KYBERNETIKA — VOLUME *52* (2016), NUMBER 6, PAGES 967–987

SWITCHING LPV CONTROL DESIGN WITH MDADT AND ITS APPLICATION TO A MORPHING AIRCRAFT

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In flight control of a morphing aircraft, the design objective and the dynamics may be different in its various configurations. To accommodate different performance goals in different sweep wing configurations, a novel switching strategy, mode dependent average dwell time (MDADT), is adopted to investigate the flight control of a morphing aircraft in its morphing phase. The switching signal used in this note is more general than the average dwell time (ADT), in which each mode has its own ADT. Under some simplified assumptions the control synthesis condition is formulated as a linear matrix optimization problem and a set of mode-dependent dynamic state feedback controllers are designed. Afterwards the proposed approach is applied to a morphing aircraft with a variable sweep wing to demonstrate its validity.

Keywords: switching linear parameter-varying system, flight control, morphing aircraft, mode dependent average dwell time

Classification: 93C95, 93D09

1. INTRODUCTION

With the development of new material and technology, aircraft could improve the flight performance by morphing wings, in which “morphing” means the aircraft can change aerodynamic shape to obtain optimal flight performance [2, 3]. One morphing concept is using variable-sweep wing to optimize flight performance. During the morphing aircraft′s wing shape-varying process, the dynamic responses will be governed by time-varying aerodynamic forces and moments, which will be related to the wing′s shape. Due to the significant wing reconfiguration, aerodynamic parameters, which are varied dramatically, will make the morphing aircraft be a complicated system with strong non-linearity and uncertainties. Therefore, analysis and control of morphing aircraft are more challenging than those for traditional flight vehicles [10, 15]. On the other hand, compared with conventional wing-fixed aircraft, the morphing aircraft has multi-objective adaptability, wider flight envelop and higher combat effectiveness [4]. Moreover, in the flight control of a morphing aircraft, different performance goals are often desirable for different wing configurations. In such a circumstance, it is sometimes difficult to design a single controller to satisfy different performance in its all configurations. Typically, the controller is designed by compromising the performance in some wing configuration.

DOI: 10.14736/kyb-2016-6-0967

In recent years, the issue of LPV system has been widely investigated due to its merits of compensating for the shortages of traditional gain-scheduling techniques (see for example [1, 18, 21, 23, 24] and references therein). LPV control theory, whose state matrices depend on (measurable) time-varying parameters [8, 16], provides a systematic gain-scheduling design technique [1] and has been extensively used in the fields ranging from aerospace to process control industries [1, 22]. In Ref. [17], the conditions that guarantee stability, robustness and performance properties of the global gain-scheduled designs are given using a quadratic Lyapunov functions, but the quadratic Lyapunov function, which is independent of the scheduled parameters, showed its conservatism. In Ref. [20], a parameter-dependent Lyapunov function method, which leads to a less conservative result, has been proposed to analyze and synthesize the LPV system. However, for an LPV system with a large parameter variation range, a single Lyapunov function, quadratic or parameter-dependent, may not exist, even if it does exist, it is often necessary to sacrifice the performance in some parameter subregions in order to obtain a single LPV controller over the entire parameter region. In such a case, the concept of switching LPV system, by generalizing the switched LTI systems to LPV ones, is put forward [9, 12, 13, 21]. In Ref. [13], two parameter-dependent switching logics, hysteresis switching and average dwell time (ADT) switching, are applied to an F-16 aircraft model with different design objectives and aircraft dynamics in its low and high angle of attack regions. In Ref. [14], a switching LPV controller is used to regulate the air-fuel ratio of an internal combustion engine, and all of them have improved system performance in certain extent. Meanwhile, for the purpose of improving design performance and transient responses as switching occurs, a smooth switching strategy has also been developed and various successful control applications have been reported [5, 6, 10].

On another research front, switching signal, which is used to distinguish the switched systems from the other systems, has played a vital role to the system performance [19]. As a type of switching signals, ADT switching logic means that the number of switches is bounded in a finite interval and the average time between consecutive switching is not less than a constant [11], which is more general than Dwell Time (DT) switching logic [7]. However, it has been recognized that the property in the ADT switching is still not anticipated, since the average time interval between any two consecutive switching is at least τ_a, which is independent of the system mode. To release the restrictions of ADT to the switched control system, a mode-dependent ADT switching strategy is proposed by providing two mode-dependent parameters to ADT switching strategy [26].

So far there is no result available yet on control of switching LPV systems with MDADT based on parameter-dependent Lyapunov functions, which will reduce the conservatism, enhance flexibility and improve the disturbance attenuation performance in the analysis and synthesis of a switched LPV system. This motives us for this investigation. The main contribution of this paper is that a novel notion of parameter-dependent MDADT switching scheme and a group of parameter-dependent Lyapunov functions are used to investigate the problem of control of switching LPV systems, and then the proposed result is applied to a switching LPV representation of the morphing aircraft to accommodate multiple control objectives in different sweep wing configurations.

This paper is organized as follows. The system description and some preliminaries are given in section 2. In section 3 we investigate switching LPV control design problem under a novel notion of MDADT switching approach and the switching control synthesis condition will be formulated as matrix optimization problem, whereas in section 4, the LPV model of a sweepback morphing aircraft is deduced at first, and then the flight control is designed by the presented method, at last the corresponding simulation illustrates the effectiveness. Finally the conclusion remarks are drawn in section 5.

The notation in this paper is standard. R stands for the set of real numbers and R_+ for the nonnegative real numbers. $R^{m\times n}$ is the set of $m \times n$ real matrices. The transpose of a real matrix M is denoted by M^T. $\ker(M)$ is used to denote the orthogonal complement of M. $S^{n\times n}$ is used to denote the real symmetric matrices and if $M \in S^{n\times n}$, then $M > 0 (M \geq 0)$ indicates that M is positive definite (positive semidefinite) and $M < 0 (M \leq 0)$ denotes a negative definite (negative semidefinite) matrix. For $x \in R^n$, its norm is defined as $\|x\|_2 = \sqrt[2]{x^T x}$. The space of square integrable function is denoted by, that is, for any $u(t) \in l_2$, $\|u(t)\|_2 = \sqrt[2]{u^T(t)u(t)}$ is finite.

2. PRELIMINARIES

An open-loop LPV system to be investigated is described as:

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A_{i(\rho)} & B_{1,i(\rho)} & B_{2,i(\rho)} \\ C_{1,i(\rho)} & D_{11,i(\rho)} & D_{12,i(\rho)} \\ C_{2,i(\rho)} & D_{21,i(\rho)} & D_{22,i(\rho)} \end{bmatrix} \begin{bmatrix} x \\ \omega \\ u \end{bmatrix} \rho \in P_i \tag{1}$$

where x, $\dot{x} \in R^n$, $z \in R^{n_z}$ is the controller output, and $\omega \in R^{n_\omega}$ is the disturbance input, $y \in R^{n_y}$ is the measurement for control, $u \in R^{n_u}$ is the control input. All of the statespace data are continuous functions of the parameter ρ. It is assumed that ρ is in a compact set $P \subset R^s$ with its parameter variation rate bounded by $\underline{v}_k \leq \dot{\rho}_k \leq \overline{v}_k$, for $k = 1, 2, \ldots, s$, and the parameter value is measurable in real-time. The following assumptions are also needed.

A1. $(A_{(\rho)}, B_{2(\rho)}, C_2(\rho))$ triple is parameter-dependent stabilizable and detectable for all ρ;

A2. The matrix functions $[B_2^T(\rho) \;\; D_{12}^T(\rho)]$ and $[C_2(\rho) \;\; D_{21}(\rho)]$ have full row ranks for all ρ;

A3. $D_{22(\rho)} = 0$.

