discarded electric bikes.	--	
New–old transition handling capability s_{10}	The capability to replace and handle old bikes during the introduction of new "split lock" bikes.	--	

5. An MCGDM Method for Shared Electric Bike Green Recycling Supplier Selection

In this study, a novel basic uncertain information generalized power weighted average (BUIGPWA) operator is proposed based on the generalized power weighted average (GPWA) operator, showcasing its relevant properties. Subsequently, this study introduces the basic uncertain information-based best–middle–worst method with the TOPSIS (BUI-BMW-TOPSIS) model, considering the best, middle, and worst reference points to further enhance decision accuracy and reliability. Finally, a new multi-criteria group decision-making (MCGDM) method is developed for evaluating green recycling suppliers for shared electric bikes by utilizing the BUIGPWA operator for aggregating group information and combining it with the BUI-BMW-TOPSIS model for MCGDM.

5.1. The Basic Uncertain Information Generalized Power Weight Average Operator

The BUIGPWA operator extends the capabilities of the GPWA operator [12] by incorporating the handling of uncertain information. Through the incorporation of expected value and uncertainty information, BUIGPWA enhances the accuracy and reliability of decision-making models. In comparison with the GPWA operator, BUIGPWA not only integrates multiple factors for evaluation but also effectively addresses data uncertainty. The method's flexible parameter settings allow for its application in a wide range of decision-making scenarios. In summary, BUIGPWA offers a more comprehensive and reliable MCGDM tool by taking into account weights, expected values, and uncertainty information. The definition of the BUIGPWA operator is as follows:

Definition 6. *Let* $\alpha_i = \langle x_i; c_i \rangle (i = 1, 2, \ldots n)$ *be a set of BUI (basic uncertain information), and let the weight vector of the array be* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$*, where* $\omega_i > 0$, $\sum\limits_{i=1}^{n} \omega_i = 1 (i = 1, 2, \cdots, n)$*. The BUIGPWA operator is defined as the mapping* $BUIGPWA : \Omega^n \to \Omega$*, such that it satisfies*

$$BUIGPWA(\alpha_1, \ldots, \alpha_n) = (GPWA(x_1, \ldots, x_n); GPWA(c_1, \ldots, c_n)) \tag{6}$$

where $\lambda \geq 0$ *and* $T(\alpha_i) = \sum_{j=1; j \neq i}^{n} \omega_j Supp(\alpha_i, \alpha_j)$*.* $Supp(\alpha_i, \alpha_j)$ *represents the support of* α_i *by* α_j*, and it satisfies the following properties:*

(1) $Supp(\alpha_i, \alpha_j) \in [0, 1]$*;*

(2) $Supp(\alpha_i, \alpha_j) = Supp(\alpha_j, \alpha_i)$*;*

(3) If $d(\alpha_i, \alpha_j) < d(\alpha_s, \alpha_t)$*, then* $Supp(\alpha_i, \alpha_j) > Supp(\alpha_s, \alpha_t)$*, where* $d(\alpha_i, \alpha_j)$ *is the distance between the BUI values* α_i *and* α_j*.*

Theorem 1. *Let* $\alpha_i = \langle x_i; c_i \rangle (i = 1, 2, \ldots n)$ *be a set of BUI, and let the weight vector of the array be* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$*, where* $\omega_i > 0$, $\sum\limits_{i=1}^{n} \omega_i = 1 (i = 1, 2, \cdots, n)$*. Then*

$$BUIGPWA(\alpha_1, \ldots, \alpha_n) = \left(\left(\frac{\sum\limits_{i=1}^{n} (1 + T(\alpha_i)) x_i^{\lambda}}{\sum\limits_{i=1}^{n} (1 + T(\alpha_i))} \right)^{1/\lambda} ; \left(\frac{\sum\limits_{i=1}^{n} (1 + T(\alpha_i)) c_i^{\lambda}}{\sum\limits_{i=1}^{n} (1 + T(\alpha_i))} \right)^{1/\lambda} \right) \tag{7}$$

where $T(\alpha_i)$ *refers to the formula defined in definition 6, and* $BUIGPWA(\alpha_1, \ldots, \alpha_n)$ *remains a BUI value.*

Proof. (1) According to Definition 5 and Definition 6,

$$BUIGPWA(\alpha_1, \ldots, \alpha_n) = \left(\left(\frac{\sum\limits_{i=1}^{n} (1 + T(\alpha_i)) x_i^{\lambda}}{\sum\limits_{i=1}^{n} (1 + T(\alpha_i))} \right)^{1/\lambda} ; \left(\frac{\sum\limits_{i=1}^{n} (1 + T(\alpha_i)) c_i^{\lambda}}{\sum\limits_{i=1}^{n} (1 + T(\alpha_i))} \right)^{1/\lambda} \right) \tag{8}$$

(2) Since $\alpha_i = \langle x_i; c_i \rangle (i = 1, 2, \ldots n)$ is a BUI value, then

$$\left(\frac{\sum\limits_{i=1}^{n} (1 + T(\alpha_i)) x_i^{\lambda}}{\sum\limits_{i=1}^{n} (1 + T(\alpha_i))} \right)^{1/\lambda} \in [0, 1], \left(\frac{\sum\limits_{i=1}^{n} (1 + T(\alpha_i)) c_i^{\lambda}}{\sum\limits_{i=1}^{n} (1 + T(\alpha_i))} \right)^{1/\lambda} \in [0, 1] \tag{9}$$

Given that $GPWA(x_1, \ldots, x_n) \in [0, 1]$, $GPWA(c_1, \ldots, c_n) \in [0, 1]$, it follows from definition 1 that $BUIGPWA(\alpha_1, \ldots, \alpha_n)$ is also a BUI value. □

Theorem 2. *Let* $\alpha_i = \langle x_i; c_i \rangle (i = 1, 2, \ldots n)$ *be a BUI value. If the weight vector is* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$*, satisfying* $\omega_i > 0$, $\sum\limits_{i=1}^{n} \omega_i = 1 (i = 1, 2, \cdots, n)$*, then*

(1) Idempotency: If $\alpha_i = \alpha = \langle x; c \rangle (i = 1, 2, \cdots, n)$*, then*

$$BUIGPWA(\alpha_1, \ldots, \alpha_n) = \alpha = (x; c) \tag{10}$$

(2) Monotonicity: Let $\beta_i = \langle x_i^*; c_i^* \rangle (i = 1, 2, \ldots n)$ *be a set of BUIs, satisfying* $x_i \leq x_i^*, c_i \leq c_i^*$*, and* $T(\alpha_i) = T(\beta_i) = T(i), (i = 1, 2, \ldots n)$*. Then*

$$BUIGPWA(\alpha_1, \ldots, \alpha_n) \leq BUIGPWA(\beta_1, \ldots, \beta_n) \tag{11}$$

(3) Boundedness: Let $\alpha^- = \langle x^-; c^- \rangle, \alpha^+ = \langle x^+; c^+ \rangle$*, satisfying*

$$\begin{array}{c} x^- = \min_i\{x_i\}, c^- = \min_i\{c_i\}; \\ x^+ = \max_i\{x_i\}, c^+ = \max_i\{c_i\}, \end{array} \tag{12}$$

Then

$$\alpha^- \leq BUIGPWA(\alpha_1, \ldots, \alpha_n) \leq \alpha^+ \tag{13}$$

Proof. (1) Given $\alpha_i = \langle x_i; c_i \rangle = \langle x; c \rangle (i = 1, 2, \cdots, n)$ and

$$\sum_{i=1}^{n} \frac{\omega_i(1 + T(\alpha_i))}{\sum_{j=1}^{n} \omega_j (1 + T(\alpha_j))} = 1 \tag{14}$$

Therefore,

$$\begin{cases} GPWA(x_1, \ldots, x_n) = x \\ GPWA(c_1, \ldots, c_n) = c \end{cases} \tag{15}$$

According to Theorem 1, we have

$$BUIGPWA(\alpha_1, \ldots, \alpha_n) = (GPWA(x_1, \ldots, x_n); GPWA(c_1, \ldots, c_n)) = \langle x; c \rangle \tag{16}$$

(2) Given $T(\alpha_i) = T(\beta_i) = T(i)(i = 1, 2, \cdots, n)$, it is easy to see that

$$\begin{cases} \omega_i(1 + T(\alpha_i)) / \sum_{i=1}^{n} \omega_i(1 + T(\alpha_i)) = \omega_i(1 + T(i)) / \sum_{i=1}^{n} \omega_i(1 + T(i)) \\ \\ \omega_i(1 + T(\beta_i)) / \sum_{i=1}^{n} \omega_i(1 + T(\beta_i)) = \omega_i(1 + T(i)) / \sum_{i=1}^{n} \omega_i(1 + T(i)) \end{cases} \tag{17}$$

Let $v_i = \omega_i(1 + T(i)) / (\sum_{i=1}^{n} \omega_i(1 + T(i))), i = 1, \cdots, n$.
Given $x_i \leq x_i^*, c_i \leq c_i^*$, it is easy to obtain

$$\begin{array}{c} GPWA(x_1, x_2, \ldots x_n) \leq GPWA\left(x_1^*, x_2^*, \ldots x_n^*\right) \\ \\ GPWA(c_1, c_2, \ldots c_n) \leq GPWA\left(c_1^*, c_2^*, \ldots c_n^*\right) \end{array} \tag{18}$$

Case 1: If $x_i = x_1^*, c_i = c_i^*$ for all $i = 1, 2, \ldots, n$ holds, then $\alpha_i = \beta_i = (x; c)$. Therefore, $BUIGPWA(\alpha_1, \ldots, \alpha_n) = BUIGPWA(\beta_1, \ldots, \beta_n)$.

