RADIAL SPATIAL DIVISION BASED ON $Qi(x_i, y_i)$ AND RESTRAINED EDGE MOSAIC IN CONSTRUCTED TIN

Hua QI [a, b], Deren LI [a]

[a] The State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University, Wuhan, Hubei, China, 430079
[b] Dept. of Surveying Engineering, School of Civil Engineering, Southwest Jiaotong University, Chengdu, Sichuan, China, 610031
Email: qi-3dgis@sohu.com

Commission PS, WG V/2

KEY WORDS: TIN, Algorithm, Time complexity

ABSTRACT:

To insert restrained edges into TIN is of great necessity in many fields. At present, for the need of mass data processing, it is expected that the time efficiency of the algorithm be higher and higher. As a result, the paper proposes the method of radial spatial division based on $Qi(x_i, y_i)$ to realize the restrained edges mosaic in constructed TIN. First of all, it introduces the basic principle of radial spatial division based on $Qi(x_i, y_i)$. After that, on the basis of the principle the algorithm to realize restrained edges mosaic is given in detail. A spatial division tree is proposed as an efficient implementation method in the aspect of reconstruction of triangles and their spatial relationship after the division. And then, the triangle obtained with this method is sure in the sub-zone is proved. The analysis of time complexity shows that the time complexity to execute $Qi(x_i, y_i)$ is lower than that to compute the distance from a point to a line. Finally, the conclusions are presented. It is shown that the radial spatial division algorithm proposed in this paper has more advantages in time efficiency than the spatial division algorithm based on distance.

1. INTRODUCTION

Triangulated Irregular Network plays great role in many fields such as finite element analysis, curved face approaching, computer vision, digital elevation model and mapping, three-dimensional simulation, which need to conduct a restrained edge (feature line) mosaic into TIN. For example, in the application of DEM and mapping, it is necessary to inset terrain feature line into TIN in order to enhance the approach degree between TIN and the actual terrain. In high accuracy mapping with the LIDAR data for highway corridor mapping projects requiring 0.6 meter contours at 1:1200 scale, beyond hard surface corridor, photogrammetry support (terrain breaklines) is required to maintain the true shape of the terrain (Renslow, 2001). There are two approaches: (1) combine restrained edge with TIN construction, or judge whether the triangle intersect with restrained edge when construct TIN (Zhu and Chen, 1998); (2) mosaic the restrained edge in constructed TIN (Lu, Wu and Lu, 1994). This paper mainly refers to the relative method of the second approach.

Lu and etc. introduced an algorithm, 'm+2 borders cater corner exchange' (Lu, Wu and Lu, 1994), which is a typical algorithm among the second approach mentioned above for it can resolve all complex cases. But this algorithm needs to compare the distances from points to line which resulted in redundant code and hard to record adjacent relation among triangles, etc (Li and Tan, 1999; Wang, 2001). Therefore with analysis of 'track creation' algorithm (Zhou and Liu, 1996), Li and Tan found that 'track creation' algorithm might fail when affected area is a concave polygon. And they proposed that transform concave affected area to convex polygon before apply 'track creation' algorithm to mosaic restrained edge in any affected area, and then 'descending algorithm' and 'looping algorithm' were proposed (Li and Tan, 1999). Wang provided some special graphic patterns which may make 'descending algorithm' fail and suggested that 'looping algorithm' can be used to assure the validity of exchange in any situations. In terms of implementation method, there are lots of differences between the algorithm given by Lu and the follow-up algorithms (Wang, 2001).