Supposing the parameter set P is covered by a number of closed subsets $\{P_i\}_{i\in Z_N}$ by means of a family of switching surfaces S_{ij}, where the index set $Z_N = \{1, 2, \ldots, N\}$, and $\cup_{i=1}^{Z_N} P_i = P$, $P_i \cap P_j = S_{ij}$, $\forall (i,j) \in Z_N \times Z_N, i \neq j$. In this paper, we are interested in the problem of designing a group of LPV controllers in the form of

$$\begin{bmatrix} \dot{x_k} \\ u \end{bmatrix} = \begin{bmatrix} A_{k,i(\rho,\dot{\rho}} & B_{k,i(\rho)} \\ C_{k,i(\rho)} & D_{k,i(\rho)} \end{bmatrix} \begin{bmatrix} x_k \\ y \end{bmatrix}, i \in Z_N \tag{2}$$

and each of them is suitable for a specific parameter subset P_i. The state dimension of each controller is $x_k \in R^{n_k}$. The control design requirement at each parameter subregion

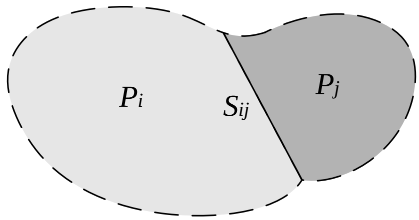

Fig. 1. Switching regions with dwell time.

could be different and even conflicting for different parameter regions. Each controller, also a function of the parameter ρ, stabilizes the open-loop system with best achievable performance in a specific parameter region, and meanwhile maintains the closed-loop system stability under the given switching strategy.

The switching event occurs when the parameter trajectory hits the switching surfaces, so it is obvious that the switching event is parameter-dependent. A switching signal σ is defined as a piecewise constant function. It is assumed that σ is continuous from the right everywhere, and only limited number of switches occur in any finite time interval.

Then the switching closed-loop LPV system can be described by:

$$\begin{bmatrix} \dot{x_{cl}} \\ u \end{bmatrix} = \begin{bmatrix} A_{cl,\sigma(\rho,\dot{\rho}} & B_{cl,\sigma(\rho)} \\ C_{cl,\sigma(\rho)} & D_{cl,\sigma(\rho)} \end{bmatrix} \begin{bmatrix} x_{cl} \\ \omega \end{bmatrix}, \rho \in P_i, i \in Z_N \tag{3}$$

where $x_{cl}^T = [x^T \ x_k^T] \in R^{n+n_k}$. It is straightforward to show that the resulting closed-loop system is a switched LPV system, which could have discontinuity and multiple state space gains at switching surfaces due to the use of multiple LPV controllers.

In this paper, the aim is to design a set of switching signal ρ with mode-dependent average dwell time (MDADT) property based on parameter-dependent Lyapunov functions, such that

C1. When $\omega = 0$,the switched LPV system (3) is parameter-dependent quadratically stable;

C2. When $x_0 = 0$ and $\omega \neq 0.\omega \in l_2,\|z\|_2 < \gamma\|\omega\|_2$.

For this purpose, the definition of the MDADT switching is given as follow:

Definition 2.1. (Zhao et al. [26]) For a switching signal σ and any $0 \leq t \leq T$, let $N_{\sigma p}(T,t)$ be the switching numbers that the p^{th} subsystem is activated over the time interval $[t,T]$ and $T_p(T,t)$ denote the total running time of the p^{th} subsystem over the time interval $[t,T], p \in Z_N$, we say thatσ has a mode-dependent average dwell time τ_{ap} if there exist positive numbers N_{0p} and τ_{ap} such that

$$N_{\sigma p}(T,t) \leq N_{0p} + \frac{T_p(T,t)}{\tau_{ap}}, \ \forall T \geq t \geq 0. \tag{4}$$

Remark 2.2. As can be seen, Definition 2.1 constructs a different set of switching signals from that with the ADT property [7], that is, if there exist positive numbers $\tau_{ap}, p \in S$, such that a switching signal has the MDADT property, it only requires that the average time among the intervals associated with the p^{th} subsystem is larger than τ_{ap}. This switching law is less strict than the ADT switching scheme in that each mode has its own ADT.

Definition 2.3. (Zhao et al. [25]) For $\beta > 0$,$\eta > 0$ and $\gamma > 0$, system (3) is said to be globally uniformly exponentially stable with a weighted H_∞ performance γ, if it is exponentially stable with $\omega(t) = 0$, and under zero initial condition, it holds for any non-zero $\omega(t) \in l_2[0, +\infty)$, that

$$\beta \int_0^\infty e^{-\eta s} z^T(s) z(s)\, \mathrm{d}s \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s)\, \mathrm{d}s. \tag{5}$$

3. SWITCHING CONTROL BASED ON MDADT

Supposing that there exists a family of positive definite matrix functions $\{X_i(\rho)\}_{i \in Z_N}$ and each of them is smooth over the corresponding parameter subset P_i, then the multiple parameter-dependent Lyapunov function can be defined as

$$V_\sigma(x_{cl,\rho}) = x_{cl}^T X_\sigma(\rho) x_{cl} \tag{6}$$

where the value of switching signal σ represents the active operating region P_i and thus determines the corresponding matrix function $X_i(\rho)$. Switching occurs only when parameter ρ leaves its current subregion.

In order to guarantee stability, it must be ensured that the parameter-dependent Lyapunov function decreases at each switching event. In other words, when the scheduling parameter moves from one subregion to another, the Lyapunov function defined within the subregion it is leaving must be greater than the Lyapunov function defined within the subregion it is entering. As a consequence, the average switching frequency over a finite time interval is limited to $\frac{1}{\tau_a}$ to compensate for possible increase of Lyapunov functions. Then we have the following lemma:

Lemma 3.1. Given scalars λ_0, $\mu > 0$, an open-loop system (1),the parameter set P and its partition $P_i(\rho)$,if there exist a family of positive definite matrix functions $R_i, S_i : R_+^{n*n}, i \in Z_N$ such that for any $\rho \in P_i$

$$N_{R,i(\rho)}^T \begin{bmatrix} \begin{matrix}\{R_i(\rho)A_i(\rho) + A_i(\rho)R_i(\rho) \\ -\sum_{k=1}^s \{\underline{v}_k, \overline{v}_k\} \frac{\partial R_i(\rho)}{\partial \rho_k} + \lambda_0 R_i(\rho)\}\end{matrix} & * & * \\ C_{1,i(\rho)} R_i(\rho) & -\gamma_i I_{n_z} & * \\ B_{1,i(\rho)}^T & D_{11,i(\rho)}^T & -\gamma_i I_{n_\omega} \end{bmatrix} N_{R,i(\rho)}(\rho) < 0 \tag{7}$$

$$N_{S,i(\rho)}^T \begin{bmatrix} \begin{matrix}\{S_i(\rho)A_i(\rho) + A_i^T(\rho)S_i(\rho) \\ -\sum_{k=1}^s \{\underline{v}_k, \overline{v}_k\} \frac{\partial S_i(\rho)}{\partial \rho_k} + \lambda_0 S_i(\rho)\}\end{matrix} & * & * \\ C_{1,i(\rho)} S_i(\rho) & -\gamma_i I_{n_z} & * \\ B_{1,i(\rho)}^T & D_{11,i(\rho)}^T & -\gamma_i I_{n_\omega} \end{bmatrix} N_{S,i(\rho)}(\rho) < 0 \tag{8}$$