Case 2: If there exists $i_0 \in \{1, 2, \ldots, n\}$ that does not satisfy $x_{i_0} = x_{i_0}^*$ or $c_{i_0} = c_{i_0}^*$. Assume $x_{i_0} \prec x_{i_0}^*$, then $GPWA(x_1, x_2, \ldots x_n) \leq GPWA\left(x_1^*, x_2^*, \ldots x_n^*\right)$, according to Definition 3

$$GPWA(x_1, x_2, \ldots x_n) \cdot GPWA(c_1, c_2, \ldots c_n) \leq GPWA(x_1^*, x_2^*, \ldots x_n^*) \cdot GPWA(c_1^*, c_2^*, \ldots c_n^*) \tag{19}$$

Therefore, $BUIGPWA(\alpha_1, \ldots, \alpha_n) \leq BUIGPWA(\beta_1, \ldots, \beta_n)$.
(3) Boundedness can be derived from monotonicity. □

5.2. Selection of Green Recycling Suppliers for Shared Electric Bikes Using the MCGDM Method Based on the BUIGPWA Operator and BUI-BMW-TOPSIS Model

The BMW-TOPSIS model [34] enhances the traditional TOPSIS method by integrating a middle reference point, thereby improving the MCGDM model. This approach provides a more comprehensive reflection of the overall performance of decision alternatives, making it more practical and effective for addressing complex decision problems and enhancing the accuracy and reliability of decisions. In the context of real decision-making processes, dealing with ambiguity and uncertainty in information is often inevitable. To address this challenge, this paper proposes the integration of the BUI framework into the BMW-TOPSIS model, resulting in the development of the BUI-BMW-TOPSIS model. This model is specifically designed to manage and represent uncertain information effectively within decision-making processes by leveraging the strengths of BUI in conjunction with the BMW-TOPSIS model.

Furthermore, in the context of selecting green recycling suppliers for shared electric bikes, this paper proposes an MCGDM method that utilizes the BUIGPWA operator and the BUI-BMW-TOPSIS model. Firstly, the BUIGPWA operator is employed to aggregate expert information on qualitative attributes, effectively addressing uncertainties in the evaluation process. Subsequently, the BUI-BMW-TOPSIS model is utilized to evaluate and rank green recycling suppliers comprehensively by integrating the best, middle, and worst reference points. The specific steps of these two stages are as follows:

Stage 1: Define the qualitative information decision matrix as $U_1^k(k = 1, 2, \ldots g)$ to form a comprehensive qualitative information decision matrix $U_1 = \left(\alpha_{ij}\right)_{m\times n_1}$. Use the BUIGPWA operator to calculate the qualitative index of expert information aggregation. The specific steps are as follows:

Step 1.1: Calculate the support degree between expert e_k and expert e_l

$$Supp\left(\alpha_{ij}^k, \alpha_{ij}^l\right) = 1 - d\left(\alpha_{ij}^k, \alpha_{ij}^l\right), k, l = 1, \cdots, t \tag{20}$$

where $d\left(\alpha_{ij}^k, \alpha_{ij}^l\right)$ represents the distance between the information provided by expert e_k and expert e_l, specifically shown as

$$d\left(\alpha_{ij}^k, \alpha_{ij}^l\right) = \frac{1}{2}\left(\left|x_{ij}^k - x_{ij}^l\right| + \left|c_{ij}^k - c_{ij}^l\right|\right) \tag{21}$$

Step 1.2: Calculate the support degree $T\left(\alpha_{ij}^k\right)$ of α_{ij}^k

$$T\left(\alpha_{ij}^k\right) = \sum\nolimits_{l=1;l\neq k}^{g} v_l Supp\left(\alpha_{ij}^k, \alpha_{ij}^l\right) \tag{22}$$

Then, calculate the aggregation weight $\xi_{ij}^k(k = 1, \cdots, t)$ of α_{ij}^k

$$\begin{gathered}\xi_{ij}^k = \frac{v_k\left(1+T\left(\alpha_{ij}^k\right)\right)}{\sum_{k=1}^{g} v_k\left(1+T\left(\alpha_{ij}^k\right)\right)}(k = 1, \cdots, t) \\ \xi_{ij}^k \geq 0, \sum\nolimits_{k=1}^{g} \xi_{ij}^k = 1\end{gathered} \tag{23}$$

Step 1.3: Use the BUIGPWA operator to calculate the aggregated group information and obtain comprehensive information

$$BUIGPWA(\alpha_1, \ldots, \alpha_n) = \left(\left(\sum_{i=1}^{n} \xi_{ij}^k x_i^\lambda\right)^{1/\lambda}; \left(\sum_{i=1}^{n} \xi_{ij}^k c_i^\lambda\right)^{1/\lambda}\right) \tag{24}$$

Thus, obtain the comprehensive qualitative decision matrix

$$U_1 = \left(\alpha_{ij}\right)_{m\times n_1}, \alpha_{ij} = \langle x_{ij}; c_{ij}\rangle, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n_1 \tag{25}$$

Stage 2: Construct the BUI-BMW-TOPSIS model to solve the evaluation problem of green recycling suppliers for shared electric bikes. The specific steps are as follows:

Step 2.1: Aggregate the qualitative decision matrix $U_1 = \left(\alpha_{ij}\right)_{m\times n_1}$ and the quantitative decision matrix $U_2 = \left(\alpha_{ij}\right)_{m\times n_2}$ to obtain the comprehensive decision matrix

$$U = \left(\alpha_{ij}\right)_{m\times n}, \alpha_{ij} = \langle x_{ij}; c_{ij}\rangle, (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n). \tag{26}$$

Step 2.1.1: The quantitative values x_{ij}^k for the quantitative indicator decision information R_{ij}^k are derived from the expert's expectation interval for the quantitative attribute indicator s_j. Assuming that expert e_k specifies an expectation interval $\left[x^k\min_{s_j}, x^k\max_{s_j}\right]$ for the quantitative attribute indicator s_j with a credibility factor c_j^k, the quantitative value x_{ij}^k for the quantitative indicator decision information R_{ij}^k can be calculated using the following transformation formula:

$$x_{ij}^k = \begin{cases} 0 & R_{ij}^k \le x^k\min_{s_j} \\ \frac{R_{ij}^k - x^k\min_{s_j}}{x^k\max_{s_j} - x^k\min_{s_j}} & x^k\min_{s_j} < R_{ij}^k < x^k\max_{s_j} \\ 1 & R_{ij}^k \ge x^k\max_{s_j} \end{cases} \tag{27}$$

Step 2.1.2: Employ the BUIGPWA operator obtained in Step 1 to aggregate the group information (assuming $\lambda = 1$). This aggregation process yields the quantitative values x_{ij}^k, forming the quantitative decision matrix U_2. Subsequently, integrate the qualitative decision matrix U_1 with the quantitative decision matrix U_2 to obtain the comprehensive decision matrix U.

Step 2.2: Find the best, worst, and middle reference points (B, M, W) of the comprehensive decision matrix U, and calculate the distance between scheme A_i and these three reference points.

Step 2.2.1: Find the best reference point B, expressed as follows:

$$B = \alpha^+ = \left(\alpha_1^+, \alpha_2^+, \ldots, \alpha_n^+\right) \tag{28}$$

where

$$\alpha_j^+ = \begin{cases} \max_i\{\alpha_{ij}\} = \left(\max_i\{x_{ij}\}; \max_i\{c_{ij}\}\right), j \in J_+ \\ \\ \min_i\{\alpha_{ij}\} = \left(\min_i\{x_{ij}\}; \min_i\{c_{ij}\}\right), j \in J_- \end{cases} \tag{29}$$

where $J_+ = \{1, 2, \ldots, k\}$ is the set of benefit indicators and $J_- = \{k+1, k+2, \ldots, n\}$ is the set of cost indicators.

