KYBERNETIKA — VOLUME 54 (2018), NUMBER 4, PAGES 648–663

DEPENDENCE OF HIDDEN ATTRACTORS ON NON-LINEARITY AND HAMILTON ENERGY IN A CLASS OF CHAOTIC SYSTEM

Ge Zhang, Chunni Wang, Ahmed Alsaedi, Jun Ma and Guodong Ren

Non-linearity is essential for occurrence of chaos in dynamical system. The size of phase space and formation of attractors are much dependent on the setting of nonlinear function and parameters. In this paper, a three-variable dynamical system is controlled by different nonlinear function thus a class of chaotic system is presented, the Hamilton function is calculated to find the statistical dynamical property of the improved dynamical systems composed of hidden attractors. The standard dynamical analysis is confirmed in numerical studies, and the dependence of attractors and Hamilton energy on non-linearity selection is discussed. It is found that lower average Hamilton energy can be detected when intensity of nonlinear function is enhanced. It indicates that non-linearity can decrease the energy cost triggering for dynamical behaviors.

Keywords: Helmholtz theorem, chaos, hidden attractor,bifurcation, Hamilton energy

Classification: 37B25, 37L30

1. INTRODUCTION

In the end of 19th century, Poincaré discovered the emergence of intrinsic stochastic properties in the deterministic systems by handling three-body problems [6]. In 1963, Lorenz presented the first realistic example for chaotic solution approached from a dissipative system described by deterministic dynamical equations during the predicting of weather change. It is confirmed that weather forecast with long period becomes difficult because the results are much dependent on the initial setting, and this phenomenon is called as butterfly effect [35]. As a result, sensitivity to initial setting is thought as one distinct property of chaotic systems [35, 43]. Furthermore, transition from periodical behavior to chaos and more statistical analysis on chaotic system become more attractive [8, 9, 13, 52]. Indeed, the application of chaos is often appreciated when its dynamical properties and behaviors are known for circuit implement. For example, signal oscillator [1, 23] can be designed by using chaotic circuits [39, 48, 51, 53, 62, 66], chaotic neural network is used to process and estimate signals [2, 3, 54], secure communication and image encryption [11, 25, 30, 32, 61, 65, 68] can be realized by using kinds of chaotic systems.

DOI: 10.14736/kyb-2018-4-0648

Inspired by the exact description on chaos, which period three implies chaos, the chaos emergence and ways to chaos occurrence are discussed [34]. For example, the ecologist May R M [46] found the occurrence of chaos via bifurcation in Logistic map. Feigenbaum [12] found two generic constants from period doubling bifurcation to chaos [22, 31]. Shaw et al. [55] investigated the droplet from dripping faucet and chaotic attractors were confirmed from the sampled time series. Within the topics associated with chaos and hyperchaos, many new chaotic systems are designed and improved to generate specific chaotic attractors [5, 19, 21, 26]. Furthermore, many schemes are proposed for realizing chaos control and synchronization [40, 45, 42, 60, 41]. The formation and profile of attractors tell more information about dynamical systems. Leonov and Kuznettsov proposed the concept about hidden attractors [27, 28, 29], which a basin of attraction that does not intersect with small neighborhoods of any equilibrium points. Dudkowski et al. [10] discussed the dynamical behaviors in a class of nonlinear systems composed of hidden attractors. In fact, the dynamics of attractors could be dependent on the shape of equilibrium points, for example, Wang et al. [59] discussed the attractors in system composed arbitrary number of equilibrium points, while Sprott et al. [20, 47] investigated the dynamical system that equilibrium points are located on line shape. Most of the well-known dynamical systems can generate finite number of attractors, and some dynamical systems can be controlled to generate infinite number of attractors by generating infinite equilibrium points, which can be realized by using nonlinear function such as step function, Jerk function. As a result, multi-scroll attractors [17, 38, 67] can be induced in the dynamical systems. More interesting, hidden attractors have been paid much attention and the dynamics transition is investigated by modulating the constraint formula on equilibrium points [14, 15, 16, 17, 50, 58, 69]. In fact, the dynamics of system is much dependent on the parameter region and nonlinear interaction function as well. Indeed, standard analysis [64, 70] such bifurcation calculation and Lyapunov exponent approach are available for detecting emergence of chaos.

In this paper, the nonlinear terms in a three-variable dynamical system composed of hidden attractors are modulated by different types, and the Hamilton energy, phase portrait and sampled time series are analyzed to understand the transition of hidden attractors and energy dependence on non-linearity selection.

2. MODEL AND SCHEME

In Ref. [49], a chaotic system with different shapes of equilibria is presented to discuss the dynamics of hidden attractor. For example, the nonlinear function is selected carefully so that the system can exhibit chaotic attractor with circular equilibrium, ellipse equilibrium, square-shaped equilibrium, and/or rectangle-shaped equilibrium. As mentioned in Ref. [33], multi-scroll attractors can be triggered in the chaotic system coupled by Jerk function, which can geneate a lot of equilirbium points in the chaotic system. A generic dynamical system composed of hidden attractors can be described by

$$\begin{cases} \dot{x} = az + f \\ \dot{y} = bxz + cz^3 \\ \dot{z} = x^2 + y^2 - r^2 + dxz, \end{cases} \tag{1}$$

where x, y, z are output variables, the parameters are often fixed at $a = -0.1$, $b = 1$, $c = -1.2$, $r = 1.1$, f describes the type of nonlinear function. It is confirmed that the equilibrium points are located in annulus defined by $x^2 + y^2 = r^2$. The position of equilibrium points are modulated when each kind of nonlinear functions f is applied, respectively. For simplicity, four kinds of nonlinear function are considered for following discussion.