$$\begin{bmatrix} R_i(\rho) & I \\ I & S_i(\rho) \end{bmatrix} \geq 0 \tag{9}$$

where $N_{R,i}(\rho) = \ker([\ B_{2,i}^T(\rho)\ \ D_{12,i}^T(\rho)\ \ 0\ ])$, and $N_{S,i}(\rho) = \ker([\ C_{2,i}(\rho)\ \ D_{21,i}(\rho)\ \ 0\ ])$ and for any $\rho \in S_{ij}$,$\forall(i,j) \in Z_N * Z_N, i \neq j$

$$\frac{1}{\mu} R_j(\rho) \leq R_i(\rho) \leq \mu R_j(\rho), \tag{10}$$

$$\frac{1}{\mu}(S_j(\rho) - R_j^{-1}(\rho)) \leq R_i(\rho) - S_j^{-1}(\rho) \leq \mu R_j(\rho) - S_j^{-1}(\rho), \tag{11}$$

then the closed-loop LPV system (3) is exponentially stabilized by switching LPV controllers over the entire parameter set P for every switching signal σ with average dwell time

$$\tau_a > \frac{\ln \mu}{\lambda_0} \tag{12}$$

and its performance is maintained as $\|z\|_2 < \gamma_0 \|z\|_2$ with $\gamma_0 = \max\{\gamma_i\}_{i \in Z_N}$.

As we know, the minimum of admissible ADT τ_a is computed by two mode-independent parameters, which will give rise to certain conservatism. Contrary to the ADT switching strategy, the MDADT switching strategy allows the change of Lyapunov function by μ_i times of its value before the ith subsystem is activated, i. e.

$$V_i(x(t)) \leq \mu_i V_j(x(t)) \tag{13}$$

then, for the compensation of the increasing of the Lyapunov function, the average switching frequency of the ith subsystem over a finite time interval is limited to $\frac{1}{\tau_{ai}}$ [28]. Followed by the above description, we have the following conclusion:

Theorem 3.2. Given an open-loop LPV system (1), the parameter set P and its partition $P_i(\rho)$, if there exist a family of positive definite matrix functions $R_i, S_i : R_+^{n*n}, i \in Z_N$, such that for any $\rho \in P_i$

$$N_{R,i(\rho)}^T \begin{bmatrix} \begin{array}{c}\{R_i(\rho)A_i^T(\rho) + A_i(\rho)R_i(\rho) \\ + \sum_{k=1}^s \{\underline{v}_k, \overline{v}_k\} \frac{\partial R_i(\rho)}{\partial \rho_k} + \lambda_i R_i(\rho)\}\end{array} & * & * \\ C_{1,i(\rho)} R_i(\rho) & -\gamma_i I_{n_z} & * \\ B_{1,i(\rho)}^T & D_{11,i(\rho)}^T & -\gamma_i I_{n_\omega} \end{bmatrix} N_{R,i(\rho)}(\rho) < 0 \tag{14}$$

$$N_{S,i(\rho)}^T \begin{bmatrix} \begin{array}{c}\{S_i(\rho)A_i(\rho) + A_i^T(\rho)S_i(\rho) \\ + \sum_{k=1}^s \{\underline{v}_k, \overline{v}_k\} \frac{\partial S_i(\rho)}{\partial \rho_k} + \lambda_i S_i(\rho)\}\end{array} & * & * \\ B_{1,i(\rho)}^T S_i(\rho) & -\gamma_i I_{\omega} & * \\ C_{1,i(\rho)} & D_{11,i(\rho)} & -\gamma_i I_{n_z} \end{bmatrix} N_{S,i(\rho)}(\rho) < 0 \tag{15}$$

$$\begin{bmatrix} R_i(\rho) & I \\ I & S_i(\rho) \end{bmatrix} \geq 0 \tag{16}$$

where $N_{R,i}(\rho) = \ker([\ B_{2,i}^T(\rho) \ \ D_{12,i}^T(\rho) \ \ 0\ ])$,and $N_{S,i}(\rho) = \ker([\ C_{2,i}(\rho) \ \ D_{21,i}(\rho) \ \ 0\ ])$ and for any $\rho \in S_{ij}$,$\forall (i,j) \in Z_N * Z_N, i \neq j$

$$\begin{cases} \mu_i R_i(\rho) \geq R_i(\rho) \\ \mu_i(S_i(\rho) - R_i^{-1}(\rho)) \geq S_j(\rho) - R_j^{-1}(\rho), \end{cases} \tag{17}$$

then the closed-loop LPV system (3) is exponentially stabilized by switching LPV controllers over the entire parameter set P for every switching signal σ with average dwell time

$$\tau_{ai} > \frac{\ln \mu_i}{\lambda_i}, \tag{18}$$

and its performance is maintained as $\|z\|_2 < \gamma_0 \|z\|_2$ with $\gamma_0 = \max\{\gamma_i\}_{i \in Z_N}$.

Proof. See the appendix. □

Remark 3.3. It can be seen from 3.2 that the parameters μ_i and λ_i are mode-dependent. When μ_i and λ_i have the same values respectively, it turns out to be the ADT switching logic, so the ADT can be viewed as a special case of the MDADT switching logic. Therefore the MDADT switching logic is more general than the ADT switching logic. In practice, it is sometimes too rigid to design a switching logic satisfying ADT requirement. Taking a sweepback morphing aircraft for example, it is desirable that the aircraft has different variation rate in different configurations, as a consequence, the morphing aircraft has different dwell time in its different configurations during the phase of morphing. Hence, the more relaxed and flexible MDADT switching is of considerably importance in engineering implementation.

Remark 3.4. As was discussed before, MDADT is a special case of ADT which corresponds to the case of $\lambda_p = \lambda, \mu_p = \mu, \tau_{ap} = \tau_a, \forall p \in Z_N$, so we can conclude that the weighted H_∞ performance criterion on MDADT approach covers that on ADT approach, and MDADT approach is flexible to avoid dwelling on the subsystems whose performance indices are not good.

Set the matrices $X_i(\rho)$and $X_i^{-1}(\rho)$ as

$$X_i(\rho) = \begin{bmatrix} S_i(\rho) & N_i(\rho) \\ N_i^T(\rho) & ? \end{bmatrix}, \ X_i^{-1}(\rho) = \begin{bmatrix} R_i(\rho) & M_i(\rho) \\ M_i^T(\rho) & ? \end{bmatrix}$$

where $M_i(\rho) N_i^T(\rho) = I - R_i(\rho) S_i(\rho)$ and "*"means the elements we do not care. Applying elimination lemma, it can be seen that condition (30) is equivalent to conditions (14) − (16). Moreover, if we choose

$$X_i(\rho) = \begin{bmatrix} S_i(\rho) & R_i^{-1}(\rho) - S_i(\rho) \\ R_i^{-1}(\rho) - S_i(\rho) & S_i(\rho) - R_i^{-1}(\rho) \end{bmatrix},$$

and then each $X_i(\rho)$ can be decomposed to

$$X_i(\rho) = \begin{bmatrix} I & -I \\ 0 & I \end{bmatrix} \begin{bmatrix} R_i^{-1}(\rho) & 0 \\ 0 & S_i(\rho) - R_i^{-1}(\rho) \end{bmatrix} \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix}.$$