Calculate the Euclidean distance $d(A_i, B)$ between scheme A_i and reference point B

$$d(A_i, B) = d(A_i, \alpha^+) = \sqrt{\sum_{j=1}^{n} d\left(\alpha_{ij}, \alpha_j^+\right)^2} \tag{30}$$

where $d\left(\alpha_{ij}, \alpha_j^+\right)$ is expressed as

$$d\left(\alpha_{ij}, \alpha_j^+\right) = \begin{cases} \left(\frac{1}{2}\left(\left|x_{ij} - x_j^+\right|\right) + \frac{1}{2}\left(\left|c_{ij} - c_j^+\right|\right)\right), j \in J_+ \\ \\ \left(\frac{1}{2}\left(\left|x_{ij} - x_j^-\right|\right) + \frac{1}{2}\left(\left|c_{ij} - c_j^-\right|\right)\right), j \in J_- \end{cases} \tag{31}$$

Step 2.2.2: Find the worst reference point W, expressed as follows:

$$W = \alpha^- = \left(\alpha_1^-, \alpha_2^-, \ldots, \alpha_n^-\right) \tag{32}$$

where

$$\alpha_j^- = \begin{cases} \min_i\{\alpha_{ij}\} = (\min_i\{x_{ij}\}; \min_i\{c_{ij}\}), j \in J_+ \\ \max_i\{\alpha_{ij}\} = (\max_i\{x_{ij}\}; \max_i\{c_{ij}\}), j \in J_- \end{cases} \tag{33}$$

where $J_+ = \{1, 2, \ldots, k\}$ is the set of benefit indicators and $J_- = \{k+1, k+2, \ldots, n\}$ is the set of cost indicators.

Calculate the Euclidean distance $d\left(\alpha_{ij}, \alpha_j^-\right)$ between scheme A_i and reference point W

$$d(A_i, W) = d(A_i, \alpha^-) = \sqrt{\sum_{j=1}^{n} d\left(\alpha_{ij}, \alpha_j^-\right)^2} \tag{34}$$

where $d\left(\alpha_{ij}, \alpha_j^-\right)$ is expressed as

$$d\left(\alpha_{ij}, \alpha_j^-\right) = \begin{cases} \left(\frac{1}{2}\left(\left|x_{ij} - x_j^-\right|\right) + \frac{1}{2}\left(\left|c_{ij} - c_j^-\right|\right)\right), j \in J_+ \\ \left(\frac{1}{2}\left(\left|x_{ij} - x_j^+\right|\right) + \frac{1}{2}\left(\left|c_{ij} - c_j^+\right|\right)\right), j \in J_- \end{cases} \tag{35}$$

Step 2.3: Find the middle reference point M, expressed as follows:

$$M = \alpha^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*) \tag{36}$$

where

$$\begin{gathered} \alpha_j^* = middle_i\{\alpha_{ij}\} = (middle_i\{x_{ij}\}; middle_i\{c_{ij}\}) \\ (i = \{1, 2, \ldots m\}, j = \{1, 2, \ldots n\}) \end{gathered} \tag{37}$$

Calculate the Euclidean distance $d(A_i, M)$ between scheme A_i and reference point M

$$d(A_i, M) = d(A_i, \alpha^*) = \sqrt{\sum_{j=1}^{n} d\left(\alpha_{ij}, \alpha_j^*\right)^2} \tag{38}$$

where $d\left(\alpha_{ij}, \alpha_j^*\right)$ is expressed as follows:

$$d\left(\alpha_{ij}, \alpha_j^*\right) = \frac{1}{2}\left(\left|x_{ij} - x_j^*\right|\right) + \frac{1}{2}\left(\left|c_{ij} - c_j^*\right|\right), j = 1, 2, \ldots n \tag{39}$$

Step 2.2.3: The decision-making scheme is divided into two parts as follows: $B - M$ and $M - W$. The $B - M$ part indicates that the scheme is better than M, while the $M - W$ part indicates that the scheme is worse than M, but the sorting result in each part is B-M, indicating $A_{\sigma(i)}^+$, and M-W, indicating $A_{\theta(i)}^-$. We can obtain a unified result as follows: $\left\{A_{\sigma(i)}^+\right\} \geq \left\{A_{\theta(i)}^-\right\}$, the specific division rules are as follows:

$$\text{(BM) If } \underset{j=1}{\overset{n}{\#}}\left(\alpha_{ij} \geq \alpha_j^*\right) > \frac{n}{2}, \text{ then } A_i \succcurlyeq M;$$

$$\text{(MW) If } \underset{j=1}{\overset{n}{\#}}\left(\alpha_{ij} \geq \alpha_j^*\right) \leqslant \frac{n}{2}, \text{ then } A_i \prec M;$$

where the symbol # represents "the number of", indicating the number of α_{ij} better than or equal to α_j^*. α_j^* is the attribute s_j corresponding to the evaluation value of reference point M.

Step 2.4: Use the BUI-BMW-TOPSIS model to rank the decision schemes in the same segment based on their relative closeness. The specific process is as follows:

$$(r_i^*)_c = \begin{cases} \frac{d_i^M}{d_i^B + d_i^M}, & A_i \text{ in segment } B-M, \\ \\ \frac{d_i^W}{d_i^M + d_i^W}, & A_i \text{ in segment } M-W. \end{cases} \tag{40}$$

where $d_i^\square$ represents the distance between the selected scheme A_i and the reference point $\square(\square = B, M, W)$.

Step 2.5: Rank the relative closeness $\left(r_i^*\right)_c$ of schemes in the $B-M$ segment from large to small, and rank the relative closeness $\left(r_i^*\right)_c$ of schemes in the $M-W$ segment from small to large. Obtain the internal ranking results of each segment and then combine the ranking results of the two segments to obtain the final overall ranking result

$$A_{\sigma(1)} \succ A_{\sigma(2)} \succ \ldots \succ A_{\sigma(m)}.$$

The process of the MCGDM approach for shared electric bike green recycling supplier selection is depicted in Figure 1.

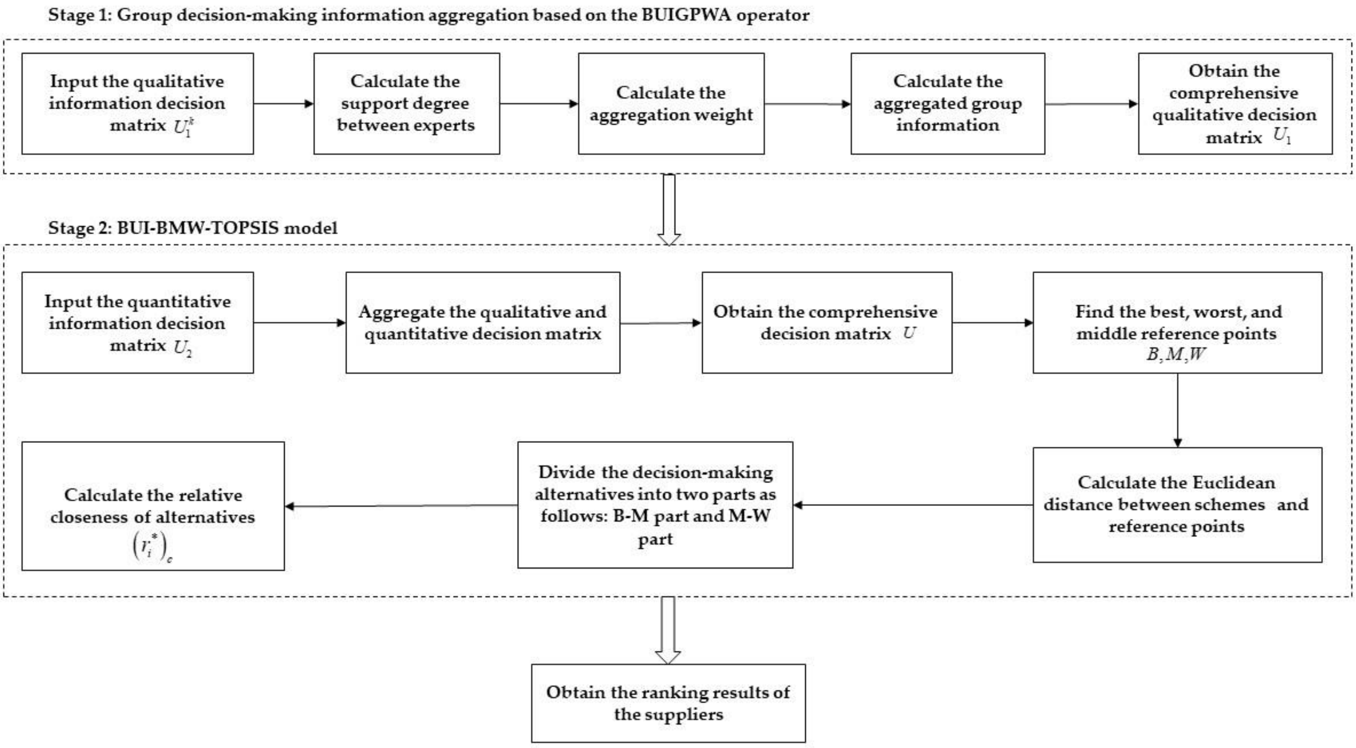

Figure 1. Illustrative process of the MCGDM approach.