$$
\begin{aligned}
&f_1 = A\sin(\omega y + \varphi); \qquad && f_4 = k(\alpha + 3\beta y^2)x; \\
&f_2 = \begin{cases} A & \omega y + \varphi > 0 \\ 0 & \omega y + \varphi = 0 \\ -A & \omega y + \varphi < 0 \end{cases}; \qquad && f_3 = \begin{cases} A(\omega y + \varphi) & \omega y + \varphi > 0 \\ 0 & \omega y + \varphi = 0 \\ -A(\omega y + \varphi) & \omega y + \varphi < 0 \end{cases}
\end{aligned} \tag{2}
$$

where nonlinear function f_1 encodes the sampled time series and can input quasi-perodical signals, f_2 is a discontinuous sign function and the switch depends on the outputs, f_3 inputs positive modulation, φ is initial phase value. f_4 is memritor-based function [4, 7, 18, 36, 57] with memory effect dependent on the variable y. As mentioned in Ref. [36], the memory effect based on memristor could be available for selection of multiple modes in electrical activities of neuron. At first, the equilibrium points are estimated when the amplitude of nonlinear function f_1 is set as $A \neq 0$, it is approached as follows

$$
\begin{cases}
A\sin(\omega y + \varphi) = 0 \to y = \dfrac{n\pi - \varphi}{\omega} \\
z(bx + cz^2) = 0 \to x \in R \\
x^2 + (\dfrac{n\pi - \varphi}{\omega})^2 = r^2 \\
z = 0.
\end{cases} \tag{3}
$$

As a result, the equilibrium points can be estimated by

$$
\begin{cases}
x = \pm r \\
y = 0, n\pi - \varphi = 0 \\
z = 0
\end{cases}; \qquad
\begin{cases}
y \neq 0, n\pi - \varphi \neq 0 \\
x^2 + (\dfrac{n\pi - \varphi}{\omega})^2 = r^2. \\
z = 0
\end{cases} \tag{4}
$$

While the equilibrium points under $z \neq 0$ are controlled to be free in a large region, it is approached as follows

$$
\begin{cases}
A\sin(\omega y + \varphi) = -az \to y = \dfrac{1}{\omega}\sin^{-1}(-\dfrac{az}{A}) - \dfrac{\varphi}{\omega} \\
z(bx + cz^2) = 0 \to x = -\dfrac{c}{b}z^2 \\
c^2z^4 + b^2[\dfrac{1}{\omega}\sin^{-1}(-\dfrac{az}{A}) - \dfrac{\varphi}{\omega}]^2 - b^2r^2 - bcdz^3 = 0.
\end{cases} \tag{5}
$$

When f_4 is applied for stability analysis, the equilibrium points will be controlled by $y = r$, $-r$ at $x = 0$, $z = 0$ while it will be dependent on the variable x, z as $x^2 + dxz = r^2$ at $y = 0$; otherwise, it can be dependent on all the variables. As is well known, continuous power supply is helpful to keep the oscillating behavior in the dynamical system. The standard Hamilton energy [33, 42, 56, 63] in dimensionless dynamical system can be approached by using Helmholtz theorem [24], which the dynamical system can be mapped into two vector field named as gradient vector f_d and rotational field f_c as follows

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = (J(x,y,z) + R(x,y,z))\nabla H = f_c(x,y,z) + f_d(x,y,z). \tag{6}$$

When the nonlinear function is described by f_1, f_2, f_3, the vector field is defined by

$$\begin{cases} f_c = (x,y,z) = J\,(x,y,z)\,\nabla H = \begin{pmatrix} az + f_{1,2,3} + \frac{ac}{br^2}xz^3 \\ bxz + cz^3 - \frac{c}{r^2}y^2z^3 + \frac{b}{a}xf_{1,2,3} \\ x^2 + y^2 - r^2 \end{pmatrix} \\ f_d = (x,y,z) = R\,(x,y,z)\,\nabla H = \begin{pmatrix} -\frac{ac}{br^2}xz^3 \\ \frac{c}{r^2}y^2z^3 - \frac{b}{a}xf_{1,2,3} \\ dxz \end{pmatrix}. \end{cases} \tag{7}$$

Where the subsrcipt 1, 2, 3 defines different nonlinear functions, and the vector field should meet the criterion [24, 63] as follows

$$\begin{cases} \nabla H^T f_c(x,y,z) = 0 \\ \nabla H^T f_d(x,y,z) = \mathrm{d}H/\mathrm{d}t = \dot{H}. \end{cases} \tag{8}$$

As a result, the Hamilton function is approached by

$$H = H_{1,2,3} = -\frac{b}{2a}x^2 + y + \frac{c}{4r^2}z^4. \tag{9}$$

That is, the Hamilton energy holds the same formula when nonlinear terms are controlled by the functions f_1, f_2, f_3. Furthermore, the derivative of Hamilton energy function with respect to time is verified by

$$\mathrm{d}H/\mathrm{d}t = \dot{H} = \dot{H}_{1,2,3} = -\frac{b}{a}xf_{1,2,3} + \frac{c}{r^2}x^2z^3 + \frac{c}{r^2}y^2z^3 + \frac{c}{r^2}dxz^4. \tag{10}$$