It is then obvious that condition (17) is the same as condition (13).
Since the coefficient involved in (17), the synthesis condition for switching control with MDADT is non-convex, and the non-convex switching LPV synthesis condition is usually difficult to solve. However, if the matrix variables $R_i(\rho)$ is selected to satisfy the condition

$$R_j(\rho) = \mu_i R_i(\rho), \tag{19}$$

on the switching surfaces [12], then for any $\rho \in S_{ij}, \forall (i,j) \in Z_N * Z_N, i \neq j$ we have

$$\mu_i S_j(\rho) \geq S_i(\rho). \tag{20}$$

The equality constraint (19) can be rewritten as an LMI condition through a relaxed process

$$-\varepsilon I \leq R_j(\rho) - \mu_i R_i(\rho) \leq \varepsilon I, \tag{21}$$

where ϵ is a small positive number. After solving matrix functions $R_i(\rho)$ and $S_i(\rho)$ the gains of switched LPV controllers can be constructed as

$$K_i(\rho) = \begin{bmatrix} A_{k,i}(\rho) & B_{k,i}(\rho) \\ C_{k,i}(\rho) & D_{k,i}(\rho) \end{bmatrix} = func(R_i(\rho), S_i(\rho)), i \in Z_N \tag{22}$$

where $func(R_i(\rho), S_i(\rho))$ denotes a function of $R_i(\rho)$ and $S_i(\rho)$, which can be defined as

$$\begin{aligned} A_{k,i}(\rho) = & - N_i^{-1}(\rho)\Big\{ A_i^T(\rho) - S_i(\rho)\frac{\mathrm{d}R_i(\rho)}{\mathrm{d}t} - N_i(\rho)\frac{\mathrm{d}M_i^T(\rho)}{\mathrm{d}t} \\ & + S_i(\rho)[A_i(\rho) + B_{2,i}(\rho)F_i(\rho) + L_i(\rho)C_{2,i}(\rho)]R_i(\rho) \\ & + \frac{1}{\gamma_i}S_i(\rho)[B_{1,i}(\rho) + L_i(\rho)D_{21,i}(\rho)]B_{1,i}^T(\rho) \\ & + \frac{1}{\gamma_i}C_{1,i}^T(\rho)[C_{1,i}(\rho) + D_{12,i}(\rho)F_i(\rho)]R_i(\rho)\Big\} M_i^{-T}(\rho) \end{aligned} \tag{23}$$

$$B_{k,i}(\rho) = N_i^{-1}(\rho)S_i(\rho)L_i(\rho) \tag{24}$$

$$C_{k,i}(\rho) = F_i(\rho)R_i(\rho)M_i^{-T}(\rho) \tag{25}$$

$$D_{k,i}(\rho) = 0 \tag{26}$$

where

$$F_i(\rho) = -(D_{12,i(\rho)}^T D_{12,i(\rho)})^{-1}[\gamma_i B_{2,i(\rho)}^T R_{i(\rho)}^{-1} + D_{12,i(\rho)}^T C_{1,i(\rho)}], \tag{27}$$

$$L_i(\rho) = -[-\gamma_i S_i(\rho)^{-1} C_{2,i(\rho)}^T + B_{1,i(\rho)} D_{21,i(\rho)}^T](D_{12,i(\rho)}^T D_{12,i(\rho)})^{-1}. \tag{28}$$

Remark 3.5. It is noted that the term (1,1) in (14) and (15) implies that the open-loop plant can be thought as a shifted system with its A matrix changing to $A + \frac{\lambda_i}{2}I$, so the same for controller A_k matrix. Therefore, if the matrix functions $R_i(\rho)$ and $S_i(\rho)$ can be solved, then the gains of the switching LPV controllers can be constructed by replacing A and A_k in the standard formula by $A + \frac{\lambda_i}{2}I$ and $A_k + \frac{\lambda_i}{2}I$.

4. APPLICATION TO MORPHING AIRCRAFT

The system to be controlled is the longitudinal short-period nonlinear dynamic mode of a morphing aircraft with variable-wing sweep. Suppose that the wing sweep angle χ can be changed to accommodate different mission requirements. Accordingly, some parameters such as mean aerodynamic chord, span, and wing area, will change with the wing sweep angle. In that scenario, the response of the aircraft will be governed by the time-varying aerodynamic forces and moments, which are functions of the wing's shape changes. It is assumed that the wing sweep angle can be changed from 15° to 60° continuously, which are corresponding to loiter and dash configurations.As known, it is desired fast and accurate responses in its dash configuration, while in its loiter configuration the emphasis of the aircraft is the maintainability of the stability. The longitudinal short-period nonlinear dynamic model of the morphing aircraft can be described as

Fig. 2. Variable sweep wing morphing aircraft.

$$\begin{cases} \dot{\alpha} = \dfrac{1}{mV}(-L - T\sin\alpha + mg\cos\gamma + F_{l_z}), \\ \dot{q} = -\dfrac{\dot{I}_y}{I_y}q + \dfrac{1}{I_y}(-S_x g\cos\theta + M_A + TZ_T + M_{I_y}), \end{cases} \tag{29}$$

where α and q are the angle of attack and the pitch rate, γ and θ are the flight-path angle and pitch angle, m is the aircraft mass, V is the total aircraft velocity, g is gravitational constant, T is thrust vector, I_y is the moment of inertia about the y body axis, Z_T is the position of the power, L and M_A are lift and pitching moment, respectively, which can be approximately expressed as

$$\begin{cases} L = QS_w(C_{L0} + C_{L\alpha}\alpha + C_{L\delta_e}\delta_e), \\ M_A = QS_w c_A(C_{m0} + C_{m\alpha}\alpha + C_{m\delta_e}\delta_e + C_{mq}\dfrac{qc_A}{2V}), \end{cases} \tag{30}$$

where Q is the dynamic pressure and can be expressed as $Q = 0.5\rho V^2$, S_w is the wing area, C_{L0} is the lift coefficient at $\alpha = 0$,$C_{L\alpha} = \frac{\partial C_L}{\partial \alpha}$,$C_{L\delta_e} = \frac{\partial C_L}{\partial \delta_e}$,$c_A$ is the length of mean aerodynamic chord, C_{m0} is the pitching-moment coefficient at $\alpha = 0$,$C_{m\alpha} = \frac{\partial C_m}{\partial \alpha}$, $C_{m\delta_e} = \frac{\partial C_m}{\partial \delta_e}$,$C_{mq}$ is the damp in pitch and can be expressed as $C_{mq} = \frac{\partial C_m}{\partial q}$. F_{I_Z} and M_{I_y} denote the inertia force and moment in the wing transition which can be expressed as

$$\begin{cases} F_{I_z} = S_x(\dot{q}\cos\alpha - q^2\sin\alpha) + 2\dot{S}_x q\cos\alpha + \ddot{S}_x\sin\alpha, \\ M_{I_y} = S_x(\dot{V}\sin\alpha + V\dot{\alpha}\cos\alpha - Vq\cos\alpha), \end{cases} \tag{31}$$

where S_x denotes the x-component of static momentum $S = \int r \times \mathrm{d}m$ in fixed-body axes. The flight condition of interest is selected as the altitude $H = 1000m$, velocity $V = 25\,\mathrm{m/s}$, and other parameters in different configurations as listed in Table 1. To develop an LPV representation of the morphing aircraft, it is needed to find the wings-level equilibrium points at several configurations first. In order to cover the whole work area, the equilibrium points are selected as $\chi = 15°, 30°, 45°$ and $60°$, then the work area is divided into 3 subregions, by two switching surfaces $\chi = 30°$ and $\chi = 45°$, corresponding to low sweep angle configuration, transitional configuration and high sweep angle configuration, respectively.

χ/deg	15	30	45	60
S_w/m^2	1.6	1.391	1.168	0.958
c_A/m	0.486	0.437	0.411	0.418
$I_y/kg \cdot m^2$	6.49	6.998	7.882	8.606

Tab. 1. Parameter of the morphing aircraft configurations.