6. Case Analysis and Comparative Analysis

6.1. Case Analysis

To evaluate the selection of green recycling suppliers for shared electric bikes, six suppliers were chosen as the evaluation objects $A_i (i = 1, 2, \ldots, 6)$. A panel of four field experts, denoted as $E = \{e_1, e_2, e_3, e_4\}$, was organized to conduct the evaluation. The weight vector for the experts is represented by $v = (0.25, 0.25, 0.25, 0.25)^T$. The positions and years of work experience of the four experts are shown in Table 2. The experts evaluated the suppliers based on the following ten criteria: weight of recycled bikes s_1, financial capability s_2, search capability s_3, repair and redeployment capability s_4, distribution and organization capability s_5, government public relations capability s_6, battery recycling capability s_7, helmet recycling program s_8, waste reutilization capability s_9, and new–old transition handling capability s_{10}. Among these, s_1 is a quantitative indicator, while s_2 through s_{10} are qualitative indicators, with the weight vector for the indicators represented by $\omega_j = \frac{1}{10}(j = 1, 2, \ldots, 10)$.

Table 2. The positions and years of work experience of experts.

	Position	Years of Experience
Expert 1	Environmental Scientist	12 years
Expert 2	Government environmental department personnel	21 years
Expert 3	Manager of shared electric bike recycling suppliers	15 years
Expert 4	Staff member of shared electric bike company	10 years

Stage 1: Solving the expert information aggregation problem in MCGDM using the BUIGPWA operator.

Step 1.1: By leveraging the basic uncertain information pairs $\alpha_{ij}^k = \left\langle x_{ij}^k; c_{ij}^k \right\rangle$ from different experts, the evaluation matrix $U_1^k = \left(\alpha_{ij}^k\right)_{6\times 9}$ for the set of cities under consideration $A = \{A_1, A_2, \ldots, A_6\}$ across various qualitative indicators is provided in the form of BUI pairs. The specific evaluation information is as follows:

$$U_1^1 = \begin{pmatrix} \langle 0.5; 0.8\rangle & \langle 0.5; 0.7\rangle & \langle 0.0; 0.6\rangle & \langle 1.0; 0.6\rangle & \langle 0.7; 0.4\rangle & \langle 1.0; 0.2\rangle & \langle 0.6; 0.4\rangle & \langle 0.8; 0.5\rangle & \langle 0.3; 0.2\rangle \\ \langle 0.6; 0.7\rangle & \langle 0.6; 0.8\rangle & \langle 0.3; 0.7\rangle & \langle 0.8; 0.7\rangle & \langle 0.5; 0.5\rangle & \langle 0.9; 0.7\rangle & \langle 0.2; 0.1\rangle & \langle 1.0; 0.6\rangle & \langle 0.4; 0.5\rangle \\ \langle 0.5; 0.6\rangle & \langle 0.4; 0.3\rangle & \langle 0.7; 0.5\rangle & \langle 0.4; 0.2\rangle & \langle 0.8; 0.9\rangle & \langle 0.5; 0.6\rangle & \langle 0.2; 0.2\rangle & \langle 0.6; 0.5\rangle & \langle 0.5; 0.3\rangle \\ \langle 0.7; 0.9\rangle & \langle 0.7; 0.5\rangle & \langle 0.1; 0.6\rangle & \langle 0.6; 0.9\rangle & \langle 0.3; 0.5\rangle & \langle 0.8; 0.5\rangle & \langle 0.9; 0.7\rangle & \langle 0.5; 0.7\rangle & \langle 0.5; 0.8\rangle \\ \langle 0.6; 0.8\rangle & \langle 0.2; 0.8\rangle & \langle 0.5; 0.4\rangle & \langle 0.2; 0.6\rangle & \langle 0.7; 0.6\rangle & \langle 0.8; 0.5\rangle & \langle 0.8; 0.8\rangle & \langle 0.6; 0.5\rangle & \langle 0.7; 0.6\rangle \\ \langle 0.5; 0.8\rangle & \langle 0.8; 0.5\rangle & \langle 0.6; 0.8\rangle & \langle 0.5; 0.4\rangle & \langle 0.8; 0.7\rangle & \langle 0.2; 0.8\rangle & \langle 0.4; 0.9\rangle & \langle 0.5; 0.8\rangle & \langle 0.8; 0.8\rangle \end{pmatrix}$$

$$U_1^2 = \begin{pmatrix} \langle 0.4; 0.3\rangle & \langle 0.7; 0.2\rangle & \langle 0.5; 0.4\rangle & \langle 0.8; 0.8\rangle & \langle 0.6; 0.8\rangle & \langle 0.8; 0.6\rangle & \langle 0.3; 0.4\rangle & \langle 0.5; 0.6\rangle & \langle 0.6; 0.8\rangle \\ \langle 0.8; 0.4\rangle & \langle 0.7; 0.5\rangle & \langle 0.3; 0.2\rangle & \langle 0.6; 0.6\rangle & \langle 0.4; 0.6\rangle & \langle 0.7; 0.8\rangle & \langle 0.5; 0.5\rangle & \langle 0.5; 0.5\rangle & \langle 0.8; 0.8\rangle \\ \langle 0.6; 0.2\rangle & \langle 0.5; 0.1\rangle & \langle 0.4; 0.6\rangle & \langle 0.5; 0.9\rangle & \langle 0.7; 0.7\rangle & \langle 0.6; 0.5\rangle & \langle 0.8; 0.2\rangle & \langle 0.7; 0.8\rangle & \langle 0.7; 0.5\rangle \\ \langle 0.7; 0.6\rangle & \langle 0.7; 0.5\rangle & \langle 0.6; 0.3\rangle & \langle 0.8; 0.3\rangle & \langle 0.2; 0.7\rangle & \langle 0.7; 0.7\rangle & \langle 0.6; 0.2\rangle & \langle 0.6; 0.8\rangle & \langle 0.7; 0.9\rangle \\ \langle 0.5; 0.3\rangle & \langle 0.6; 0.6\rangle & \langle 0.2; 0.1\rangle & \langle 0.7; 0.5\rangle & \langle 0.6; 0.8\rangle & \langle 0.9; 0.6\rangle & \langle 0.7; 0.4\rangle & \langle 0.4; 0.7\rangle & \langle 0.3; 0.4\rangle \\ \langle 0.8; 0.7\rangle & \langle 0.8; 0.4\rangle & \langle 0.3; 0.5\rangle & \langle 0.8; 1.0\rangle & \langle 0.7; 0.9\rangle & \langle 0.5; 0.9\rangle & \langle 0.6; 0.3\rangle & \langle 0.5; 0.9\rangle & \langle 0.5; 0.6\rangle \end{pmatrix}$$