This paper does not discuss the diversity of the algorithms mentioned above but probe into the method of spatial division in order to provide a more effective approach. It is shown that 'spatial division based on distance' is the basis of the 'm+2 borders cater corner exchange' algorithm. It means that affected region are divided into two independent sub-zone by restrained edge before spatial segmentation based on distance is conducted on the two sub-zone repeatedly till all sub-zones emerge as a triangle. According to the method, to find a point among the boundary point which is the nearest to any baseline (such as restrained edge), the triangle constructed by that point and the baseline must be in the sub-zone. But, the disadvantageous of this approach is the cost of calculating the distance. Therefore, this paper proposes a so-called radial spatial division method based on function $Qi(x_i, y_i)$, and apply the method to construct initial triangulation in the region affected by restrained edge for restrained edge mosaic.

In Section 2 some basic conventions are given for discuss. In Section 3 the principle of radial spatial division method based on function $Qi(x_i, y_i)$ is presented. Section 4 and Section 5 describe our algorithm in detail. A algorithm, the radial spatial division algorithm, for implementing restrained edge mosaic in constructed TIN based on the principle of radial spatial division method with function $Qi(x_i, y_i)$ is proposed, and that the triangle created by radial spatial division algorithm proposed in this paper must be in the sub-zone is justified, and that an exceptional case is discussed, and that the spatial division tree which will be great benefit for the reconstruction of triangles and their topology in region affected by restrained edge after spatial division is proposed. Section 6 gives analysis about the time complexity of algorithms. Section 7 presents the conclusions.

2. BASIC CONVENTION

Some basic conventions are listed below for discuss:

- no position coincide points in data set, or such points having been cleared off;
- supposing that vertexes of triangle numbered clockwise;
- restrained edge: defined as a vector,from the initial position to the end;
- cross edge: the edge of triangle intersected with restrained edge;
- medium triangle: the triangle which have two cross edges;
- region affected by restrained edge: region composed by triangles which have at least one cross edge;
- diagonal vertex: two triangles, ΔA and ΔB , are contiguity with cross edge. In the two triangles, the vertexs which are not on the cross edge are diagonal vertexs, and the diagonal vertex in ΔB is called the diagonal vertex of ΔA ;
- left adjecent edge of diagonal vertex: edge is composed of diagonal vertex in ΔB and its previous vertex;
- right adjecent edge of diagonal vertex: edge is composed of diagonal vertex in ΔB and its next vertex;
- expanding edge: the standard for region spatial division, which is also an edge of divided triangle.

See Figure 1(a), for example, ΔA is $\Delta p_1 p_7 p_8$ and ΔB is $\Delta p_1 p_2 p_7$; p_2 is the diagonal vertex of ΔA ; $p_1 p_2$ is the left adjecent edge of diagonal vertex p_2 ; $p_2 p_3$ is the right adjecent edge of diagonal vertex p_2 .

3. RADIAL SPATIAL DIVISION METHOD BASED ON $Qi(x_i, y_i)$ FUNCTION

3.1 Radial spatial division theory based on $Qi(x_i, y_i)$ function

For the point set P in the plane, a radial cluster is composed of a point at random together with all other points. Every radial's azimuth angle α_i is calculated and ascending sorted with α_i as a key word, and then an ordered radial sequence in which radials with the same azimuth angle are counted as one radial is developed.

At least there is one point besides the base point in every radial. Occasionally there are several points in the same radial. Here the spatial adjacent relation among the points at the same direction can be get as follow:

When $x = 0$: ascending sort with $|y|$ as key word is down for the points; and

When $x \neq 0$: ascending sort with $|x|$ as key word is down for the points;

In this case, this sequence's important property is that no point exists between any two adjacent radials.

