3D BUILDING RECONSTRUCTION FROM POINT PRIMITIVES: A GEOMETRIC GRAPH APPROACH

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ABSTRACT:

Building reconstruction is one of the most difficult and important 3D modelling problems for urban environment. In this paper, we develop an approach using minimum number of basic geometric elements to reconstruct the buildings. The basic element is a number of unstructured 3D points representing the topological identity of a building roof. Such point primitives can be measured by human or computational operators from images either semi-automatically or automatically. The proposed approach reconstructs the roof surface from connecting points to generate line segments and then grouping the line segments to form faces. The outcome is a set of polygons representing the roof surface, i.e., the building model. The first step determines the roof boundary based on the assumptions of angles. The second step finds the roof ridges with conditions of parallelism. The third step reconstructs the roof surface by Delaunay triangulation. The forth step applies the constrained Delaunay triangulation for boundary and roof ridge to be embedded in the triangles. The last step merges coplanar triangles to form roof faces. The outcome is a set of polygons representing the roof surface. Finally, the building will be reconstructed as a polyhedron object expressed by vertices and displayed for 3D visualization and spatial analysis.

1. INTRODUCTION

Due to recent progress of 3D data collection techniques, advancement of computational capability, and the inception of economic large data storage, real world objects can be easily and rapidly rendered in virtual 3D environment. The virtual 3D can be a powerful tool for people to explore, analyze and plan the reality. In a virtual 3D urban scene, building objects attract most attention because of the variety of applications. The representation of buildings in 3D geospatial information system or CAD system can support urban planning, decision-making process, on-line positioning, environment monitor and management, virtual tour, and more. Therefore, 3D building modeling becomes a very active research area in computer vision, CAD, computer graphics, and photogrammetry. Remondino (2006) defined the complete process of 3D object modeling that starts from data acquisition and ends with a 3D virtual model visually interactive on a computer. The process is complicated and complex, involving detection, extraction and reconstruction of the object. Detection refers to locate an object of interest, extraction comprises methods for delineating interested features, and reconstruction refers to determine the shape of an object and establishing the relations among its components.

Despite the difference of reconstruction methods and outputs, they are all guided by data sources which are influenced by budget, scale, time and labor issue. Considering all factors, including resolution, efficiency, accuracy, stable, level of detail and cost, imagery is the primary data source in this research. The scale of the images depends on the quality required for the 3D model and is typically larger than 1:5000. Using aerial images, many building details can be measured, and the measurement error is maximal 0.2 meter in height (Ulm, 2003). The traditional way, human-based image interpretation would be adopted in this research. The operators need to interact with images to obtain the object information for each geometric model of buildings. The drawback of this process is that it is a costly and labor intensive operation. On the other hand, automatic process towards fully machine-based image interpretation is highly demanded, though it is a difficult task to achieve at present. Due to occlusions, imperfect image quality, invisible features, mixture with adjacent objects, small structures on the roof, human interaction is still unavoidable in practical use. The most common way to collect features from aerial image is using stereo image pairs. The operators manually drive the 3D mouse to measure the roof structures and building footprints. In many commercial photogrammetric systems, operators delineate the building footprints and roof polygons.

This research develops a semi-automatic approach, which is meant to minimize the required interaction and maximize the capability of automation. Operators are only asked to measure a minimum number of 3D points for each building object. Such 3D points are the topologic primitives that define the building structure. Considering the difficult of delineating footprints from aerial images, only roofs will be measured. A point of roof surface is a topological node representing the intersection of line segments. Each line is the edge of the planar surface of roof structure. To reduce the task loading to operators, they only need to measure such topological points and measure them without specific order. This easy process yields the collected point cloud as a completely unstructured dataset.