$$U_1^3 = \begin{pmatrix} \langle 0.6; 0.9\rangle & \langle 0.7; 0.8\rangle & \langle 0.3; 0.4\rangle & \langle 0.5; 0.2\rangle & \langle 0.6; 0.4\rangle & \langle 0.7; 0.6\rangle & \langle 0.5; 0.9\rangle & \langle 0.2; 0.4\rangle & \langle 0.5; 0.5\rangle \\ \langle 0.5; 1.0\rangle & \langle 0.6; 0.9\rangle & \langle 0.7; 0.8\rangle & \langle 1.0; 0.5\rangle & \langle 0.9; 0.2\rangle & \langle 0.4; 0.2\rangle & \langle 0.3; 0.8\rangle & \langle 0.8; 0.6\rangle & \langle 0.6; 0.2\rangle \\ \langle 0.4; 0.8\rangle & \langle 0.5; 0.7\rangle & \langle 0.8; 0.7\rangle & \langle 0.7; 0.4\rangle & \langle 0.5; 0.6\rangle & \langle 0.6; 0.1\rangle & \langle 0.5; 0.8\rangle & \langle 0.5; 0.1\rangle & \langle 0.4; 0.4\rangle \\ \langle 0.8; 0.6\rangle & \langle 0.6; 0.5\rangle & \langle 0.2; 0.8\rangle & \langle 0.8; 0.3\rangle & \langle 0.6; 0.5\rangle & \langle 0.9; 0.5\rangle & \langle 0.8; 0.6\rangle & \langle 0.4; 0.2\rangle & \langle 0.8; 0.2\rangle \\ \langle 0.7; 0.5\rangle & \langle 0.8; 0.7\rangle & \langle 0.6; 0.9\rangle & \langle 0.9; 0.7\rangle & \langle 0.2; 0.2\rangle & \langle 0.5; 0.3\rangle & \langle 0.6; 0.7\rangle & \langle 0.6; 0.5\rangle & \langle 0.5; 0.3\rangle \\ \langle 0.6; 0.6\rangle & \langle 0.4; 1.0\rangle & \langle 0.7; 0.6\rangle & \langle 0.7; 0.2\rangle & \langle 1.0; 0.1\rangle & \langle 0.8; 0.4\rangle & \langle 0.7; 0.6\rangle & \langle 0.9; 0.4\rangle & \langle 1.0; 0.4\rangle \end{pmatrix}$$

$$U_1^4 = \begin{pmatrix} \langle 0.7; 0.8\rangle & \langle 0.6; 0.7\rangle & \langle 0.5; 0.6\rangle & \langle 0.8; 0.9\rangle & \langle 0.8; 0.8\rangle & \langle 0.4; 0.9\rangle & \langle 0.6; 0.6\rangle & \langle 0.6; 0.8\rangle & \langle 0.4; 0.8\rangle \\ \langle 0.4; 0.9\rangle & \langle 0.8; 0.6\rangle & \langle 0.9; 0.9\rangle & \langle 0.6; 0.7\rangle & \langle 0.7; 0.9\rangle & \langle 0.7; 1.0\rangle & \langle 0.5; 0.8\rangle & \langle 0.7; 0.7\rangle & \langle 0.8; 0.9\rangle \\ \langle 0.6; 0.6\rangle & \langle 0.5; 0.8\rangle & \langle 0.8; 1.0\rangle & \langle 0.4; 0.5\rangle & \langle 0.7; 0.6\rangle & \langle 0.6; 0.8\rangle & \langle 0.9; 0.6\rangle & \langle 0.5; 0.6\rangle & \langle 0.9; 0.9\rangle \\ \langle 0.9; 0.5\rangle & \langle 0.7; 0.5\rangle & \langle 0.4; 0.3\rangle & \langle 0.5; 0.5\rangle & \langle 0.4; 0.2\rangle & \langle 0.9; 0.3\rangle & \langle 1.0; 0.4\rangle & \langle 0.4; 0.2\rangle & \langle 0.6; 0.2\rangle \\ \langle 0.5; 0.3\rangle & \langle 0.5; 0.4\rangle & \langle 0.8; 0.4\rangle & \langle 0.9; 0.3\rangle & \langle 0.8; 0.1\rangle & \langle 0.5; 0.4\rangle & \langle 0.6; 0.5\rangle & \langle 0.6; 0.1\rangle & \langle 0.7; 0.3\rangle \\ \langle 0.8; 0.1\rangle & \langle 0.9; 0.2\rangle & \langle 0.5; 0.5\rangle & \langle 0.6; 0.3\rangle & \langle 0.6; 0.5\rangle & \langle 0.7; 0.6\rangle & \langle 0.4; 0.5\rangle & \langle 0.8; 0.2\rangle & \langle 0.7; 0.2\rangle \end{pmatrix}.$$

The specific steps for aggregating expert information are as follows:

Step 1.2: Calculate the support matrix $T^k = \left(T\left(\alpha_{ij}^k\right)\right)_{6\times 9}(k = 1, 2, 3, 4)$ for the experts as follows:

$$T^1=\begin{pmatrix}0.5262 & 0.5625 & 0.5500 & 0.4375 & 0.6125 & 0.4250 & 0.6125 & 0.5500 & 0.4875\\ 0.5125 & 0.6500 & 0.4250 & 0.5625 & 0.5750 & 0.4625 & 0.4375 & 0.6000 & 0.5000\\ 0.5500 & 0.5875 & 0.5875 & 0.5500 & 0.6000 & 0.6125 & 0.4250 & 0.6125 & 0.5500\\ 0.5875 & 0.7375 & 0.5375 & 0.4875 & 0.6250 & 0.6125 & 0.5750 & 0.5750 & 0.5125\\ 0.5500 & 0.5000 & 0.5625 & 0.4500 & 0.5125 & 0.4875 & 0.5875 & 0.6500 & 0.5750\\ 0.5375 & 0.5750 & 0.5875 & 0.5625 & 0.5625 & 0.4875 & 0.5250 & 0.5250 & 0.5250\end{pmatrix}$$

$$T^2=\begin{pmatrix}0.5250 & 0.5625 & 0.6250 & 0.5125 & 0.6125 & 0.5750 & 0.5625 & 0.6000 & 0.5625\\ 0.4875 & 0.6500 & 0.4500 & 0.6125 & 0.5500 & 0.5125 & 0.5625 & 0.5750 & 0.5500\\ 0.5750 & 0.5375 & 0.5375 & 0.5000 & 0.6500 & 0.6375 & 0.5000 & 0.5375 & 0.5750\\ 0.6625 & 0.7375 & 0.5125 & 0.5875 & 0.5500 & 0.5875 & 0.5000 & 0.5250 & 0.5125\\ 0.6250 & 0.6000 & 0.4125 & 0.5750 & 0.4875 & 0.5375 & 0.6125 & 0.5500 & 0.5750\\ 0.5875 & 0.5750 & 0.5875 & 0.4125 & 0.5125 & 0.5375 & 0.5500 & 0.5000 & 0.5250\end{pmatrix}$$

$$T^3=\begin{pmatrix}0.5255 & 0.5625 & 0.6250 & 0.4375 & 0.6125 & 0.5750 & 0.5375 & 0.5000 & 0.5875\\ 0.5125 & 0.5750 & 0.4500 & 0.5625 & 0.4250 & 0.4375 & 0.5625 & 0.6500 & 0.4750\\ 0.5000 & 0.6125 & 0.6125 & 0.5500 & 0.6250 & 0.5375 & 0.4500 & 0.5125 & 0.5500\\ 0.6625 & 0.7125 & 0.5125 & 0.5875 & 0.5750 & 0.6375 & 0.6000 & 0.5750 & 0.5125\\ 0.6000 & 0.5500 & 0.4375 & 0.5500 & 0.4125 & 0.5375 & 0.6375 & 0.6500 & 0.6250\\ 0.5875 & 0.3500 & 0.6125 & 0.5625 & 0.4125 & 0.4875 & 0.5750 & 0.5000 & 0.5250\end{pmatrix}$$

$$T^4=\begin{pmatrix}0.5250 & 0.5875 & 0.6000 & 0.4875 & 0.5875 & 0.4250 & 0.6125 & 0.5500 & 0.5875\\ 0.5125 & 0.6250 & 0.3750 & 0.5875 & 0.5000 & 0.4625 & 0.5625 & 0.6250 & 0.5250\\ 0.5750 & 0.5875 & 0.5375 & 0.6000 & 0.6750 & 0.5875 & 0.4750 & 0.6125 & 0.4250\\ 0.6125 & 0.7375 & 0.5625 & 0.5625 & 0.5500 & 0.5875 & 0.5750 & 0.5750 & 0.5375\\ 0.6250 & 0.5500 & 0.5125 & 0.5250 & 0.4625 & 0.5625 & 0.6375 & 0.5500 & 0.6250\\ 0.4625 & 0.5000 & 0.6375 & 0.5875 & 0.5375 & 0.5625 & 0.6000 & 0.4750 & 0.5250\end{pmatrix}$$