It is found that the Hamilton energy function with respect to time is dependent on the setting for non-linearity completely. In the case for setting function f_4 with multiplication between different variables, the vector is calculated by

$$\begin{cases} f_c = (x,y,z) = \begin{pmatrix} az + \frac{ac}{br^2}xz^3 \\ bxz + cz^3 - \frac{c}{r^2}y^2z^3 \\ x^2 + y^2 - r^2 \end{pmatrix} \\ f_d = (x,y,z) = \begin{pmatrix} f_4 - \frac{ac}{br^2}xz^3 \\ \frac{c}{r^2}y^2z^3 \\ dxz \end{pmatrix}. \end{cases} \tag{11}$$

According to the criterion shown in Eq.(8), the Hamilton function can be calculated as follows

$$\begin{cases} H = H_4 = -\dfrac{b}{2a}x^2 + y + \dfrac{c}{4r^2}z^4 \\ \mathrm{d}H/\mathrm{d}t = \dot{H}_4 = -\dfrac{b}{a}xf_4 + \dfrac{c}{r^2}x^2z^3 + \dfrac{c}{r^2}y^2z^3 + \dfrac{c}{r^2}dxz^4. \end{cases} \tag{12}$$

That is, the Hamilton energy function is dependent on the variables, parameter setting and nonlinear terms as well. As a result, restriction of variable and phase compression can modulate the Hamilton energy, and then the attractors can be controlled in effective way.

3. NUMERICAL RESULTS AND DISCUSSION

The fourth order Runge–Kutta algorithm is used to find solutions from the dynamical system with time step h=0.01, and ODE45 is carried out on Matlab tool. The initial values are selected as $(x_0, y_0, z_0) = (0.1, 0.2, 0.3)$, parameters are selected as $a = -0.1$, $b = 1$, $c = -1.2$, $r = 1.1$. In fact, the improved system (1) driven by $f_1 = A\sin(\omega y + \varphi)$ is similar to the dynamical system mapped from the Jerk circuit that the outputs are encoded by quasi-periodical function and then the dynamical system can be controlled completely. It is important to detect the emergence of chaos in this dynamical system when different nonlinear functions are applied. There is a lot of different exact definitions of the chaos phenomenon like the definition in the Smale's sense, Devaney's sense, Li-Yorke's sense, ergodic definition. In this paper, we use the Li–Yorke's definition to observe the occurrence of chaos by calculating the Lyapuonv exponent spectrum reported by Wolf in Ref. [64]. In Figure 1, the phase portrait is calculated to observe the dependence of attractors on bifurcation parameters setting.

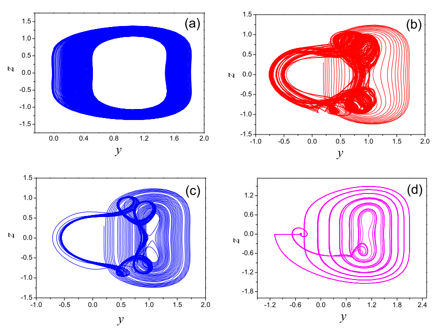

Fig. 1. Different attractors are plotted by setting different parameters d, A. For (a) $d = 0.005$, $A = 0$; (b) $d = -0.06$, $A = 0.01$; (c) $d = -0.06$, $A = 0$; (d) $d = -0.6$, $A = 0.01$; and $\omega = \pi/4$, $\varphi = 0.0$.

It is found that chaotic attractors can be tamed to trigger multi-scroll attracors, and chaos can be suppressed under applying appropriate parameter setting. Furthermore, the bifurcation analysis and largest Lyapunov exponent [64] spectrum are calculated to detect the occurrence of chaos for parameter region, the results are plotted in Figure 2.

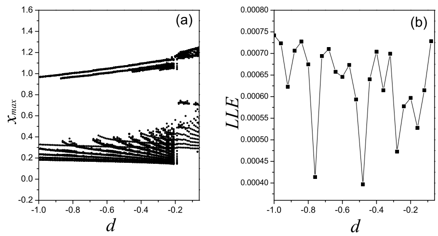

Fig. 2. Bifurcation diagram and largest Lyapunov exponents are calculated by changing the bifurcation parameter d, the other parameters are fixed at $A = 0.01$,$\omega = \pi/4$, $\varphi = 0$. $x_{\max}$ represents the maximal value in the sampled time series for variable x, and it is detected as $x(t-1) < x_{\max}(t) < x(t+1)$.

That is, the largest Lyapunov exponent is much close to zero with increasing the parameter d, and the bifurcation analysis for maximal variable confirms that the first variable can show multiple peaks to induce multi-periodical mode in the sampled time series. Furthermore, the evolution of the variables is calculated to show the transition and stability of attractors in Figure 3.

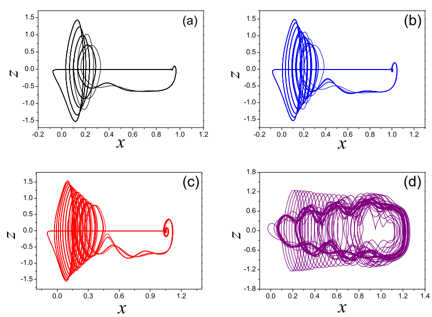

Fig. 3. Formation of attractors, for (a) $d = -1$; (b) $d = -0.6$; (c) $d = -0.3$; (d) $d = -0.03$; the other parameters are fixed at $A = 0.01$, $\omega = \pi/4$, $\varphi = 0$. The nonlinear function is selected by f_1.

That is, multi-scroll attractors can be suppressed to behave periodical oscillation when nonlinear modulation f_1 is applied. On the other hand, the modulation from f_1 can also be suppressed by another nonlinear term dxz via setting appropriate parameter d, so that multi-scroll attracors can be enhanced greatly. Furthermore, the effect of initial phase setting is investigated in Figure 4.