There are two methods to model buildings from points, model-based method and constraint-based method. In model-based method, the building is determining by fitting pre-defined models into points to determine the shape, dimension, position and orientation. In constraint-based method, the building is reconstructed based on geometric properties, such as planarity, alignment, parallelism, orthogonality, etc (Grossmann, Santo-Victor, 2005). The former method is limited by pre-defined models and the latter one has less limitation, as long as there are enough geometric properties to define a unique building. To maximize the capability of reconstruction, we exploit algorithms with geometric constraints. Moreover, the reconstruction process from 3D point primitives to a building model will be fully performed by algorithms. We devote ourselves to develop algorithms which can model building objects from the most primitive elements. The complete building object will be represented as a polyhedron model which assumes the building to be bounded by planar surfaces. The roof contains to no curved surfaces and the walls stand vertical. Each building object contains geometric information and topological structures which support spatial, topological and thematic studies.

2. RELATED WORK

We briefly review four semi-automatic related studies that also apply distinct point clouds to model 3D buildings. Early work can be found in Grün and Dan (1997) that proposed a model-based topology builder - TOBAGO. The system is designed to classify the unorganized point cloud into generic roof database based on the number of ridge points. The roof planar faces are determined through geometric criteria and enclosed the point cloud as a complete building model. After TOBAGO, Grün and Wang (1998) proposed a second generation approach called CyberCity-Modeler. This approach works for weakly structured point cloud which requires the boundary point in sequential order. Finding the planar faces and generating roof through all points is considered as a consistent labeling problem and solved by probabilistic relaxation. Köehl and Grussenmeyer (1998) developed a technique to generate 3D city objects with geometric, topological and thematic modeling at the same time. The operator has to decompose complex object into basic geometric elements and then associate the point cloud with a generic model library with its geometry and theme. Zlatanova et al. in 1998 presented a process restricting eave points to follow certain sequence and projected them onto the DTM to construct vertical walls. The operator can digitize roof points arbitrarily and then superimpose roof points into pre-define graphics from database in the stereo model.

3. BUILDING OBJECT AND MODEL

There is no common way to define a “building” object, and there is no common model to represent the “building”. Building is a 3D object and we need rules to describe and represent this object. In this research, we generally describe a building as a polyhedron. The polyhedron is composed of planar faces. The planar face consists of straight line segments between two points, and planar faces surrounded by line segments. This is a general description for 3D object and this description could fit most buildings in reality. There are no specific knowledge and parameters related to building such as length, width, height and roof-height in the description. Therefore, the generic assumption should apply to a huge variety of building styles, such as houses in residential areas with very different geometry.

In this research, our main concern is the determination of roofs. A building is considered as a polyhedron, and the roof is the surface of the polyhedron from top view. The roof is defined as a connected simple planar graph with *n* vertices. A graph is noted as *G=(V, E)* where *V* is a finite set of *n* vertices in R^2, and *E* is a set of edges determined by the vertices in *V*. A vertex is a point, an edge represents the line segment in a graph. A graph is called a geometric graph if the vertices or edges are associated with geometric objects in a Euclidean space.

Planar graph:

Let *V* be a finite set of *n* vertices in R^2, and *E* be a set of edges determined by the vertices in *V*. A planar graph is defined as:

- for each edge $ab \in E,\ ab \cap V = \varnothing$
- for each edge pair $ab \neq cd$ in *A*, $ab \cap cd = \varnothing$

A connected graph is any two vertices a, b of *G* are connected if there is a path from a to b in *G*.

A building can be a simple polyhedron or a combination of several polyhedrons. While collecting the building roof, an operator has to separate a complicate building roof into several roof units. Each roof unit is an individual point dataset which has to satisfy the roof definition (Figure 1). In this paper, we call each roof unit just as “roof” for short. The reconstruction processes only run one roof each time.