3.2 Spatial relation analysis methods between point at any radial and some other radial

In order to analyse spatial relation between point p_i on any radial i (azimuth α_i) and some radial k (azimuth α_k), $\Delta\alpha$ which start from radial i to radial k widdershins should be calculated, see Formula (1), and the spatial relationship between point p_i and radial k can be deduced with Formula (2).

$$\Delta\alpha = \begin{cases} \alpha_i - \alpha_k & \alpha_i - \alpha_k \geq 0 \\ \alpha_i - \alpha_k + 360^0 & \alpha_i - \alpha_k < 0 \end{cases} \qquad (1)$$

$$\begin{array}{llll} if & \Delta\alpha < 180^0 & then & p_i \ \ right \ \ of \ \ k \\ if & \Delta\alpha = 0^0 \ , \ 180^0 & then & p_i \ \ and \ \ k : collinear \\ if & \Delta\alpha > 180^0 & then & p_i \ \ left \ \ of \ \ k \end{array} \qquad (2)$$

Usually $\arctan(x)$ is used to calculate azimuth α_i . In analyzing the spatial relationship between radials, the Qi algorithm turns the spatial adjacent relationship between radials into the adjacent relationship between the point of intersection of radials and side of the externally tangent rectangle of the unit circle. Then, the azimuth angle computation is shifted to the Qi length computation (Qi, Li and Zhu, 2003). So in practice for decreasing time complexity of algorithm $\arctan(x)$ could be replaced by $Qi(x_i, y_i)$, see Formula (6), to calculate distance of Qi (Qi, 1996). All those analysis are implemented with function $Qi(x_i, y_i)$. Formula (3) and (4) correspond to Formula (1) and (2), respectively.

$$\Delta Q_i = \begin{cases} Q_{ii} - Q_{ik} & Q_{ii} - Q_{ik} \geq 0 \\ Q_{ii} - Q_{ik} + 8 & Q_{ii} - Q_{ik} < 0 \end{cases} \qquad (3)$$

$$\begin{array}{llll} if & \Delta Q_i < 4 & then & p_i \ \ right \ \ of \ \ k \\ if & \Delta Q_i = 0, 4 & then & p_i \ \ and \ \ k : collinear \\ if & \Delta Q_i > 4 & then & p_i \ \ left \ \ of \ \ k \end{array} \qquad (4)$$

3.3 Spatial analysis method of confirming adjacent radials of each radial in radial cluster

According to Formula (3), ΔQ_i can be worked out. When $\Delta Q_i = MAX$, corresponding radial is the left adjacent edge of the basic radials, and when $\Delta Q_i = MIN$, corresponding radial is the right adjacent edge of the basic radial.

4. RADIAL SPATIAL DIVISION WITH RESTRAINED EDGE AS EXPANDING EDGE AND RESTRAINED EDGE MOSAIC

4.1 Confirmation of region affected by restrained edge

4.1.1 Confirmation of the first triangle

The algorithm steps of confirmation of the first triangle in the region affected by restrained edge is the following:

- Record the starting point of restrained edge into left edge queue *Q* and right edge stack *S*;
- Calculate $Qi(x_i, y_i)$ of every triangular edge with the the starting point of restrained edge as a center;
- Use Qi as a key word to get two edges each adjacent with left and right of restrained edge. Starting points of restrained edge are one end of the two edges respectively. The triangle which contains the two edges is the first triangle in the region affected by restrained edge;
- Record the *ID* of the other point of the left adjacent edge and ΔQi for the left adjacent edge and the restrained edge into the left edge queue *Q* of the affected region;
- Record the ID of the other point of the right adjacent edge and ΔQi for the edge and the restrained edge into the right edge stack *S* of the affected region;
- Record the *ID* of the triangle.

In the process mentioned above, if restrained edge has the same value of Qi as that of either edge of triangle, which means that restrained edge coincide with either edge of triangle, then the mosaic of the next restrained edge of restrained edge string can be performed directly. Because a triangle associated with the starting point of the next restrained edge has been gained, all other triangles which make starting point as vertex can be gained through the triangular adjacent relationship, and the first triangle can be get in the same way mentioned above.