Division details of the parameter set are listed in the Table 2. The local linear models are then obtained by linearizing the nonlinear equations of motion at those points. Based on these linear small perturbation equations we can get the LPV model of the variable sweep wing morphing aircraft in the following form

$$\dot{x} = A(\rho)x + B(\rho)u \tag{32}$$

where the state variables $x = (\Delta\alpha \quad \Delta q)^T$ and the control input $\Delta\delta_e$. The control design objective is to track the command of the angle of attack. It is formulated as a model-following problem, where the ideal model to be followed is chosen as a second-order low-pass filter based on desired flying qualities. A block diagram of the system interconnection to synthesize the switched LPV controller is shown in Figure 3, where $P(\rho)$ is the model set of linearized aircraft dynamics at different trim points, and has two outputs: the angle of attack α, and the pitch rate q. The inputs of the open-loop system include the 2-dimensional sensor noise signal n, the angle of attack command α_{cmd} and the control input u. The outputs of the open-loop system are weighted error signals α_p and z_a, and the measurement y.

As mentioned before, pilots desire fast and accurate responses in the high sweep angle configuration, while in the low sweep angle configuration region, the requirement for the flying quality is not so critical, and the emphasis of the control design is the maintainability of the stability. Therefore, we choose different performance weighting functions in the different sweep angle regions to reflect design requirements

$$W_{p_1} = \frac{16s+80}{10s+1},\ W_{p_2} = \frac{18s+90}{s+1},\ W_{p_3} = \frac{20s+100}{0.1s+1}$$

where the subscripts 1, 2 and 3 denote the index of the subregion, respectively. Noting that the tracking error in the steady state is 1.25%, 1.11% and 1%, and the bandwidth in the region 1 is the biggest, meanwhile, in region 3 the bandwidth is the smallest, this is consistent with the design objective. The control input is elevator deflection $\Delta\delta_e$, and the positions and rates of control input are fed into to penalize the control effort.

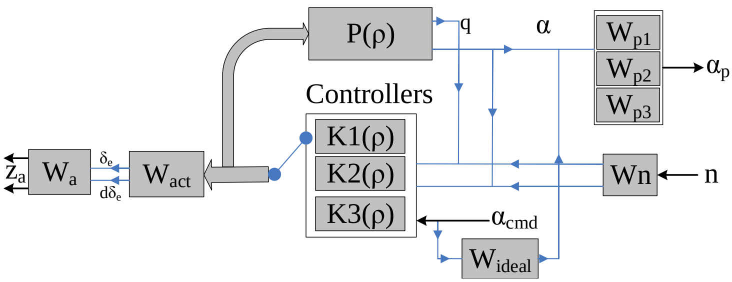

Fig. 3. Weighted interconnection of morphing aircraft.

The related weighting functions and are given as

$$W_a = diag(0.1,\ 0.2), \qquad W_{act} = \left[\begin{array}{r|r} -20 & 20 \\ \hline 1 & 0 \\ -20 & 20 \end{array}\right],$$

the other common weighting functions are chosen as

$$W_n = diag(0.6,\ 0.1), \qquad W_{ideal} = \frac{\alpha_{ideal}}{\alpha_{cmd}} = \frac{144}{s^2+19.2s+144},$$

where the ideal model is a second-order system with the natural frequency 12 rad/s and the damp ratio 0.8.The multiple Lyapunov at each subset are specified as affine functions of parameter ρ, that is, we have

$$R_i(\rho) = R_i^0 + \rho R_i^1,\ S_i(\rho) = S_i^0 + \rho S_i^1, \quad i = 1,2,3, \tag{33}$$

subset	q_1	q_2	q_3
sweep angle	$[15°, 30°]$	$[30°, 45°]$	$[45°, 60°]$

Tab. 2. Parameter of the morphing aircraft configurations.

where the matrices $R_i^k, S_i^k, k = 0, 1$ are new optimization variables to be determined. To eliminate solving the non-vex problem, we enforce the constraint (19) on the switching surfaces. So, the matrix function $R_i(\rho)$ and $S_i(\rho)(i = 1, 2, 3)$ can be obtained by (14)–(17) and the controllers can be constructed by (23).

Remark 4.1. From (33) it can be seen that the Lypunov functions are in a parameter-dependent form, so they are more general than quadratic Lyapunov functions of the form $V(x) = x^T Px$. Furthermore, the concept of parameter-dependent Lyapunov function can be exploited to lead to some potential advantages in the system synthesis.

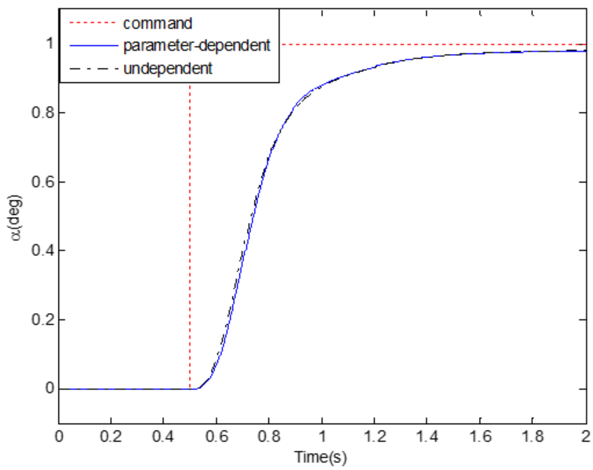

Fig. 4. Step response of angle of attack.

In subregion 1, a simple simulation was done to show the superiority of the parameter-dependent Lyapunov function method. As the performance weighting function is W_{p1}, so the fast response is required. Though the trajectory produced by the parameter-dependent method is close to the independent-method-trajectory in Figure 3, it superiority is showed in its amplified counterpart in Figure 4. This is the difference between the two methods in a single point, when the process of interpolation in classical gain-scheduling is concerned, the advantage of the parameter-dependent-Lyapunov-method is prominent.

Two switched LPV controllers corresponding to ADT and MDADT switching logics are designed using the condition in 3.1 and 3.2. As the matrices $R_i^k, S_i^k, k = 0, 1\&i = 1, 2, 3$ are optimization variables to be determined. The general objective function to optimize is defined as $\min \sum_{i=1}^{Z_N} \omega_i\gamma_i$, where ω_i is the weight to penalize γ_i and $\sum_{i=1}^{Z_N} \gamma_i = 1$. So we define the objective function as $\min \max\{\omega_1, \omega_2\}$ which means minimizing the "worst-case" performance level γ_i.

The switching surfaces $\chi = 30°$ and $\chi = 45°$ divide the whole parameter region into $Z_N = 3$ subregions, which means a complete morphing process has two switching events.

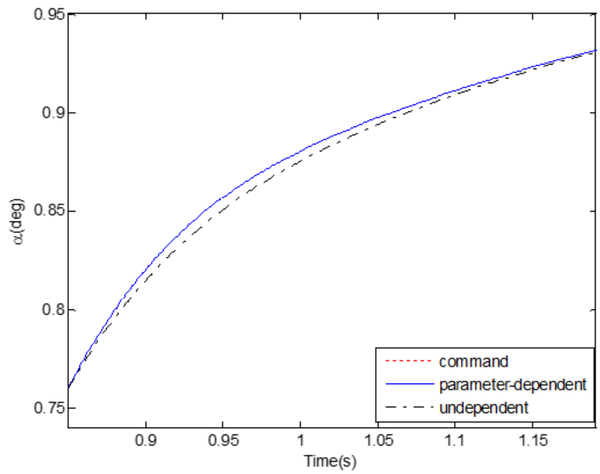

Fig. 5. Amplified counterpart.

Suppose that the variation rate of the sweep angle is less than $15deg/s$, that is, $\underline{v} = -15$ and $\overline{v} = 15$. Under the ADT switching logic, let $\lambda_0 = 0.04$ and $\mu = 1.1$, it can be reached from (12) that $\tau_a^* = 2.4s$, so we choose $\tau_a = 3s$ to satisfy the requirement of the ADT, which means the average dwell time of each subregion of the sweep angle is 3s, without loss of generality, let the switches occurs at 3s, 6s respectively and the sweep angle has a constant variation rate of $5°/s$.