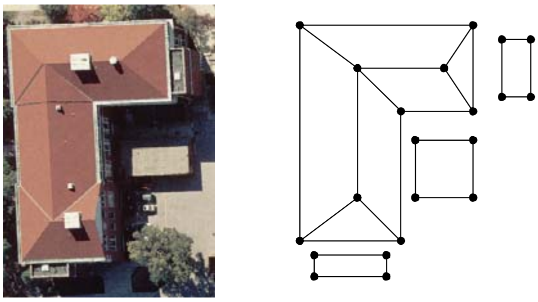
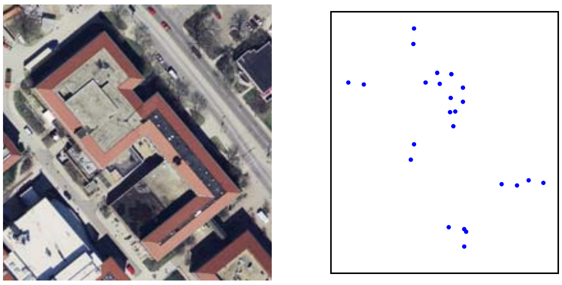

(a) (b)

Figure 1. (a) A building image. (b)A building is composed of four separated units.

To present the building polyhedron as a 3D virtual object in computer graphics is based on the data structure. There are three common models to present 3D object: wireframe model, surface model, and volume model. A wireframe model represents an object by a collection of 3D edges. A surface model is a collection of surfaces, and a volume model is a solid object with mathematical description. In this paper, we are mainly concerned that our work can present the building as groups of edges and groups of surfaces to satisfy either wireframe models or surface models.

4. METHODOLOGY

The aerial images serve more like a roof landscape than a building landscape. Our main concern in this paper is the determination of roofs. The initial information of the roof is a point dataset. Figure 2 illustrates the point cloud obtained from an aerial image pairs. The result is a surface defined by planar faces. These points define exact faces and the spatial relations of the faces. Roof reconstruction is equivalent to find a polyhedron surface. Disregarding the metric properties of a roof, a roof surface can be embedded as a planar graph. Our approach starts from finding the line segments and then determining the faces by line segments.



(a) (b)

Figure 2. (a) Aerial image of a building. (b) Roof topologic point primitives

The roof terminology discussed in this research is illustrated in Figure 3. The boundary is formed by all eaves of a roof and equivalently the orthographic projection of all eaves onto a horizontal plane. The points of the roof boundary are called corner points. The remaining points of the roof are called roof points. The ridges parallel to the boundary are roof ridge, and the remaining ridges are hip ridges.

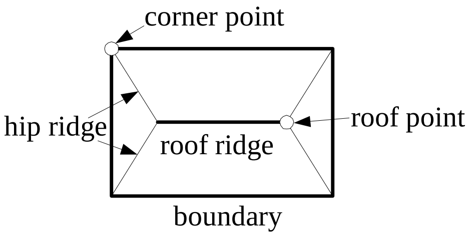


Figure 3. Roof elements from a top view.

We only adopt two constraint rules as our building knowledge.

- Most buildings tend to have an orthogonal boundary. If the angle is not a right angle, then it must be an obtuse angle. This is applied to both interior angles and exterior angles.
- A roof ridge is the axis of symmetry, if these ridges are parallel to the boundary.

The process is to determine significant line segments to recover the boundary and roof ridges. The suggested five steps for roof reconstruction include:

1.

1. Determine roof boundary.
2. Determine roof ridge edges.
3. Triangulate the point set by Delaunay triangulation.
4. Modify edges by constrained Delaunay triangulation.
5. Merge coplanar triangles.

2.

4.1 Boundary determination

The roof boundary is a set of edges representing the eaves, and the vertical walls can be constructed by projecting the eaves to the ground. Each intersection of two edges includes an interior angle and an exterior angle. If the interior angle or exterior angle is not a right angle, both of them should likely avoid acute angles. The first step of modifying the triangulated graph is to find a boundary satisfying this rule. The boundary can be considered a cycle composed of a sequence of polylines, and each edge is adjoined two vertices. In the 2D plane, we consider this cycle as a polygon. The constraint of the polygon is the angle of each adjacent edge pairs has to be as close to a right angle as possible and no acute angles. The problem is finding a polygon that contains a set of points lying in a plane with angle constraints. To find a polygon under this constraint, we start from an initial convex hull. Then we work on angles larger than 90 degree individually. The idea is breaking edges associated with obtuse angles into more pieces to find possible edges that minimize the angles. An edge $\langle a,b \rangle$ connects two vertices *a* and *b*. Breaking the edge means to find a vertex *p* and generate two edges $\langle a,p \rangle$ and $\langle p,b \rangle$. Points inside the polygon will be selected for breaking edges. The selection is based on two geometric characters: distance and angle. The shape of polygon will be changed gradually until no edges could be modified. Therefore, finding the vertex to break the edge is the key of the algorithm. The process is like squeezing the polygon toward inside until it satisfied our requirement. The output is optimal but may not be unique. The algorithm is outlined below.