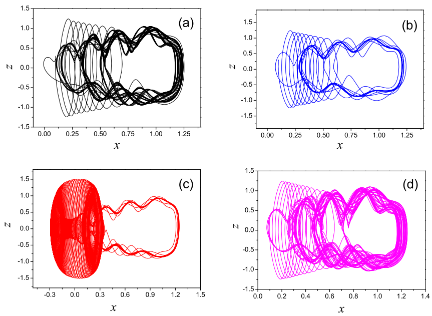

Fig. 4. Formation of attractors are calculated by setting different initials phase φ, for (a)$\varphi = \pi/8$; (b) $\varphi = \pi/2$; (c)$\varphi = 3\pi/4$; (d)$\varphi = 2\pi$; the other parameters are fixed at $A = 0.01$, $\omega = \pi/4$, $d = -0.06$.The nonlinear function is selected by f_1.

It is found that the attractors in the dynamical system are much dependent on the initial setting for phase in the nonlinear function f_1, and the scroll number of attractors can be modulated completely. Extensive numerical results confirmed that chaotic attractors can also be suppressed by setting appropriate initial phase, e. g. $\varphi = -\pi/4$, and $A = 0.01$, $\omega = 2$, $d = -0.10$. In fact, nonlinear function f_1 changes the dynamical system modulated by Jerk function-like function that more equilibrium points can be formed to generate scroll-attractors . The evolution of response behavior of this system is also dependent on the nonlinear term dxz, however, the properties of attactors in this system is mainly controlled by the nonlinear function f_1 when parameter d is fixed. It is also interesting to investigate the case when nonlinear term $f_2 = Asign(\omega y + \varphi)$, $f_3 = A|\omega y + \varphi|$ are used to change the dynamical properties in this system. Similar numerical algorithm is carried out to find solutions from the system driven by discontinuous nonlinear functions as f_2, f_3, then the attractors and sampled time series for the first variables are calculated in Figure 5.

It is found that similar chaotic attractors can be formed by applying different nonlinear modulation f_1, f_2, f_3 on the dynamical system when the amplitude A is fixed the same, and the sampled time series for variable x also show similar oscillating properties. The mechanism could be that the nonlinear modulation from f_1, f_2, f_3 all depends on the second variable y completely and modulation function are switched with certain rhythm. In fact, the three kinds of nonlinear functions can modulate the dynamical behaviors with certain periodicity.

In the following discussion, we investigate the dynamical response of system by applying memristor-based function as $f_4 = k(\alpha + 3\beta y^2)x$, and the stability of attractors will be discussed. Memristor [57] is a new electric device and it is often used in the

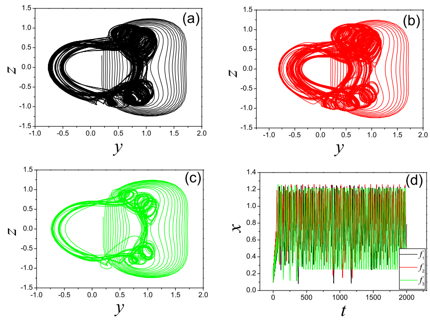

Fig. 5. Formation of attractors by applying nonlinear terms, for (a)f_1; (b)f_2; (c)f_3; sampled time series for variable x are calculated (d); the other parameters are fixed at $A = 0.01$, $\omega = \pi/4$, $\varphi = 0$, $d = -0.06$.

nonlinear circuit to support the chaotic behaviors, and its memductance is dependent on the inputs current. As a result, memory effect is found in the memristor. As mentioned in Ref. [43], switch and resetting initial values for the dynamical system can select different periodical and chaotic attractors when other parameters are fixed in the initial-dependent system with memory. Furthermore, the memristor is used to describe the effect of electromagnetic induction in neuron and it can bridge the coupling between magnetic flux and membrane potential of neuron. Indeed, this improved neuron model [37, 44] can explain the emergence of multiple modes and response to external electromagnetic radiation on neuronal activities. In Figure 6, the attractors are calculated by setting different feedback gains k in the memristor function f_4.

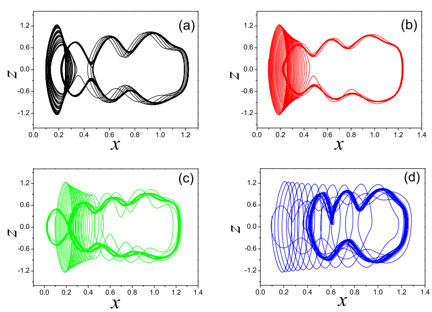

Fig. 6. Formation of attractors in the system modulated by memristor function f_4, for (a)$k = 0.001$; (b)$k = 0.005$; (c)$k = 0.01$; (d)$k = 0.03$; the other parameters are fixed at $\alpha = 0.2$, $\beta = 0.2$, $d = -0.06$.

That is, the chaotic attractors are modulated with increasing the feedback gain in the memristor function and the scrolls are enhanced. Furthermore, we also investigate the case when feedback gain d is changed when gain k in memristor function is fixed at appropriate values; the results are shown in Figure 7.

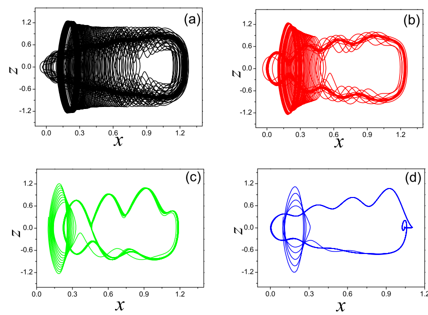

Fig. 7. Formation of attractors in the system modulated by memristor function f_4, for (a)$d = -0.01$; (b)$d = -0.03$; (c)$d = -0.2$; (d)$d = -0.6$, the other parameters are fixed at $\alpha = 0.2$, $\beta = 0.2$, $k = 0.005$.