4.1.2 Confirmation of the middle and the last triangle

The middle and the last triangle can be confirmed by analysis of spatial relationship between diagonal vertex and restrained edge. If the diagonal vertex lies to the left (or right) of the restrained edge, right adjacent edge (or left adjacent edge) of the diagonal vertex is certain to be the cross edge, and ΔB is certain to be middle triangle; if the diagonal vertex happen to be situated in the restrained edge, ΔB is certain to be the last triangle in the region affected by restrained edge. So the middle or the last triangle in region affected by restrained edge can be acheieved through analysing the spatial relationship between the diagonal vertex of the first triangle and restrained edge in the region affected by restrained edge. Such analysis should be continued till find the last triangle ,when the whole region affected by restrained edge can be confirmed.

The spatial relationship analysis mentioned above can be acheieved by constructing radial with starting point and diagonal vertex and applying Formula (3) and (4). The step go as :

- If $\Delta Qi > 4$, the diagonal vertex of ΔA must lie in the left of restrained edge. Record ΔQi into left edge queue *Q* of the affected region;
- If $\Delta Qi < 4$, the diagonal vertex of ΔA must lie in the right of restrained edge. Record ΔQi into right edge stack *S* of the affected region;
- If $\Delta Qi = 0$ or 4, the diagonal vertex coincide with the restrained edge;
- Record the ID of ΔB.

Repeat the process mentioned above till the diagonal vertex of ⊿ A coincided with restrained edge. Record the end point of the restrained edge into the left edge queue *Q* and right edge stack *S*, respectively.

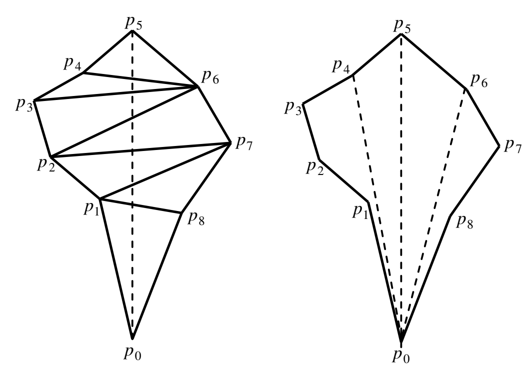
The sequence of the triangular *ID* recorded by radial spatial division process is the affected region of the restrained edge.

4.2 Restrained edge mosaic

In fact the process of confirming affected region of restrained edge is the first step of radial spatial division of the region affected by restrained edge. Being found from *Q* as an edge point which meets the condition of $\Delta Qi = MAX$, the edge point separate *Q* into two parts, the radial wherein the point lies is the left adjacent of the restrained edge. Then connecting this point with the two ends of restrained edge can form a triangle. Similarly in *S* to find an edge point that satisfies the condition of $\Delta Qi = MIN$ and separate *S* into two parts, the radial wherein the point lies is the right adjacent of the restrained edge. And then connecting the edge point with the two ends of restrained edge can form a triangle. The region affected by restrained edge is divided into two triangles that have a restrained edge as common line and four independent sub-zones. In Figure 1, $p_0 p_5$ is the restrained edge. Two triangles, $\Delta p_0 p_4 p_5$ and $\Delta p_5 p_6 p_0$, and four independent sub-zone, (p_0, p_1, Λ, p_4) , (p_4, p_5) , (p_5, p_6) and (p_6, p_7 , p_8, p_0), can be obtained after restrained edge mosaic process. Figure 1(b) shows the results.

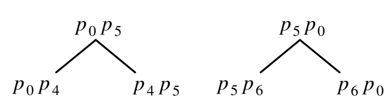
For example, the whole division process shows in figure 2 (a), (b).

If the affected region is composed of two triangles, implementing diagonal processing of convex quadrangle exchange directly can do restrained edge mosaic.