Boundary construction. *Let V be a set of vertices in* $\mathbb{R}^3$. *Find a cycle in* $\mathbb{R}^2$ *with angle constraints.*

Input: A set of vertices *v* in $\mathbb{R}^2$.
Output: A set of polygon P' with no acute angles and most right angles.

1. Start with an initial polygon for the vertex set. The initial polygon $P = (V', E')$ is a convex hull.

2. Find interior and exterior angles of each edge pair. If any of the angles is equal to 90 degree, we no longer deal with it. If the angle is larger than 90 degree, we try to make the angler smaller until it get close to 90 degree. To do that:

> Calculate interior and exterior angles $\angle abc$ of each edge pair $e_{ab}, e_{bc} \in E'$. For each pair:
> - If the interior or exterior angle is $90 \pm \mu$, the edge pair is marked as *fixed edges* (*e$_f$*). μ is the tolerance. The *fixed edges* will no longer be modified.
> - If the interior angle is $> 90 \pm \mu$, the edge pair is marked as *breakable edges* (*e$_b$*). The *breakable edges* need to be further modified.

3. Consider each *breakable edge* individually to find vertices associated with it. With these vertices, we can divide the *breakable edge* into two edges. These vertices are called *candidate vertices*.

```
for each $e_b$
    • $e_b = \langle v_i, v_j \rangle$, find a set of vertices
      $V_b = \{v_1, v_2, ...v_n\},\ V_b \cap V' = \varnothing$ and form a new
      edge pair $E_i = \{\langle e_{ix}, e_{xj} \rangle \mid x \in V_b\}$.
    • New edge set of each breakable edge is $E_i$. The
      new polygon set is
      $P' = (V'', E_l),\ V'' = V' \cup V_b,\ E_l = E_f \cup E_i,$
      where $l = 1, 2, ....m,\ E_f$ is fiexed edge set.
end
```

4. The new edge associates with *breakable edge* and *candidate vertices* have to pass validity checks.

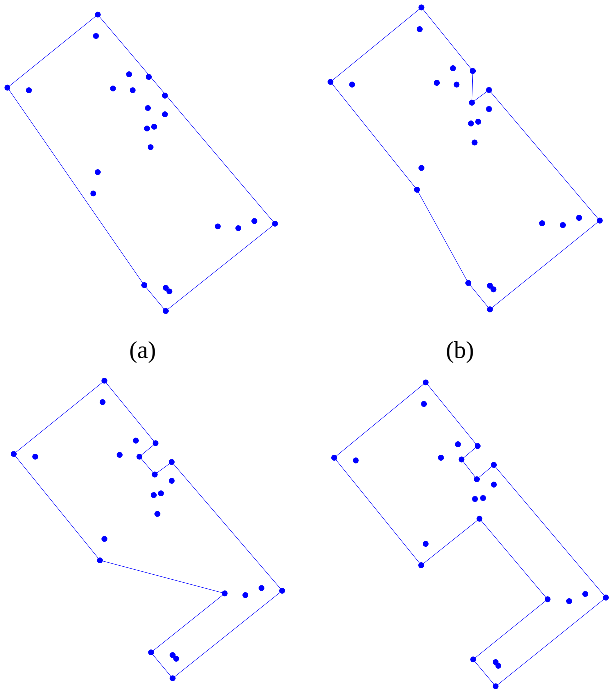
```
for each polygon
    • For each polygon $P'$, any vertex $v$ must inside the polygon.
    • For each edge pair $e_{ab}, e_{bc} \in E_l$, the interior angle $\angle abc$ is $\geq 90 \pm \mu$ degree.
end
```

5. If a *candidate vertex* is contributed to more than one *breakable edge;* $v \in E_l$ where $l = 1, 2, \ldots m$, $|\mathrm{m}| \geq 2$. Select the vertex associated with *breakable edge* by highest score. Score $\propto \theta \propto dis$; where θ is interior angle $\angle abc$ and *dis* is the distance from *candidate vertex* to *breakable edge*.