Under the MDADT switching logic let $\lambda_1 = 0.04$, $\lambda_2 = 0.04$, $\lambda_3 = 0.03$, $\mu_1 = 1.1$, $\mu_2 = 1.05$, $\mu_3 = 1.1$, note that via (18) we have $\tau_{a1}^* = 2.4s$,$\tau_{a2}^* = 1.3s$,$\tau_{a3}^* = 3.2s$, so we choose $\tau_{a1} = 3s$,$\tau_{a2} = 1.5s$, $\tau_{a3} = 4.5s$, which means the average dwell time of each subregion of the sweep angle are 3s, 1.5s and 4.5s. In order to satisfy the requirements $\tau_{ai} > \tau_{ai}^*(i = 1, 2, 3)$, it is assumed that the variation rate of the wing sweep angle is $5deg/s$ at first, 3s later the variation rate is $10deg/s$, and 1.5s later the variation rate turn to $3.3deg/s$ until the morphing is finished, by then the sweep angle is not changing any more. So the switching occurs at 3s, 4.5s and thus the dwell time of each subsystem satisfying the requirement of the MDADT switching scheme.

Based on the partition of the entire parameter region and the definition of the switching signal, the variation of the wing sweep angle and the corresponding switching signal are showed in Figure 6. As can be seen from Figure 6, both methods have same sweep signal and switching signal at first, but 3s later the difference occurs, and the MDADT trajectory enhances its variation rate for the sake of reducing its MDADT in subregion 2, on the contrary, in subregion 3 the variation decreases in order to enhance its dwell time. As shown in Figure 6, the switching surfaces of the two switching strategy is different, so the switching signal is different from each other.

Responses of the actuators are shown in Figure 7−Figure 9. The dotted lines in Figure 7 represent the angle of attack response of the ideal mode, the dashed and solid lines represent the responses using ADT switching and MDADT switching, respectively.

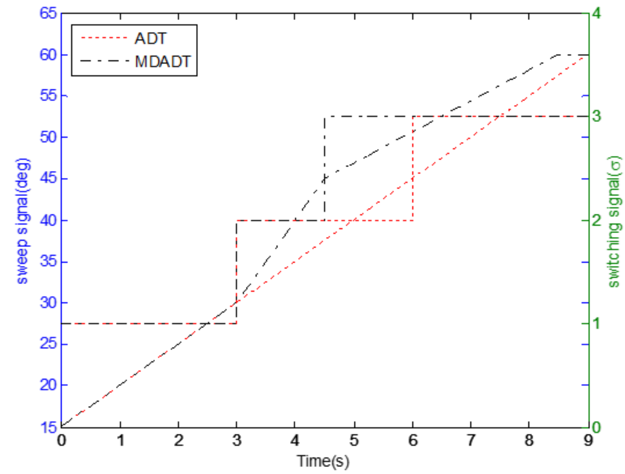

Fig. 6. Sweep angle and switching signal.

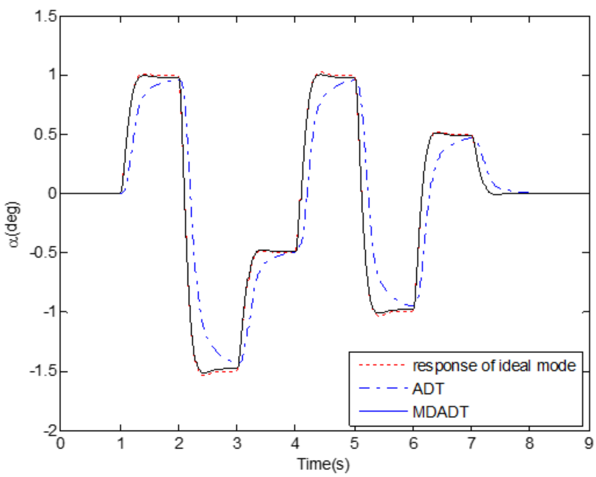

Fig. 7. Command signal and responses.

Figure 8 represents the nonlinear response to the command input and the response of the actuators are shown in Figure 9.

Remark 4.2. It is noted that the sweep signals in Figure 6 are different from each other. In ADT switching strategy, the sweep signal, which is in a dotted form, has the same variation in different subregions. Whereas the dashed line has different variation

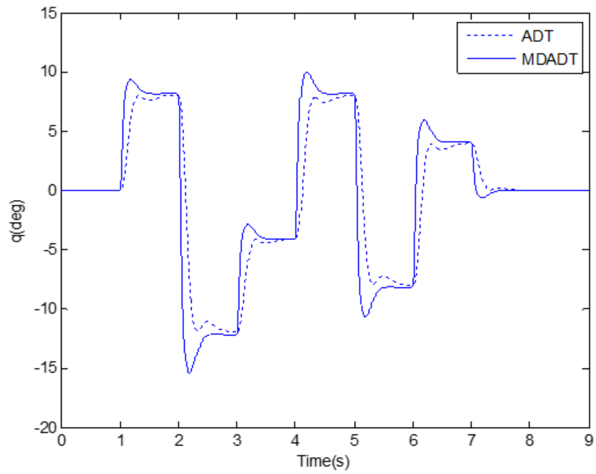

Fig. 8. Pitch rate response.

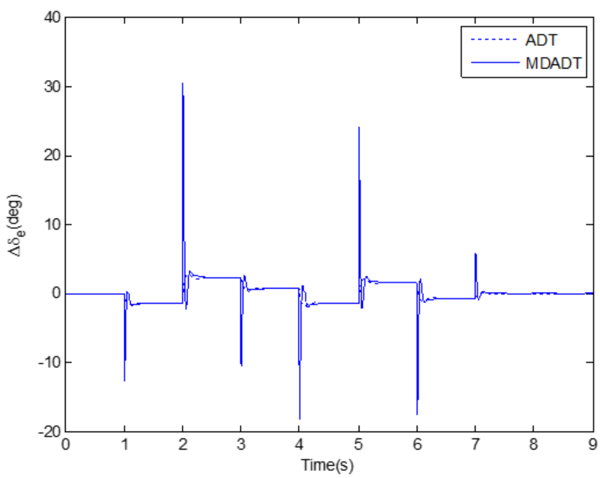

Fig. 9. Elevator deflection.

rate in different region in order to maintain different dwell time in its subregion, as the dwell time is mode-dependent, the switching signal, which is determined by the sweep signal, is mode-dependent. It is by this cause we can deliberately adjust the average dwell time in each subregion, i. e. MDADT.

5. CONCLUSIONS

Due to the large variation of the parameters and multiple control objectives in different configurations for a morphing aircraft, a switched LPV model was obtained and different performance weighting functions were selected to reflect design requirements in its different configurations. Under the framework of ADT and MDADT switching logics, the controller design problem for the switched LPV model of a morphing aircraft were investigated and the sufficient conditions were given in the form of linear matrix inequalities (LMIs). A family of LPV controllers were constructed according to the LMIs' solutions and each of them was suitable for the corresponding parameter region. At last, the proposed switching LPV control method was applied to a morphing aircraft with a variable wing sweep angle and the simulation results demonstrated the effectiveness of the approach.

ACKNOWLEDGEMENT

This work was partially supported by the National Natural Science Foundation of China under grants 61303311, 61374032, 61573286 and Aeronautical Science Foundation of China under grant 20140753012.