It is found that chaotic attractors can be suppressed and a periodical oscillating behavior is induced when memristor function is applied on the dynamical systems. To discern the effect of feedback gains k, d in the nonlinear function, the bifurcation diagram is calculated in Figure 8.

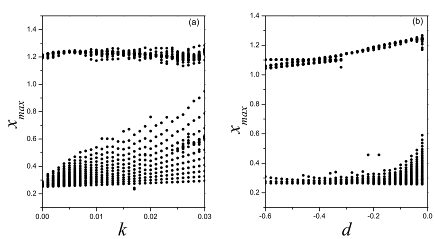

Fig. 8. Bifurcation diagram is calculated from maximal value x with increasing the feedback gains. (a) k is changing at fixed $d = -0.06$; (b) d is changing at fixed $k = 0.006$. The other parameters are fixed at $a = -0.1$, $b = 1$, $c = -1.2$, $r = 1.1$, $\alpha = 0.2$, $\beta = 0.2$.

The results in Figure 8 confirmed that periodical behaviors can be switched to chaotic states by applying appropriate setting on feedback gains k in memristor function, and d in the nonlinear terms of the dynamical systems. Extensive numerical results found that the profile of attractors is much dependent on the feedback gains in the memristor function and nonlinear function xz. Finally, it is important to detect the evolution of Hamilton energy by setting and triggering different attractors in the dynamical system.

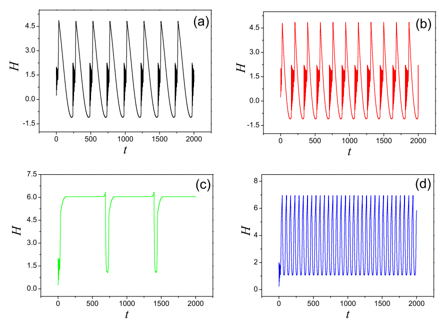

Fig. 9. Evolution of Hamilton function is calculated by applying different nonlinear modulation function, for (a)f_1;(b)f_2;(c)f_3;(d)f_4, the parameters are fixed at $A = 0.01$, $\omega = \pi/4$, $\varphi = 0$, $d = -0.6$, $k = 0.05$, $\alpha = 0.2$, $\beta = 0.2$.

It was ever confirmed that chaotic state can consume a lower average Hamilton energy, and the average Hamilton energy will be decreased with increase the scroll number of attractors [33]. The nonlinear function f_3 always imposes positive stimulus and thus the dynamical system can hold higher maximal Hamilton energy than the system driven by nonlinear function f_1 and f_2, which generates switch between negative and positive perturbation on the dynamical system. The memristor function enlarges the fluctuation of Hamilton energy greatly. It is also found that periodical oscillation can cost higher Hamilton energy. To further estimate the dependence of Hamilton energy on amplitude of nonlinear function, the average Hamilton energy is calculated by

$$\langle H \rangle = \frac{1}{T}\int_0^T H(t)\,\mathrm{d}t \tag{13}$$

where T is the transient period for calculating the average Hamilton energy. From the view of dynamical control, these nonlinear function f_1, f_2, f_3, f_4 can input continuous energy to change the dynamical states. As a result, appropriate amplitude is useful to enhance the oscillating behavior in the dynamical system driven by nonlinear function. However, setting some amplitude can cause breakdown and no solution can be approached in these dynamical systems because no stable equilibrium point can be detected. In Figure 10, the average Hamilton energy is estimated within a transient period 2000 time units.

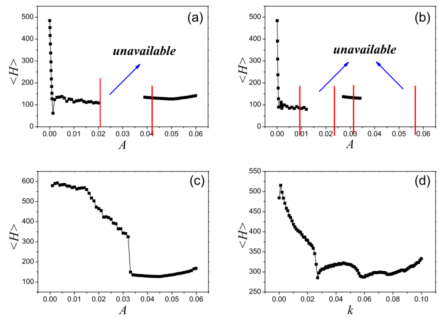

Fig. 10. Evolution of average Hamilton energy function by applying different nonlinear modulation function, for (a)f_1; (b)f_2;(c)f_3;(d)f_4, calculated by changing the amplitude A and the feedback gain k, the parameters are fixed at $\omega = \pi/4$, $\varphi = 0$, $d = -0.6$, $\alpha = 0.2$, $\beta = 0.2$. Transient period about $T = 2000$ time units is used for calculating. 'Unavailable' means that no solution can be found from the dynamical system, thus no Hamilton energy can be estimated.

As shown in the curves in Figure 10, the average Hamilton energy decreases with the increase of amplitude in the nonlinear function. The potential mechanism could be that stronger non-linearity can much contribute to the oscillating behaviors in the dynamical system, continuous release and absorbing in energy will approach a lower average Hamilton energy. As a result, appropriate setting for amplitude in nonlinear functions can enhance rich dynamical behaviors and also decrease the Hamilton energy, which is also available for circuit realization when operational amplifier is required. In this way, appropriate nonlinear function and electric devices can be selected to design chaotic circuits with lower energy cost, thus it decreases the cost of commercial electric devices such as operational amplifier(OM) in circuit setting.