6. A new set of polygons will be formed with *breakable edges*. $\{P'_1, P'_2, \ldots P'_l\}$. Select the polygon P' by the combination of interior angles
    - The P' has to contain most right angles ($90 \pm \mu$ degree).
    - If the interior angle is not a right angle, it has to be obtuse and as small as possible.

The final polygon represents the boundary $\bigcup_{i=1}^{n} \langle v_{i-1} v_i \rangle$ of edge $\langle v_{i-1} v_i \rangle$, where $v_0 = v_n$

The output is the possible boundary polygon. The vertices of the polygon are corner points; the edges of the polygon are boundary. We keep possible results close to the requirement at this stage. The tolerance $\pm\mu$ is an accuracy controller, the unit is angle in degrees. The determination of μ is mainly based on image resolutions and the quality of measurement. Figure 4 illustrates the processes of boundary determination.


(a) (b)

(c) (d)

Figure 4. (a) The initial convex hull of the point dataset. (b), (c) The intermediate steps of roof boundary reconstruction. (d) The final output represents the roof boundary.

4.2 Roof ridge determination

The second step is finding the roof ridge edge set which is totally determined based on the boundary. Points which do not belong to boundary are called roof points. The process runs through all roof points to get a set of random combination edges. Select any edges parallel to boundary edges within $\pm\mu$ as roof ridge edges (Figure 5b). Four rules are applied for selecting proper roof ridges.

1 If roof ridge edges cross each other, the edges with larger height value will be selected.

2 One roof ridge may parallel more than one boundary edges. The length of roof ridge can not be longer than the total length of those boundary edges.

3 If a roof point could contribute to more than one roof ridge, select the roof ridge parallel to the ground, which is more common than a roof ridge tilted relative to horizon.

4 If a roof point contributes to two roof ridges and they all parallel to the ground, both of them are possible options. That is to say there are multiple results.

```
Input: A set $V = \{v_p, p = 1,\ldots, m\}$ of roof points, The boundary edge set $\bigcup_{i=1}^{n} \langle v_{i-1} v_i \rangle$
Output: One or more set of edges $E_r$
begin
    $V(p)$ = roof vertices;
    Each $v_p$ has 3D coordinate values;
    The boundary edge set $E(i) = \{e_{ij} \mid j \in V(i)\}$;
    Compute all combination of two vertices
    $\{e_{xy} = \langle v_x, v_y \rangle \mid x, y \in V(p)\}$;
    Remove all $e_{xy}$ cross $e_{ij}$;
    for each $e_{xy}$
        Find $e_{xy}$ parallel to the boundary edge $e_{ij}$
        Validity check.
    end
    Select candidate $e_{xy}$ according to rules.
    Get $E_r = \bigcup e_{xy}$, where $E_r$ is an edge set;

    Validity check
      1. If any two edges $\langle e_{xy}, e_{uv} \rangle$ where $x, y, u, v \in V(p)$
         cross each other, remove the $e_{xy}$ with smaller
         height value;
      2. The length $\overline{e_{xy}}$ is $\geq \sum \overline{e_{ij}}$.
end
```

4.3 Delaunay Triangulation

A roof surface can be embedded as a planar graph. The basic or minimum region of a roof surface is triangle. We also consider that each corner point follows tri-connection, i.e. all corner points are incident to exactly three edges. Two edges belong to boundary edges, and the rest edge adjacent the corner point to a roof point. We adopt the idea of triangulating the planar graph because of above properties. The triangulation method is based on the Delaunay Triangulation (Delaunay, 1934) which is designed to optimize the surfacing of a random set of points. Delaunay Triangulations maximize the minimum angle of all the angles of the triangles, which means they tend to avoid sliver triangles. A triangulation $\Delta = (E, V)$ is Delaunay triangulation if all edges satisfied the empty circumcircle property of any triangle for the *V* (Figure 5a). The result is a group of 2D triangles, which offers an initial 3D surface.