APPENDIX

Proof of 3.2

Proof. Assuming there exist a group of matrix functions $\{X_i(\rho)\}_i \in Z_N$ make the closed-loop system (3) satisfy the bounded real lemma over each parameter set $P_i, i \in Z_N$

$$\begin{bmatrix} \begin{array}{c} A^T_{cl,i}(\rho,\dot\rho)X_i(\rho) + X_i(\rho)A_{cl,i}(\rho,\dot\rho) \\ +\dot X_i(\rho) + \lambda_i X_i(\rho) \end{array} & X_i(\rho)B_{cl,i}(\rho) & C^T_{cl,i}(\rho) \\ B^T_{cl,i}(\rho)X_i(\rho) & -\gamma_i I & D^T_{cl,i}(\rho) \\ C_{cl,i}(\rho) & D_{cl,i}(\rho) & -\gamma_i I \end{bmatrix} < 0 \tag{34}$$

with performance level γ_i when $\rho \in p_i$. It is straightforward that $\forall i \in Z_N$,

$$\begin{aligned} \dot V_i(x_{cl}) &\le -\lambda_i V_i(x_{cl}) + \gamma_i^2 \omega^T(t)\omega(t) - z^T(t)z(t) \\ &\le -\lambda_i V_i(x_{cl}) + \gamma_0^2 \omega^T(t)\omega(t) - z^T(t)z(t) \end{aligned} \tag{35}$$

with $\gamma_0 = \max\{\gamma_i\}_i \in Z_N$. For any $T > 0$, let $t_0 = 0$ and assuming the sequence of finite switching time over the interval $[0,T]$ is $t_1, t_2, \ldots, t_\theta, t_{\theta+1}, \ldots, t_{N_\sigma(0,T)}$,where $N_\sigma(0,T) = \sum_{p=1}^{N} N_{\sigma p}(0,T)$ and in which N denotes the number of subregions. For any $t \in [t_\theta, t_{\theta-1})$,by integrating both sides of (35) we can get

$$V_{\sigma(t)}(x_{cl}(t)) \le e^{-\lambda_{\sigma(t_i)}t_\theta + \lambda_{\sigma(t_i)}t} V_{\sigma(t_\theta)}(x_{cl}(t)) - \int_{t_\theta}^{t} e^{-\lambda_\sigma(t_\theta)(t-\tau)}\Gamma(\tau)\,\mathrm{d}\tau \tag{36}$$

where $\Gamma(\tau) = z^T(\tau)z(\tau) - \gamma_0\omega^T(\tau)\omega(\tau)$.

Due to (13),we can obtain

$$
\begin{aligned}
V_{\sigma(T)}(x_{cl}(T)) &\leq \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - t_{N_{\sigma(0,T)}})\}V_{\sigma(t_{N_{\sigma(0,T)}})}(x_{cl}(T)) \\
&\quad - \int_{t_{N_{\sigma(0,T)}}}^{T} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - \tau)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&\leq \mu_{\sigma(t_{N_{\sigma(0,T)}})} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - t_{N_{\sigma(0,T)}})\}V_{\sigma(t_{N_{\sigma(0,T)}})}(x_{cl}(t^{-}_{N_{\sigma(0,T)}})) \\
&\quad - \int_{t_{N_{\sigma(0,T)}}}^{T} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - \tau)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&\leq \mu_{\sigma(t_{N_{\sigma(0,T)}})} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - t_{N_{\sigma(0,T)}}) - \lambda_{\sigma(t_{N_{\sigma(0,T)}-1})}(t_{N_{\sigma(0,T)}} \\
&\quad - t_{N_{\sigma(0,T)}-1})\} \times V_{\sigma(t_{N_{\sigma(0,T)}}-1)} x_{cl}(t_{N_{\sigma(0,T)}-1}) \\
&\quad - \mu_{\sigma(t_{N_{\sigma(0,T)}})} \int_{t_{N_{\sigma(0,T)}}-1}^{t_{N_{\sigma(0,T)}}} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - t_{N_{\sigma(0,T)}}) \\
&\quad - \lambda_{\sigma(t_{N_{\sigma(0,T)}}-1)}(t_{N_{\sigma(0,T)}} - \tau)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&\quad - \int_{t_{N_{\sigma(0,T)}}}^{T} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - \tau)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&\quad \cdots \\
&\leq \mu_{\sigma(t_{N_{\sigma(0,T)}})}\mu_{\sigma(t_{N_{\sigma(0,T)}-1})} \cdots \mu_{\sigma(t_1)} \\
&\quad \times \exp\{(\lambda_{\sigma(t_{N_{\sigma(0,T)}})} - \lambda_{\sigma(t_{N_{\sigma(0,T)}-1})})t_{N_{\sigma(0,T)}} \\
&\quad + (\lambda_{\sigma(t_{N_{\sigma(0,T)}-1})} - \lambda_{\sigma(t_{N_{\sigma(0,T)}-2})})t_{N_{\sigma(0,T)}-1} + \cdots \\
&\quad + (\lambda_{\sigma(t_1)} - \lambda_{\sigma(t_0)})t_1 - \lambda_{\sigma(t_{N_\sigma(0,T)})}T + \lambda_{\sigma(t_0)}t_0\}V_{\sigma(t_0)}(x_{cl}(0)) \\
&\quad - \textstyle\int_0^{t_1} \mu_{\sigma(t_{N_{\sigma(0,T)}})}\mu_{\sigma(t_{N_{\sigma(0,T)}-1})} \cdots \mu_{\sigma(t_1)} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - t_{N_{\sigma(0,T)}}) \\
&\quad - \lambda_{\sigma(t_{N_{\sigma(0,T)}-1})}(t_{N_{\sigma(0,T)}} - t_{N_{\sigma(0,T)}} - 1) - \cdots - \lambda_{\sigma(t_0)}(t_1 - \tau)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&\quad - \textstyle\int_{t_1}^{t_2} \mu_{\sigma(t_{N_{\sigma(0,T)}})}\mu_{\sigma(t_{N_{\sigma(0,T)}-1})} \ldots \mu_{\sigma(t_2)} \exp\{ \quad - \lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - t_{N_{\sigma(0,T)}}) \\
&\quad - \lambda_{\sigma(t_{N_{\sigma(0,T)}-1})}(t_{N_{\sigma(0,T)}} - t_{N_{\sigma(0,T)}} - 1) - \cdots - \lambda_{\sigma(t_0)}(t_2 - \tau)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&\quad - \cdots - \int_{t_{N_\sigma(0,T)}}^{T} \exp\{-\lambda_{\sigma(t_{N_{\sigma(0,T)}})}(T - \tau)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&= \prod_{j=0}^{N_{\sigma(0,T)}-1} \mu_{\sigma(t_{j+1})} \exp\{ \sum_{j=0}^{N_{\sigma(0,T)}-1} (\lambda_{\sigma(t_{j+1})} - \lambda_{\sigma(t_j)})t_{j+1} - \lambda_{\sigma(N_{\sigma(0,T)})}T + \lambda_{\sigma(t_0)}t_0\} \\
&\quad \times V_{\sigma(t_0)}(x_{cl}(0)) - \int_0^T \prod_{p=1}^{N} \mu_p^{N_{\sigma p}(\tau,T)} \exp\{-\sum_{p=1}^{N} \lambda_p T_p(\tau, T)\}\Gamma(\tau)\,\mathrm{d}\tau \\
&= \prod_{p=1}^{N} \mu_p^{N_\sigma p(0,T)} \exp\Big\{ - \sum_{p=1}^{N} [\lambda_p \sum_{s\in\psi(p)} (t_{s+1} - t_s)] - \lambda_{\sigma(t_{N_\sigma}(0,T))}(T - t_{N_\sigma}(0, T))\Big\} \\
&\quad \times V_\sigma(t_0)(x_{cl}(0)) - \int_0^T \prod_{p=1}^{N} \mu_p^{N_{\sigma p}(\tau,T)} \exp\left\{-\sum_{p=1}^{N} \lambda_p T_p(\tau, T)\right\}\Gamma(\tau)\,\mathrm{d}\tau
\end{aligned}
$$