4. CONCLUSIONS

Based on a three-variable dynamical system, a different nonlinear term is modulated to trigger different hidden attractors. The Hamilton energy dependence on non-linearity and parameter setting is estimated. The improved dynamical system composed of hidden attractors can be controlled to switch between periodical and chaotic attractors. Nonlinear terms are critical to trigger chaotic behaviors and the modulation amplitude is important to supply enough energy. Indeed, appropriate setting in amplitude and nonlinear terms are important to decrease Hamilton energy in the dynamical systems, thus the dynamical system can be controlled to reach target orbits. These results indicate that appropriate setting and selection in non-linear function could be helpful to design chaotic, periodical circuits with lower energy cost.

ACKNOWLEDGEMENT

This work is partially supported by National Natural Science Foundation of China under Grant Nos. 11672122, 11765011.

(Received October 7, 2017)

.

REFERENCES

[1] W. M. Ahmad and J. C. Sprott: Chaos in fractional-order autonomous nonlinear systems. Chaos *6* (2003), 339–351. DOI:10.1016/s0960-0779(02)00438-1

[2] K. Aihara, T. Takabe, and M. Toyoda: Chaotic neural networks. Phys. Lett. A *144* (2001), 333–340. DOI:10.1016/0375-9601(90)90136-c

[3] Z. Aram, S. Jafari, J. Ma et al.: Using chaotic artificial neural networks to model memory in the brain. Commun. Nonlinear Sci. Numer. Simulat. *44* (2017), 449–459. DOI:10.1016/j.cnsns.2016.08.025

[4] B. C. Bao, J. P. Xu, and Z. Liu: Initial state dependent dynamical behaviors in a Memristor based chaotic circuit. Chinese Phys. Lett. *27* (2010), 070504. DOI:10.1088/0256-307x/27/7/070504

[5] K. Barati, S. Jafari, J. C. Sprott et al.: Simple chaotic flows with a curve of equilibria. Int. J. Bifurcat. Chaos *26* (2016), 1630034. DOI:10.1142/s0218127416300342

[6] J. Barrow-Green: Poincaré and the three body problem. Amer. Math. Soc. *2* (1997).

[7] L. Chua: Memristor-the missing circuit element. IEEE Trans. Circ. Theory *18* (1971), 507–519. DOI:10.1109/tct.1971.1083337

[8] D. Dantsev: A Novel type of chaotic attractor for quadratic systems without equilibriums. Int. J. Bifurcat. Chaos *12* (2002), 659–661. DOI:10.1142/s0218127402004620

[9] W. L. Ditto, S. N. Rauseo, and M. L. Spano: Experimental control of chaos. Phys. Rev. Lett. *65* (1991), 3211–3214. DOI:10.1103/physrevlett.65.3211

[10] D. Dudkowski, S. Jafari, T. Kapitaniak et al.: Hidden attractors in dynamical systems. Phys. Rep. *637* (2016), 1–50. DOI:10.1016/j.physrep.2016.05.002

[11] I. V. Ermakov, S. T. Kingni, V. Z. Tronciu et al.: Chaotic semiconductor ring lasers subject to optical feedback: Applications to chaos-based communications. Optics Commun. *286* (2013), 265–272. DOI:10.1016/j.optcom.2012.08.063

[12] M. J. Feigenbaum: The onset spectrum of turbulence. Phys. Lett. A *74* (1979), 375–378. DOI:10.1016/0375-9601(79)90227-5

[13] A. Garfinkel, M. L. Spano, W. L. Ditto et al.: Controlling cardiac chaos. Science *257* (1992), 1230–1235. DOI:10.1126/science.1519060

[14] T. Gotthans and J. Petržela: New class of chaotic systems with circular equilibrium. Nonlinear Dyn. *81* (2015), 1141–1149. DOI:10.1007/s11071-015-2056-7

[15] T. Gotthans, J. C. Sprott, and J. Petržela: Simple Chaotic Flow with Circle and Square Equilibrium. Int. J. Bifurcat. Chaos *26* (2016), 1650137. DOI:10.1142/s0218127416501376

[16] Y. L. Guo, G. Y. Qi, and Y. Hamam: A multi-wing spherical chaotic system using fractal process. Nonlinear Dyn. *85* (2016), 2765–2775. DOI:10.1007/s11071-016-2861-7

[17] X. Hu, C. Liu, L. Liu et al.: Multi-scroll hidden attractors in improved Sprott A system. Nonlinear Dyn. *86* (2016), 1725–1734. DOI:10.1007/s11071-016-2989-5

[18] M. Itoh and L. O. Chua: Memristor oscillators. Int. J. Bifurcat. Chaos *8* (2008), 3183–3206. DOI:10.1142/s0218127408022354

[19] M. A. Jafari, E. Mliki, A. Akgul et al.: Chameleon: the most hidden chaotic flow. Nonlinear Dyn. *88* (2017), 2303–2317. DOI:10.1007/s11071-017-3378-4

[20] S. Jafari and J. C. Sprott: Simple chaotic flows with a line equilibrium. Chaos Solutons Fractals *57* (2013), 79–84. DOI:10.1016/j.chaos.2013.08.018

[21] S. Jafari, J. C. Sprott, V. T. Pham et al.: Simple chaotic 3D flows with surfaces of equilibria. Nonlinear Dyn. *86* (2016), 1349–1358. DOI:10.1007/s11071-016-2968-x

[22] B. Jia, H. G. Gu, L. Li et al.: Dynamics of period-doubling bifurcation to chaos in the spontaneous neural firing patterns. Cogn. Neurodyn. *6* (2012), 89–106. DOI:10.1007/s11571-011-9184-7

[23] M. P. Kennedy: Chaos in the Colpitts oscillator. IEEE Trans. Circ. Syst. I *41* (1994), 711–774. DOI:10.1109/81.331536