(a) (b)

Figure 1 Affecting area of restrained edge



(a) (b)

Figure 2 Spatial division trees

4.3 Discussion

Compared with the method that calculates the triangular area (Lu and Wu, 1997) or the point of intersection of radials (Wang, 2001), applying Formula (3), (4) to confirm affected region of restrained edge can improve time efficiency. The process can also be implemented by judging $\vec{A}\times\vec{B}$, which is the relationship between point and directed line. Utilizing $\vec{A}\times\vec{B}$ can set up the binary tree, see figure 3. The ordered radial spatial division sequence Q, (p_0p_2 , p_0p_3 , p_0p_1 , p_0p_4 , p_0p_5 , p_0p_6 , p_0p_8 , p_0p_7), can be established by inorder traversing of that binary tree (see Figure 3). The left adjacent edge of p_0p_5 is p_0p_4 and its right adjacent edge is p_0p_6 . The time efficiency of $\vec{A}\times\vec{B}$ is almost the same order with that of Formula (3), (4), but the time complexity of this method will increase nonlinearly with the increase of the edge points in affected region, while using the value of ΔQ_i the time complexity increase linearly (Qi, Li and Zhu, 2003). Consequently method proposed in this paper with lower time complexity than that of judging $\vec{A}\times\vec{B}$ method.

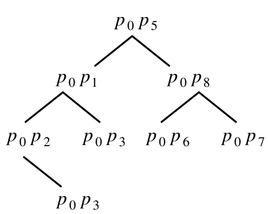


Figure 3 Binary sort trees

5. SPATIAL DIVISION OF INDEPENDENT SUB-ZONE

5.1 Spatial division method of independent sub-zone

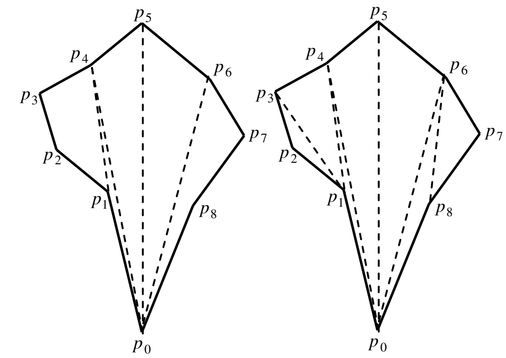
Figure 1(b), radial spatial division based on $Qi(x_i, y_i)$ are operated in independent sub-zone (p_0, p_1, Λ, p_4) with p_0p_4 as expanded edge, which is formed from the first point to the last point in the sub-zone to search the left adjacent radial of p_0p_4 . Edge point p_1 meets the condition. If there are more than one edge points in that radial, the edge point adjacent to the base point meets the condition. Then that point can be connected with expanded edge p_0p_4 to construct triangle, see figure 4(a), and $\Delta p_0p_1p_4$ including expanded edge p_0p_4 separates (p_0, p_1, Λ, p_4) into $\Delta p_0p_1p_4$ and independent sub-zone (p_0, p_1) and (p_1, p_2, p_3, p_4).

Next (p_0, p_1) and (p_1, p_2, p_3, p_4) , the two independent sub-zones, are divided with p_0p_1 and p_1p_4 as expanded edge repeatedly according the process mentioned above till there is only two edge points in the sub-zone.

Similarly the other independent sub-zones can adopt the same radial division method as in the (p_0, p_1, Λ, p_4).

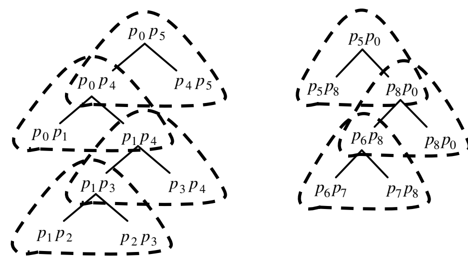
In practice, division operation have to be done only when the number of edge points in the sub-zone more than 3 otherwise the edge points can be connected to be a triangle directly.

For radial division process in region affected by restrained edge, see spatial division trees in Figure 5(a), (b). Figure 4(b) show the results.