Step 1.3: Calculate the aggregation weights $D^k=\left(\xi_{ij}^k\right)_{6\times 9}(k=1,2,3,4)$ for the decision matrix $U_1^k=\left(\alpha_{ij}^k\right)_{6\times 9}$ as follows:

$$D^1=\begin{pmatrix}0.2513 & 0.2490 & 0.2422 & 0.2447 & 0.2510 & 0.2375 & 0.2549 & 0.2500 & 0.2390\\ 0.2510 & 0.2538 & 0.2500 & 0.2470 & 0.2603 & 0.2489 & 0.2347 & 0.2481 & 0.2479\\ 0.2500 & 0.2510 & 0.2530 & 0.2500 & 0.2443 & 0.2529 & 0.2436 & 0.2570 & 0.2541\\ 0.2433 & 0.2509 & 0.2510 & 0.2390 & 0.2579 & 0.2510 & 0.2520 & 0.2520 & 0.2490\\ 0.2422 & 0.2419 & 0.2637 & 0.2377 & 0.2574 & 0.2429 & 0.2452 & 0.2578 & 0.2461\\ 0.2490 & 0.2625 & 0.2471 & 0.2551 & 0.2593 & 0.2449 & 0.2440 & 0.2542 & 0.2500\end{pmatrix}$$

$$D^2=\begin{pmatrix}0.2500 & 0.2490 & 0.2539 & 0.2574 & 0.2510 & 0.2625 & 0.2470 & 0.2581 & 0.2510\\ 0.2469 & 0.2538 & 0.2544 & 0.2549 & 0.2562 & 0.2574 & 0.2551 & 0.2442 & 0.2562\\ 0.2540 & 0.2431 & 0.2450 & 0.2419 & 0.2519 & 0.2569 & 0.2564 & 0.2450 & 0.2582\\ 0.2548 & 0.2509 & 0.2469 & 0.2550 & 0.2460 & 0.2471 & 0.2400 & 0.2440 & 0.2490\\ 0.2539 & 0.2581 & 0.2384 & 0.2582 & 0.2532 & 0.2510 & 0.2490 & 0.2422 & 0.2461\\ 0.2571 & 0.2625 & 0.2471 & 0.2306 & 0.2510 & 0.2531 & 0.2480 & 0.2500 & 0.2520\end{pmatrix}$$

$$D^3=\begin{pmatrix}0.2487 & 0.2490 & 0.2539 & 0.2447 & 0.2510 & 0.2625 & 0.2431 & 0.2419 & 0.2500\\ 0.2510 & 0.2423 & 0.2544 & 0.2470 & 0.2355 & 0.2447 & 0.2551 & 0.2558 & 0.2438\\ 0.2419 & 0.2549 & 0.2570 & 0.2500 & 0.2481 & 0.2412 & 0.2479 & 0.2410 & 0.2541\\ 0.2548 & 0.2473 & 0.2469 & 0.2550 & 0.2500 & 0.2549 & 0.2560 & 0.2520 & 0.2490\\ 0.2500 & 0.2500 & 0.2426 & 0.2541 & 0.2404 & 0.2510 & 0.2529 & 0.2578 & 0.2539\\ 0.2571 & 0.2250 & 0.2510 & 0.2551 & 0.2344 & 0.2449 & 0.2520 & 0.2500 & 0.2480\end{pmatrix}$$

$$D^4=\begin{pmatrix}0.2500 & 0.2530 & 0.2500 & 0.2532 & 0.2471 & 0.2375 & 0.2549 & 0.2500 & 0.2550\\ 0.2510 & 0.2500 & 0.2412 & 0.2510 & 0.2479 & 0.2489 & 0.2551 & 0.2519 & 0.2521\\ 0.2540 & 0.2510 & 0.2450 & 0.2581 & 0.2557 & 0.2490 & 0.2521 & 0.2570 & 0.2336\\ 0.2471 & 0.2509 & 0.2551 & 0.2510 & 0.2460 & 0.2471 & 0.2520 & 0.2520 & 0.2531\\ 0.2539 & 0.2500 & 0.2553 & 0.2500 & 0.2489 & 0.2551 & 0.2422 & 0.2422 & 0.2539\\ 0.2368 & 0.2500 & 0.2549 & 0.2592 & 0.2552 & 0.2572 & 0.2560 & 0.2458 & 0.2500\end{pmatrix}$$

Step 1.4: Use the BUIGPWA operator to aggregate the group information and obtain the comprehensive information (assuming $\lambda = 1$). This will result in the following comprehensive qualitative decision matrix $U_1 = (\alpha_{ij})_{6\times 9}$:

$$U_1 = \begin{pmatrix} \langle 0.55;0.55\rangle & \langle 0.62;0.50\rangle & \langle 0.33;0.47\rangle & \langle 0.78;0.51\rangle & \langle 0.67;0.60\rangle & \langle 0.73;0.58\rangle & \langle 0.50;0.57\rangle & \langle 0.53;0.58\rangle & \langle 0.45;0.58\rangle \\ \langle 0.57;0.65\rangle & \langle 0.68;0.62\rangle & \langle 0.55;0.50\rangle & \langle 0.75;0.53\rangle & \langle 0.62;0.58\rangle & \langle 0.68;0.55\rangle & \langle 0.38;0.56\rangle & \langle 0.75;0.60\rangle & \langle 0.65;0.60\rangle \\ \langle 0.53;0.42\rangle & \langle 0.47;0.50\rangle & \langle 0.68;0.70\rangle & \langle 0.50;0.50\rangle & \langle 0.67;0.58\rangle & \langle 0.57;0.50\rangle & \langle 0.60;0.45\rangle & \langle 0.57;0.50\rangle & \langle 0.62;0.52\rangle \\ \langle 0.77;0.65\rangle & \langle 0.68;0.50\rangle & \langle 0.32;0.50\rangle & \langle 0.68;0.49\rangle & \langle 0.38;0.48\rangle & \langle 0.85;0.58\rangle & \langle 0.83;0.48\rangle & \langle 0.47;0.47\rangle & \langle 0.65;0.52\rangle \\ \langle 0.57;0.47\rangle & \langle 0.53;0.45\rangle & \langle 0.53;0.45\rangle & \langle 0.68;0.52\rangle & \langle 0.58;0.46\rangle & \langle 0.67;0.55\rangle & \langle 0.67;0.60\rangle & \langle 0.55;0.45\rangle & \langle 0.55;0.40\rangle \\ \langle 0.67;0.56\rangle & \langle 0.74;0.60\rangle & \langle 0.53;0.60\rangle & \langle 0.65;0.46\rangle & \langle 0.77;0.56\rangle & \langle 0.55;0.68\rangle & \langle 0.53;0.57\rangle & \langle 0.67;0.58\rangle & \langle 0.75;0.50\rangle \end{pmatrix}$$

Stage 2: Construct the BUI-BMW-TOPSIS model to solve the evaluation problem of green recycling suppliers for shared electric bikes.

Step 2.1: Obtain the quantitative decision matrix.

Step 2.1.1: Utilize Formula (27) to convert the quantitative information, where the value of the quantitative index decision information R_{iq}^k is calculated as per the following formula. The corresponding values of experts' expected interval $\left[x^k \min_{s_1}, x^k \max_{s_1}\right]$ and their confidence level c_1^k are shown in Table 3.

$$R_{i1}^k = \begin{pmatrix} 19800 & 26902 & 25599 & 34050 & 31254 & 27688 \end{pmatrix}.$$

Table 3. Expected intervals and their credibility from experts.

	Expert 1 e_1	**Expert 2 e_2**	**Expert 3 e_3**	**Expert 4 e_4**
Expectation interval	$[15,000, 30,000]$	$[13,000, 32,000]$	$[14,000, 33,000]$	$[16,000, 35,000]$
Credibility c_1^k	0.9	0.8	0.4	0.6

Based on Step 1, utilizing the BUIGPWA operator to aggregate group information, we obtain the quantitative information x_1^k, resulting in the following quantitative decision matrix U_2:

$$U_2 = (\alpha_{ij})_{6\times 1} = \begin{pmatrix} \langle 0.29;0.68\rangle & \langle 0.69;0.68\rangle & \langle 0.62;0.68\rangle & \langle 0.99;0.68\rangle & \langle 0.81;0.68\rangle & \langle 0.71;0.68\rangle \end{pmatrix}^T.$$