where $\psi(p)$ denotes the set of s satisfying $\sigma(t_s) = p, t_s \in \{t_0, t_1, \dots, t_\theta, t_{\theta+1}, \dots, t_{N_\sigma-1}\}$. With the aid of (4), if there exist constants $\tau_{ap}, p \in Z_N$ satisfying the condition (18), then we can deduced that

$$\begin{aligned}
V_{\sigma(T)}(x_{cl}(T)) &\leq \exp\left\{\sum_{p=1}^{N} N_{\sigma p}(0,T)\ln\mu_p\right\}\exp\Big\{-\sum_{p=1}^{N}[\lambda_p\sum_{s\in\psi(p)}(t_{s+1}-t_s)] \\
&\quad -\lambda_{\sigma(t_{N_\sigma}(0,T))}(T-t_{N_\sigma}(0,T))\Big\}V_\sigma(t_0)(x_{cl}(0)) \\
&\quad -\int_0^T\prod_{p=1}^{N}\mu_p^{N_{\sigma p}(\tau,T)}\exp\Big\{-\sum_{p=1}^{N}\lambda_p T_p(\tau,T)\Big\}\Gamma(\tau)\,\mathrm{d}\tau \\
&\leq \exp\Big\{\sum_{p=1}^{N}[N_{0p}(0,T)\ln\mu_p\frac{T_p(0,T)}{\tau_{ap}}\ln\mu_p-\lambda_p T_p(0,T)]\Big\}V_{\sigma(t_0)}(x_{cl}(0)) \\
&\quad -\int_0^T\prod_{p=1}^{N}\mu_p^{N_{\sigma p}(\tau,T)}\exp\Big\{-\sum_{p=1}^{N}\lambda_p T_p(\tau,T)\Big\}\Gamma(\tau)\,\mathrm{d}\tau \\
&= \exp\Big[\sum_{p=1}^{N}N_{0p}(0,T)\ln\mu_p\Big]\exp\left\{\sum_{p=1}^{N}T_p(0,T)\Big[\frac{\ln\mu_p}{\tau_{ap}}-\lambda_p\Big]\right\}V_{\sigma(t_0)}(x_{cl}(0)) \\
&\quad -\int_0^T\prod_{p=1}^{N}\mu_p^{N_{\sigma p}(\tau,T)}\exp\left\{-\sum_{p=1}^{N}\lambda_p T_p(\tau,T)\right\}\Gamma(\tau)\,\mathrm{d}\tau.
\end{aligned} \tag{37}$$

Assuming the zero disturbance input to the system (3) and $\Gamma(\tau) = 0, \forall\tau \in [0,T)$, we can obtain from above that:

$$V_\sigma(T)(x(T)) \leq \exp\Big\{\sum_{p=1}^{N}N_{0p}\ln\mu_p\Big\}\exp\Big\{\max_{p\in s}(\frac{\ln\mu_p}{\tau_{ap}}-\lambda_p)T\Big\}V_{\sigma(t_0)}(x(0)). \tag{38}$$

Then, with Eq.(18), the exponential stability of the closed-loop system (3) is confirmed.

To establish the weighted H_∞ performance with MDADT switching, we assume zero initial condition for the system (3), then we can obtain from (33) that

$$V_\sigma(T)(x(T)) \leq -\int_0^T\prod_{p=1}^{N}\mu_p^{N_{\sigma p}(\tau,T)}\exp\Big[-\sum_{p=1}^{N}\lambda_p T_p(\tau,T)\Big]\Gamma(\tau)\,\mathrm{d}\tau. \tag{39}$$

Multiplying both sides of the above inequality by $\exp[-\sum_{p=1}^{N}N_{\sigma P}(0,T)\ln\mu_p]$ yields

$$\begin{aligned}
&\exp\Big[-\sum_{p=1}^{N}N_{\sigma P}(0,T)\ln\mu_p\Big]V_\sigma(T)(x(T)) \\
&\leq -\int_{t_0}^{T}\exp\Big\{\sum_{p=1}^{N}[-N_{\sigma p}(0,\tau)\ln\mu_p-\lambda_p T_p(\tau,T)]\Big\}\Gamma(\tau)\,\mathrm{d}\tau.
\end{aligned} \tag{40}$$

This gives

$$\begin{aligned}&\int_{t_0}^{T}\exp\Big\{\sum_{p=1}^{N}[-N_{\sigma p}(0,\tau)\ln\mu_p-\lambda_pT_p(\tau,T)]\Big\}z^T(\tau)z(\tau)\,\mathrm{d}\tau\\ &\le\gamma_0^2\int_{t_0}^{T}\exp\Big\{\sum_{p=1}^{N}[-N_{\sigma p}(0,\tau)\ln\mu_p-\lambda_pT_p(\tau,T)]\Big\}\omega^T(\tau)\omega(\tau)\,\mathrm{d}\tau.\end{aligned}\tag{41}$$

Then, due to the definition and the property of MDADT, we have:

$$\begin{aligned}&\int_{0}^{T}\exp\Big[\sum_{p=1}^{N}(-N_{0p}\ln\mu_p)\Big]\exp\Big\{\sum_{p=1}^{N}[-\lambda_pT_p(0,\tau)-\lambda_pT_p(\tau,T)]\Big\}z^T(\tau)z(\tau)\,\mathrm{d}\tau\\ &=\int_{0}^{T}\exp\Big[\sum_{p=1}^{N}(-N_{0p}\ln\mu_p)\Big]\exp\Big\{\sum_{p=1}^{N}[-\lambda_pT_p(0,T)]\Big\}z^T(\tau)z(\tau)\,\mathrm{d}\tau\\ &\le\gamma_0^2\int_{t_0}^{T}\exp\Big\{\sum_{p=1}^{N}[-\lambda_pT_p(\tau,T)]\Big\}\omega^T(\tau)\omega(\tau)\,\mathrm{d}\tau.\end{aligned}\tag{42}$$

Next, we integrate the above inequality from $T=0$ to ∞ to obtain (by rearranging the double-integral area)

$$\begin{aligned}&\int_{0}^{\infty}\int_{0}^{T}e^{\epsilon}e^{\delta_1T}z^T(\tau)z(\tau)\,\mathrm{d}\tau\,\mathrm{d}T=\int_{0}^{\infty}\int_{\tau}^{\infty}e^{\epsilon}e^{\delta_1T}z^T(\tau)z(\tau)\,\mathrm{d}T\,\mathrm{d}\tau\\ &=\frac{1}{\delta_1}\int_{0}^{\infty}e^{\epsilon}e^{\delta_1\tau}z^T(\tau)z(\tau)\,\mathrm{d}\tau\le\gamma_0^2\int_{0}^{\infty}\int_{0}^{T}e^{-\delta_2(T-t)}\omega^T(\tau)\omega(\tau)\,\mathrm{d}\tau\mathrm{d}T\\ &=\frac{\gamma_0^2}{\delta_2}\int_{0}^{\infty}\omega^T(\tau)\omega(\tau)\,\mathrm{d}\tau\end{aligned}\tag{43}$$

where $\epsilon=\sum_{p=1}^{N}N_{0p}\ln\mu_p$, $\delta_1=\max_{p\in Z_N}\{\lambda_p\}$,$\delta_1=\min_{p\in Z_N}\{\lambda_p\}$, therefore, the performance of closed-loop system (3) is maintained as $<\gamma_0$ with $\gamma_0=\max\{\gamma_i\}_{i\in Z_N}$.

□

(Received May 19, 2016)

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