[24] D. H. Kobe: Helmholtz's theorem revisited. Amer. J. Physics *54* (1986), 552–554. DOI:10.1119/1.14562

[25] H. S. Kwok and W. K. S. Tang: A fast image encryption system based on chaotic maps with finite precision representation. Chaos Solitons Fractals *32* (2007), 1518–1529. DOI:10.1016/j.chaos.2005.11.090

[26] G. A. Leonov, N. V. Kuznetsov, and V. I. Vagaitsev: Hidden attractor in smooth Chua systems. Physica D *241* (2012), 1482–1486. DOI:10.1016/j.physd.2012.05.016

[27] G. A. Leonov, N. V. Kuznetsov, and V. I. Vagaitsev: Localization of hidden Chua′s attractors. Phys. Lett. A *375* (2011), 2230–2233. DOI:10.1016/j.physleta.2011.04.037

[28] G. A. Leonov, N. V. Kuznetsov, and V. I. Vagaitsev: Hidden attractor in smooth Chua systems. Physica D *241* (2012), 1482–1486. DOI:10.1016/j.physd.2012.05.016

[29] G. A. Leonov, N. V. Kuznetsov, and T. N. Mokaev: Homoclinic orbit and hidden attractor in the Lorenz-like system describing the fluid convection motion in the rotating cavity. Commun. Nonlinear Sci. Numer. Simulat. *28* (2015), 166–176. DOI:10.1016/j.cnsns.2015.04.007

[30] C. Li, S. Li, M. Asim et al.: On the security defects of an image encryption scheme. Image Vision Computing *27* (2009), 1371–1381. DOI:10.1016/j.imavis.2008.12.008

[31] Y. Y. Li and H. G., Gu: The distinct stochastic and deterministic dynamics between period-adding and period-doubling bifurcations of neural bursting patterns. Nonlinear Dyn. *87* (2017), 2541–2562. DOI:10.1007/s11071-016-3210-6

[32] X. Li, C. Li, and I. K. Lee: Chaotic image. *125* (2016), 48–63. DOI:10.1016/j.sigpro.2015.11.017

[33] F. Li and C. G. Yao: The infinite-scroll attractor and energy transition in chaotic circuit. Nonlinear Dyn. *84* (2016), 2305–2315. DOI:10.1007/s11071-016-2646-z

[34] T. Y. Li and J. Y. Yorke: Period three implies Chaos. Amer. Math. Monthly *82* (1975), 985–992. DOI:10.2307/2318254

[35] E. N. Lorenz: Deterministic nonperiodic flow. J. Atmospher. Sci. *20* (1963), 130–141. DOI:10.1175/1520-0469(1963)020¡0130:dnf¿2.0.co;2

[36] M. Lv and J. Ma: Multiple modes of electrical activities in a new neuron model under electromagnetic radiation. Neurocomputing *205* (2016), 375–381. DOI:10.1016/j.neucom.2016.05.004

[37] M. Lv, C. Wang, G. Ren et al.: Model of electrical activity in a neuron under magnetic flow effect. Nonlinear Dyn. *85* (2016), 1479–1490. DOI:10.1007/s11071-016-2773-6

[38] J. Ma, X. Y. Wu, R. T. Chu et al.: Selection of multi-scroll attractors in Jerk circuits and their verification using Pspice. Nonlinear Dyn. *76* (2014), 1951–1962. DOI:10.1007/s11071-014-1260-1

[39] J. Ma, A. B. Li, Z. S. Pu et al.: A time-varying hyperchaotic system and its realization in circuit. Nonlinear Dyn. *62* (2010), 535–541. DOI:10.1007/s11071-010-9739-x

[40] J. Ma, L. Mi, P. Zhou et al.: Phase synchronization between two neurons induced by coupling of electromagnetic field. Appl. Math. Comput. *307* (2017), 321–328. DOI:10.1016/j.amc.2017.03.002

[41] J. Ma, X. L. Song, J. Tang et al.: Wave emitting and propagation induced by autapse in a forward feedback neuronal network. Neurocomputing *167* (2015), 378–389. DOI:10.1016/j.neucom.2015.04.056

[42] J. Ma, F. Wu, W. Jin et al.: Calculation of Hamilton energy and control of dynamical systems with different types of attractors. Chaos *27* (2017), 481–495. DOI:10.1063/1.4983469

[43] J. Ma, F. Q. Wu, G. D. Ren et al.: A class of initials-dependent dynamical systems. Appl. Math. Comput. *298* (2017), 65–76. DOI:10.1016/j.amc.2016.11.004

[44] J. Ma, F. Wu, and C. Wang: Synchronization behaviors of coupled neurons under electromagnetic radiation. Int. J. Mod Phys. B *31* (2017), 1650251. DOI:10.1142/s0217979216502519

[45] J. Ma, A. H. Zhang, Y. F. Xia et al.: Optimize design of adaptive synchronization controllers and parameter observers in different hyperchaotic systems. Appl. Math. Comput. 215 (2010), 3318–3326. DOI:10.1016/j.amc.2009.10.020

[46] R. M. May: Simple mathematical models with very complicated dynamics. Nature *261* (1976), 459–467. DOI:10.1038/261459a0

[47] M. Molaie, S. Jafari, J. C. Sprott et al.: Simple chaotic flows with one stable equilibrium. Int. J. Bifurcat. Chaos (2013), 1350188. DOI:10.1142/s0218127413501885

[48] B. Muthuswamy: Implementing memristor based chaotic circuits. Int. J. Bifurcat. Chaos *20* (2010), 1335–1350. DOI:10.1142/s0218127410026514