Step 2.1.2: Aggregate the comprehensive qualitative decision matrix U_1 with the quantitative decision matrix U_2 to derive the following comprehensive decision matrix U:

$$U = \begin{pmatrix} \langle 0.29;0.68\rangle & \langle 0.55;0.55\rangle & \langle 0.62;0.50\rangle & \langle 0.33;0.47\rangle & \langle 0.78;0.51\rangle & \langle 0.67;0.60\rangle & \langle 0.73;0.58\rangle & \langle 0.50;0.57\rangle & \langle 0.53;0.58\rangle & \langle 0.45;0.58\rangle \\ \langle 0.69;0.68\rangle & \langle 0.57;0.65\rangle & \langle 0.68;0.62\rangle & \langle 0.55;0.50\rangle & \langle 0.75;0.53\rangle & \langle 0.62;0.58\rangle & \langle 0.68;0.55\rangle & \langle 0.38;0.56\rangle & \langle 0.75;0.60\rangle & \langle 0.65;0.60\rangle \\ \langle 0.62;0.68\rangle & \langle 0.53;0.42\rangle & \langle 0.47;0.50\rangle & \langle 0.68;0.70\rangle & \langle 0.50;0.50\rangle & \langle 0.67;0.58\rangle & \langle 0.57;0.50\rangle & \langle 0.60;0.45\rangle & \langle 0.57;0.50\rangle & \langle 0.62;0.52\rangle \\ \langle 0.99;0.68\rangle & \langle 0.77;0.65\rangle & \langle 0.68;0.50\rangle & \langle 0.32;0.50\rangle & \langle 0.68;0.49\rangle & \langle 0.38;0.48\rangle & \langle 0.85;0.58\rangle & \langle 0.83;0.48\rangle & \langle 0.47;0.47\rangle & \langle 0.65;0.52\rangle \\ \langle 0.81;0.68\rangle & \langle 0.57;0.47\rangle & \langle 0.53;0.45\rangle & \langle 0.53;0.45\rangle & \langle 0.68;0.52\rangle & \langle 0.58;0.46\rangle & \langle 0.67;0.55\rangle & \langle 0.67;0.60\rangle & \langle 0.55;0.45\rangle & \langle 0.55;0.40\rangle \\ \langle 0.71;0.68\rangle & \langle 0.67;0.56\rangle & \langle 0.74;0.60\rangle & \langle 0.53;0.60\rangle & \langle 0.65;0.46\rangle & \langle 0.77;0.56\rangle & \langle 0.55;0.68\rangle & \langle 0.53;0.57\rangle & \langle 0.67;0.58\rangle & \langle 0.75;0.50\rangle \end{pmatrix}.$$

Step 2.2: Determine the best, worst, and middle reference points B, M, and W, and calculate the Euclidean distances d_i^B, d_i^M, and d_i^W between the alternative A_i and the reference points B, M, and W.

Step 2.2.1: Determine the best reference point $B = \alpha^+ = \left(\alpha_1^+, \alpha_2^+, \ldots, \alpha_n^+\right)$ and calculate the Euclidean distance d_i^B between the alternative A_i and the reference point B as follows:

$$B = \alpha^+ = \begin{pmatrix} \langle 0.99;0.68\rangle, \langle 0.77;0.65\rangle, \langle 0.74;0.62\rangle, \langle 0.68;0.70\rangle, \langle 0.78;0.53\rangle, \\ \langle 0.77;0.60\rangle, \langle 0.85;0.68\rangle, \langle 0.83;0.60\rangle, \langle 0.75;0.60\rangle, \langle 0.75;0.60\rangle \end{pmatrix}$$

$$d_i^B = (0.5757, 0.3934, 0.5203, 0.4620, 0.4753, 0.3384), (i = 1, 2, \ldots 6)$$

Step 2.2.2: Determine the worst reference point $W = \alpha^- = \left(\alpha_1^-, \alpha_2^-, \ldots, \alpha_n^-\right)$ and calculate the Euclidean distance d_i^W between the alternative A_i and the reference point W as follows:

$$W = \alpha^- = \begin{pmatrix} \langle 0.29; 0.68\rangle, \langle 0.53; 0.42\rangle, \langle 0.47; 0.50\rangle, \langle 0.32; 0.45\rangle, \langle 0.50; 0.46\rangle, \\ \langle 0.38; 0.46\rangle, \langle 0.55; 0.50\rangle, \langle 0.50; 0.45\rangle, \langle 0.47; 0.45\rangle, \langle 0.45; 0.40\rangle \end{pmatrix}$$

$$d_i^W = (0.3487, 0.5208, 0.4372, 0.5415, 0.3916, 0.5100), (i = 1, 2, \ldots 6)$$

Step 2.2.3: Determine the middle reference point $M = \alpha^* = \left(\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*\right)$ and calculate the Euclidean distance d_i^M between the alternative A_i and the reference point M as follows:

$$M = \alpha^* = \begin{pmatrix} \langle 0.69; 0.68\rangle, \langle 0.61; 0.55\rangle, \langle 0.62; 0.54\rangle, \langle 0.49; 0.54\rangle, \langle 0.67; 0.50\rangle, \\ \langle 0.62; 0.54\rangle, \langle 0.68; 0.57\rangle, \langle 0.59; 0.54\rangle, \langle 0.59; 0.53\rangle, \langle 0.61; 0.52\rangle \end{pmatrix}$$

$$d_i^M = (0.2795, 0.2117, 0.2690, 0.3387, 0.2013, 0.2086), (i = 1, 2, \ldots 6)$$

Step 2.3: Based on the segmentation principle for all decision alternatives, classify alternatives 2, 4, and 6 into the $B - M$ segment and alternatives 1, 3, and 5 into the $M - W$ segment. The reasons for this classification are as follows:

$$\underset{j=1}{\overset{n}{\#}}\left(\alpha_{2j} \geq \alpha_j^*\right) > \frac{n}{2}; \underset{j=1}{\overset{n}{\#}}\left(\alpha_{4j} \geq \alpha_j^*\right) > \frac{n}{2}; \underset{j=1}{\overset{n}{\#}}\left(\alpha_{6j} \geq \alpha_j^*\right) > \frac{n}{2} \Rightarrow A_i \succcurlyeq M$$

$$\underset{j=1}{\overset{n}{\#}}\left(\alpha_{1j} \geq \alpha_j^*\right) \leqslant \frac{n}{2}; \underset{j=1}{\overset{n}{\#}}\left(\alpha_{3j} \geq \alpha_j^*\right) \leqslant \frac{n}{2}; \underset{j=1}{\overset{n}{\#}}\left(\alpha_{5j} \geq \alpha_j^*\right) \leqslant \frac{n}{2} \Rightarrow A_i \prec M$$

Step 2.4: Calculate the relative closeness $\left(r_i^*\right)_c$ of the alternative A_i.

$$\left(r_i^*\right)_c = (0.5550, 0.6502, 0.6191, 0.5770, 0.6605, 0.6186), (i = 1, 2 \cdots 6)$$

Step 2.5: Rank the alternatives within the $B - M$ segment and the $M - W$ segment by sorting their relative closeness $\left(r_i^*\right)_c$ in ascending order. Combine the ranking results from both segments to obtain the final ranking. The final ranking results are as follows:

$$\begin{cases} B - M : A_4 \succ A_6 \succ A_2 \\ M - W : A_5 \succ A_3 \succ A_1 \end{cases} \Rightarrow A_4 \succ A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_1$$

6.2. Comparative Analysis

6.2.1. Comparative Analysis of Different Methods

This section conducts a comparative analysis of the ranking results obtained from the following three different methods:

Method 1: The BUI-BMW-TOPSIS model based on the BUIGPWA operator proposed in this paper;

Method 2: The traditional TOPSIS method, which does not consider the credibility of the decision-makers' evaluation information during the decision-making process;

Method 3: A modified approach in which the BUIGPWA operator is substituted with the BUIWA operator, neglecting the factor of mutual support among decision-makers.

This section assigns weights to each attribute as follows:

$$\omega = (0.06, 0.2, 0.1, 0.5, 0.05, 0.03, 0.02, 0.02, 0.01, 0.01).$$

The specific ranking results are shown in Table 4.

Table 4. Comparison of ranking results based on different decision-making methods.

Method	Decision-Making Method	Ranking Results
Method 1	The BUI-BMW-TOPSIS model proposed in this paper	$A_4 \succ A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_1$
Method 2	Traditional TOPSIS method [28,33]	$A_3 \succ A_2 \succ A_6 \succ A_5 \succ A_4 \succ A_1$
Method 3	The BUI-BMW-TOPSIS model based on the BUIWA operator [21]	$A_4 \succ A_6 \succ A_2 \succ A_1 \succ A_5 \succ A_3$

Based on the above rankings, we can draw the following conclusions:

(1) Comparison between method 1 and method 2: Method 1 ranks A_4 as the top choice, whereas method 2 evaluates A_3 as the first. There exists a notable discrepancy in their ranking results. This difference stems from the fact that method 1 incorporates middle reference points and the credibility of decision-makers' information into the decision-making process, thereby enhancing the scientificity and accuracy of the decision.