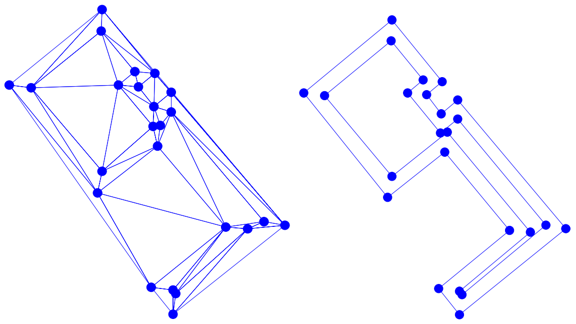

(a) (b)

Figure 5. (a) The initial result from Delaunay Triangulation. (b) Ridge edges parallel to boundary.

4.4 Constrained Delaunay Triangulation

We call edges determined by Delaunay Triangulation as initial edges. The determination of boundary and ridge edges could disagree with the initial edges. We modify these edges by using Constrained Delaunay Triangulation (CDT). The idea of CDT is to remove all edges which cross the initial edges. That will form simple polygons contained two end vertices of the initial edges, and the initial edges are diagonals. Then we re-triangulate the polygon by using "divide and conquer" algorithm (Anglada, 1997). The re-triangulation tends to be as close to Delaunay triangulation as possible (Figure 6a).

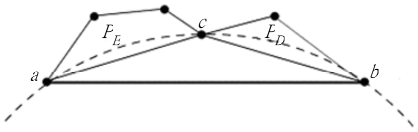
Definition (CDT):

The $G = (V, E)$ is the planar graph with $E \neq \varnothing$, where a $\Delta = (S, E \cup E')$ is a Constrained Delaunay Triangulation of *G* if the edge $e_{ab} \in E'$ are such that *a* and *b* are visible in *G* and e_{ab} fulfills the empty circumcircle property with the vertices only visible from *a* and *b*.

After running through the CDT, the last step is to remove triangles outside of boundary. Implementing height value to each triangle will make these triangles as possible roof polygons.

The algorithm outline:

1. For diagonal e_{ab}, go through all vertices to find a circumcircle of triangle Δabc which contains no any other vertex of the pseudo-polygon.
2. If currently selected vertex is *c*, then *c* divides the polygon into another two polygons, P_E and P_D.
3. Repeat until only three vertices are left.



4.5 Merge co-planar triangles

This is the only step needs to work in 3D. All adjacent triangles will be examined if they are co-planar by their normal vectors. If the normal vectors of all adjacent triangles are in approximately the same direction, they can be merged into one polygon (Figure 6b). This is the final step to reconstruct the roof surface.

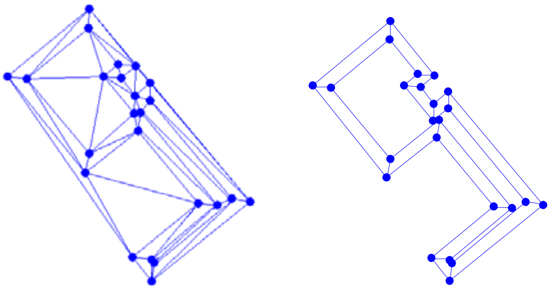

(a) (b)

Figure 6. (a) Applying CDT to remove disagreement between edges. (b) Final roof result after merging triangles.