[49] V- T. Pham, S. Jafari, X. Wang X et al.: A chaotic system with different shapes of equilibria. Int. J. Bifurcat. Chaos *26* (2016), 1650069. DOI:10.1142/s0218127416500693

[50] V. T. Pham, C. Volos, S. Jafari et al.,: A Chaotic system with different families of hidden attractors. Int. J. Bifurcat. Chaos *26* (2016), 1650139. DOI:10.1142/s021812741650139x

[51] J. R. Piper and J. C. Sprott: Simple autonomous chaotic circuit. IEEE Trans. Circ. Syst. II *57* (2010), 730–734. DOI:10.1109/tcsii.2010.2058493

[52] G. Y. Qi and G. R. Chen: A spherical chaotic system. Nonlinear Dyn. *81* (2015), 1381–1392. DOI:10.1007/s11071-015-2075-4

[53] G. D. Ren, Y. Xu, and C. N. Wang: Synchronization behavior of coupled neuron circuits composed of memristors. Nonlinear Dyn. *88* (2017), 893–901. DOI:10.1007/s11071-015-2075-4

[54] J. K. Ryeu, K. Aihara, and I. Tsuda: Fractal encoding in a chaotic neural network. Phys. Rev. E *64* (2001), 046202. DOI:10.1103/physreve.64.046202

[55] R. Shaw: The dripping faucet as a model chaotic system. Aerial Press, Santa Cruz 1984.

[56] X. L. Song, W. Y. Jin, and J. Ma: Energy dependence on the electric activities of a neuron. Chinese Phys. B *24* (2015), 604–609. DOI:10.1088/1674-1056/24/12/128710

[57] D. B. Strukov, G. S. Snider, D. R. Stewart et al.: The missing memristor found. Nature *453(* 2008), 80–83. DOI:10.1038/nature06932

[58] Z. H. Wang, S. J. Cang, E. O. Ochola et al.: A hyperchaotic system without equilibrium. Nonlinear Dyn. *69* (2012), 531-537. DOI:10.1007/s11071-011-0284-z

[59] X. Wang and G. R. Chen: Constructing a chaotic system with any number of equilibria. Nonlinear Dyn. *71* (2013), 429–436. DOI:10.1007/s11071-012-0669-7

[60] C. Wang, R. Chu, and J. Ma: Controlling a chaotic resonator by means of dynamic track control. Complexity *21* (2015), 370–378. DOI:10.1002/cplx.21572

[61] S. Wang, J. Kuang, J. Li et al.: Chaos-based secure communications in a large community. Phys. Rev. E *66* (2002), 065202. DOI:10.1103/physreve.66.065202

[62] C. N. Wang, J. Ma, Y. Liu et al.: Chaos control, spiral wave formation, and the emergence of spatiotemporal chaos in networked Chua circuits. Nonlinear Dyn. *67* (2012), 139–146. DOI:10.1007/s11071-011-9965-x

[63] C. N. Wang, Y. Wang, and J. Ma: Calculation of Hamilton energy function of dynamical system by using Helmholtz theorem. Acta Physica Sinica *65* (2016), 240501.

[64] A. Wolf, J. B. Swift, H. L. Swinney et al.: Determining Lyapunov exponents from a time series. Physica D *16* (1985), 285–317. DOI:10.1016/0167-2789(85)90011-9

[65] C. W. Wu and L. O. Chua: A simple way to synchronize chaotic systems with applications to secure communication systems. Int. J. Bifurcat. Chaos *3* (1993), 1619–1627. DOI:10.1142/s0218127493001288

[66] X. Y. Wu, J. Ma, L. H. Yuan et al.: Simulating electric activities of neurons by using PSPICE. Nonlinear Dyn. *75* (2014), 113–126. DOI:10.1007/s11071-013-1053-y

[67] M. E Yalcin: Multi-scroll and hypercube attractors from a general jerk circuit using Josephson junctions. Chaos Solutons Fractals *34* (2007), 1659–1666. DOI:10.1016/j.chaos.2006.04.058

[68] T. Yang: A survey of chaotic secure communication systems. Int. J. Comput. Cogn. *2* (2004), 81–130.

[69] A. Zarei: Complex dynamics in a 5-D hyper-chaotic attractor with four-wing, one equilibrium and multiple chaotic attractors. Nonlinear Dyn. *81* (2015), 585–605. DOI:10.1007/s11071-015-2013-5

[70] A. Zarei and S. Tavakoli: Hopf bifurcation analysis and ultimate bound estimation of a new 4-D quadratic autonomous hyper-chaotic system. Appl. Math. Comput. *291* (2016), 323–339. DOI:10.1016/j.amc.2016.07.023

Ge Zhang, Department of Physics, Lanzhou University of Technology, Lanzhou 730050. P. R. China.

e-mail: zhangge0324@163.com

Chunni Wang, Department of Physics, Lanzhou University of Technology, Lanzhou 730050. P. R. China.
e-mail: wangcn05@163.com

Ahmed Alsaedi, NAAM-Research Group, Faculty of Science, King Abdulaziz University, Jeddah 21589. Saudi Arabia.
e-mail: aalsaedi@hotmail.com

Jun Ma, Corresponding author. Department of Physics, Lanzhou University of Technology, Lanzhou 730050, P. R. China and College of Mechano-Electronic Engineering, Lanzhou University of Technology, Lanzhou 730050. P. R. China.
e-mail: hyperchaos@163.com

Guodong Ren, Department of Physics, Lanzhou University of Technology, Lanzhou 730050. P. R. China.
e-mail: rengd@lut.cn