(2) Comparison between method 1 and method 3: Although both method 1 and method 3 place A_4 and A_6 in the top two positions, they differ in the ranking of A_1 and A_5. In method 1, A_5 is ranked fourth, while A_1 is at the bottom; in contrast, method 3 ranks A_1 fourth and A_5 fifth. This discrepancy arises because method 1 employs the BUIGPWA operator, which considers the degree of support among decision-makers, thus better reflecting the integration of group information.

In summary, the BUI-BMW-TOPSIS model based on the BUIGPWA operator proposed in this paper significantly improves the accuracy and reliability of decision-making.

6.2.2. Sensitivity Analysis of Attribute Weights

To observe the impact of different attribute weights on the decision-making outcomes, we discuss the variations in the model's ranking results under 10 different weight settings. This analysis serves to validate the sensitivity of the model to changes in attribute weights. The specific ranking results under these various weight settings are presented in Table 5.

Table 5. Ranking results under different attribute weights.

Case	Attribute Weights	Ranking Results
Case 1	$\omega = \begin{pmatrix} 0.19, 0.09, 0.09, 0.09, 0.09, \\ 0.09, 0.09, 0.09, 0.09, 0.09 \end{pmatrix}$	$A_4 \succ A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_1$
Case 2	$\omega = \begin{pmatrix} 0.09, 0.19, 0.09, 0.09, 0.09, \\ 0.09, 0.09, 0.09, 0.09, 0.09 \end{pmatrix}$	$A_4 \succ A_2 \succ A_6 \succ A_5 \succ A_1 \succ A_3$
Case 3	$\omega = \begin{pmatrix} 0.09, 0.09, 0.19, 0.09, 0.09, \\ 0.09, 0.09, 0.09, 0.09, 0.09 \end{pmatrix}$	$A_6 \succ A_4 \succ A_2 \succ A_1 \succ A_5 \succ A_3$
Case 4	$\omega = \begin{pmatrix} 0.09, 0.09, 0.09, 0.19, 0.09, \\ 0.09, 0.09, 0.09, 0.09, 0.09 \end{pmatrix}$	$A_6 \succ A_4 \succ A_2 \succ A_5 \succ A_3 \succ A_1$
Case 5	$\omega = \begin{pmatrix} 0.09, 0.09, 0.09, 0.09, 0.19, \\ 0.09, 0.09, 0.09, 0.09, 0.09 \end{pmatrix}$	$A_4 \succ A_2 \succ A_6 \succ A_5 \succ A_1 \succ A_3$
Case 6	$\omega = \begin{pmatrix} 0.09, 0.09, 0.09, 0.09, 0.09, \\ 0.19, 0.09, 0.09, 0.09, 0.09 \end{pmatrix}$	$A_6 \succ A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$
Case 7	$\omega = \begin{pmatrix} 0.09, 0.09, 0.09, 0.09, 0.09, \\ 0.09, 0.19, 0.09, 0.09, 0.09 \end{pmatrix}$	$A_4 \succ A_6 \succ A_2 \succ A_5 \succ A_1 \succ A_3$
Case 8	$\omega = \begin{pmatrix} 0.09, 0.09, 0.09, 0.09, 0.09, \\ 0.09, 0.09, 0.19, 0.09, 0.09 \end{pmatrix}$	$A_4 \succ A_2 \succ A_6 \succ A_5 \succ A_3 \succ A_1$
Case 9	$\omega = \begin{pmatrix} 0.09, 0.09, 0.09, 0.09, 0.09, \\ 0.09, 0.09, 0.09, 0.19, 0.09 \end{pmatrix}$	$A_2 \succ A_6 \succ A_4 \succ A_3 \succ A_5 \succ A_1$
Case 10	$\omega = \begin{pmatrix} 0.09, 0.09, 0.09, 0.09, 0.09, \\ 0.09, 0.09, 0.09, 0.09, 0.19 \end{pmatrix}$	$A_6 \succ A_2 \succ A_4 \succ A_3 \succ A_1 \succ A_5$

Based on Table 5, we can derive the following conclusions:

(1) When the weights assigned to the weight of recycled bikes s_1, financial capability s_2, distribution and organization capability s_5, battery recycling capability s_7, and helmet

recycling program s_8 are relatively high, A_4 is the optimal alternative, indicating that A_4 performs better in these five attributes.

(2) When the weights of search capability s_3, repair and redeployment capability s_4, government public relations capability s_6, and new–old transition handling capability s_{10} are relatively high, A_6 is the optimal alternative, indicating that A_6 performs better in these four attributes.

(3) When the weight of waste reutilization capability s_9 is relatively high, A_2 is the optimal alternative, indicating that A_2 performs better in this attribute.

This analysis underscores the significant impact of varying attribute weights on the ranking of solutions. The varied performance of different solutions under different scenarios reflects the high sensitivity of the model to changes in attribute weights.

7. Conclusions

This study aims to address the MCGDM problem in selecting green recycling suppliers for shared electric bikes by introducing an innovative evaluation method. The proposed method integrates the BUIGPWA operator with the BUI-BMW-TOPSIS model. This approach effectively addresses uncertainties and challenges in MCGDM during the evaluation process. The primary conclusions drawn from this study are as follows:

7.1. Practical Implications

(1) Comprehensive indicator system construction: This paper constructs an evaluation index system covering multiple dimensions, with the following key indicators newly added: the waste reutilization capability and the new–old transition handling capability of shared e-bikes. This system also encompasses various aspects such as battery recycling capabilities and repair and redeployment capabilities, enabling enterprises to comprehensively assess the performance of suppliers in the green recycling process. By introducing these new indicators, it provides a scientific decision-making basis for green recycling suppliers of shared e-bikes, thereby effectively improving recycling efficiency, reducing waste of resources, and mitigating environmental pollution.

(2) Consideration of evaluation information credibility: This paper considers the credibility of evaluators' information in the decision-making process, making decisions in uncertain and complex environments more scientific and accurate. This approach enhances the reliability and objectivity of decision-making during the supplier selection process.

(3) Impact of different indicator weights on evaluation results: Changes in attribute weights significantly affect evaluation outcomes. The different preferences of decision-makers towards various indicators may lead to varying supplier ranking results. As such, it is crucial to flexibly adjust the weights of indicators to adapt to the specific needs of different scenarios. In conjunction with the analysis of attribute weight sensitivity in Section 6.2.2, decision-makers can set personalized weights, thereby optimizing the decision-making process, better reflecting actual conditions, and making optimal choices.

7.2. Theoretical Implications

(1) Innovation in group information aggregation methods: This paper introduces the theory of BUI, proposing a novel information representation method that offers a more diverse set of tools for information aggregation in MCGDM. The application of the BUIGPWA operator expands the applicability of the GPA operator in handling uncertain information, effectively bridging the gap of traditional methods in managing fuzzy information.

(2) Handling of MCGDM information: This study presents the BUI-BMW-TOPSIS model, which is capable of processing multi-dimensional information from different decision-makers more efficiently. This enhancement improves the accuracy and robustness of MCDM in complex scenarios, contributing fresh insights and a new framework to the development of MCGDM theory

7.3. Limitations and Future Research

Despite the satisfactory performance of the proposed BUIGPWA operator and BUI-BMW-TOPSIS model in evaluating shared e-bike green recycling suppliers, there are still limitations to consider. Firstly, the reliance on experts' subjective judgments during the evaluation process may lead to biased results. Secondly, the BUI method has limited performance in handling complex linguistic information, making it difficult to fully capture subjective or ambiguous evaluation information. To address these limitations, future research could introduce basic uncertain linguistic information (BULI) [19,35–38], which could enhance the model's ability to process linguistic expressions and uncertainty. BULI is better suited to simulate human cognition, improving the accuracy and flexibility of decision-making, and thus further optimizing the selection and management of green recycling suppliers.

In summary, this study accomplishes two main objectives. Firstly, it theoretically enriches the application scenarios of MCGDM methods. Secondly, it practically offers scientific guidance for selecting green recycling suppliers, demonstrating this study's significant practical implications and theoretical implications. Looking ahead, future research will build upon these findings to further explore this field and contribute to the realization of sustainable development goals.

Author Contributions: Conceptualization, L.L. and F.S.; methodology, L.L. and F.S.; software, F.S.; validation, C.H. and F.S.; formal analysis, L.L. and F.S.; investigation, C.H. and F.S.; data curation, L.L. and F.S.; writing—original draft preparation, L.L. and F.S.; writing—review and editing, L.L., C.H. and F.S.; visualization, L.L. and F.S.; supervision L.L. and C.H.; funding acquisition, L.L. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the National Natural Science Foundation of China (Grant no. 71991465) and the Scientific Research Project of Hunan Provincial Department of Education, China (Grant no. 20A125).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the first author.

Conflicts of Interest: The authors declare no conflicts of interest.

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