5. RESULTS

The data used to examine the proposed approach is from pair of aerial stereo images at the scale of 1:4,000. Figure 7 shows several examples from the results. Left images are the aerial imagery and the right frames show the reconstructed roof surfaces. The test shows a promising outcome. In the test, most building with orthogonal boundaries can be successfully achieved. During the measurement, operators need to estimate the location of points in hidden areas. It is almost impossible to ask the operator offering perfect measurements. Since each roof is a geometric graph, the entire process highly relies on the measurement of point coordinates. The calculation of each step is very sensitive to coordinate values, which makes the tolerance selection important. The measurement under 3D stereoscope is difficult and raises the problem of choosing the tolerance. The tolerance value can be determined by estimating of image resolution and by practically testing a few cases. In 2D plane, we need a tolerance angle value for deviation of boundary and ridge edges, respectively. In this paper, we apply $\pm 5^o$ for both x and y directions, and 0.5 m for z value. To complete building polyhedrons, vertical walls can be completed by projecting the roof boundary to the ground. We can obtain the height information of buildings from either asking the operator to collect one footprint point or from a DEM dataset.

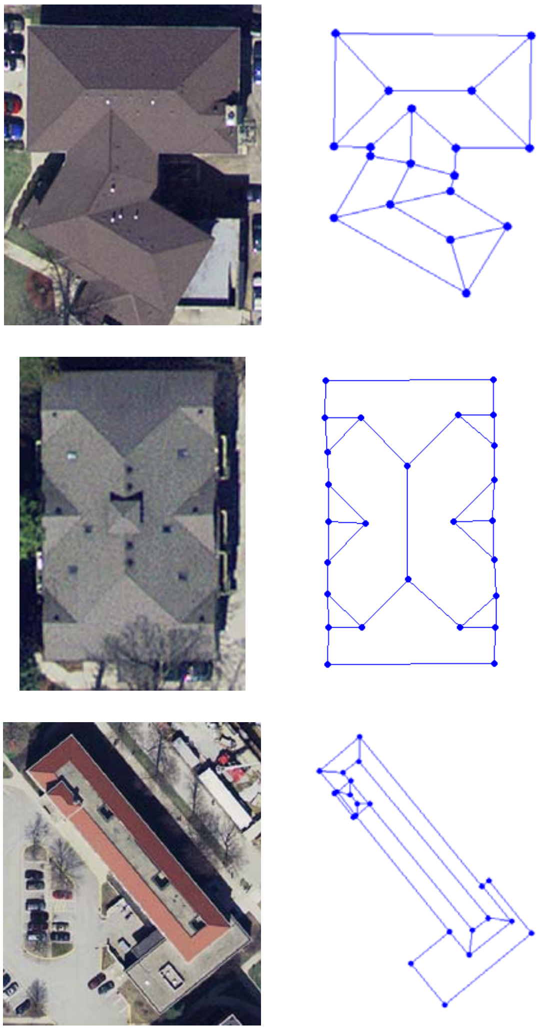


Figure 7. Examples of reconstructed roofs

6. CONCLUSIONS

In this paper, we have developed a methodology for building reconstruction from unstructured topologic point primitives. The building information is obtained from image-based data, mainly from aerial imagery. Our major effort is focusing on the reconstruction of building roofs. The underlying mechanism is using triangulation to reconstruct the roof surface. To make the surfaces more close to roof surfaces, knowledge about buildings has to be considered. One of the important building characteristics is the angle between adjacent boundary faces. For most buildings, they tend to be right angles or obtuse angles. This property plays a key rule while finding the roof boundary. The other common characteristic is the parallelism between roof ridges and boundary edges. The algorithms are contributed to find possible line segments of a building. If there are conflict between triangulation results and line segments based on building knowledge, we apply a constrained triangulation to solve the disagreement. The output represents an appropriate roof surface by a group of polygons.

The roof surface determined by this method may not be consistent with real roof. This result can be considered as the beginning of achievement of building reconstruction. To modify the result, the need of topological roof relation, post editing tool and accuracy evaluation is necessary. The topological roof relation representing most common roof shapes needs to be involved. Also, the development of topological relation of roof faces is very important for spatial analysis. An editing tool for operators to manually fix the inaccurate roof polygons would be most effective. In the future, an evaluation method needs to be developed, which is necessary for testing the performance of a reconstruction approach.

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