(a) (b)

Figure 4 Spatial division of independent area



(a) (b)

Figure 5 Spatial division trees

5.2 Justification that triangle including expanded edge must be in an independent sub-zone

It is provability that the triangles formed by the spatial division based on $Qi(x_i, y_i)$ and restrained edge mosaic must be in each independent sub-zone. In the independent sub-zone (p_0, p_1, Λ, p_4), with base point p_0 and expanded edge p_0p_4 search a triangle including expanded edge by radial spatial division theory based on $Qi(x_i, y_i)$ function, for example.

Supposed that $\Delta p_0p_1p_4$ does not in the independent sub-zone (p_0, p_1, Λ, p_4) , at least $\forall p_i \in (p_2, p_3)$ belong to $\Delta p_0p_1p_4$, because $\Delta p_0p_1p_4$ have included expanded edge, which means that at least there are one point in (p_2, p_3) lie in the right of radial p_0p_1 .

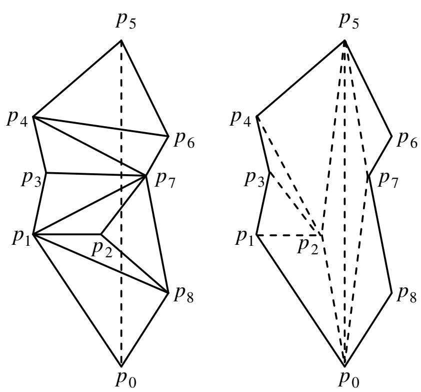
But p_0p_1 is the left adjacent radial line of p_0p_5 through radial division. According to the important property of the radial spatial division (see section 3.1 in this paper), there is not any other edge point exist among p_0p_1 and p_0p_5 .

So the hypothesis above does not tenable. And $\Delta p_0p_1p_4$ being in (p_0, p_1, Λ, p_4) can be justified.

By the same token other processes of sub-zone division and restrained edge mosaic can be authenticated.

5.3 An exceptional case

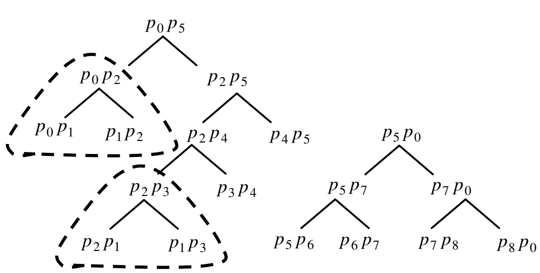
See figure 6(a), p_1 arises several times in left edge queue Q $(p_0, p_1, p_2, p_1, p_3, p_4, p_5)$ which obtained by the algorithm mentioned above (see 4.1 in this paper). Practically edge $p_1 p_2$、$p_2 p_1$ are one borderline, but logically they are two edges called 'logic edge'. Their left adjacent space is a line space without left adjacent triangle. So it is troubled for recording adjacent relationship of triangles after dividing because of no record of adjacent triangle in the record of edge adjacent relationship.



(a) (b)

Figure 6 Affecting area of restrained edge

The spatial trees showed in figure 7 (a), (b) are gained from figure 6(a) by spatial division according to the method proposed in this paper. The leaf nodes extend from left to right are the borderline of the adjacent sub-zone in the division tree, see figure 7(a), and the next leaf node of $p_1 p_2$ is $p_2 p_1$. So by searching the triangles of the two adjacent special leaf nodes (no record of adjacent triangle) are the two triangles adjacent with the special edge as the public edge. Figure 6(b) show the divided results.



(a) (b)

Figure 7 Spatial division trees

5.4 The property of spatial division tree

The properties of spatial division tree go as:

- Father node together with its two child-nodes construct a triangle. The triangle of root node includes restrained edge;
- All nodes except leaf node represent either the independent sub-zone which have those nodes as expanded edge, as well as the public edges of divided triangle in the affected region, in which the triangle of root node of tree (a) is adjacent with the triangle of root node of tree (b);
- Leaf nodes array, which formed by traversing on the leaf nodes of the tree (a) and tree (b) from left to right, is the clockwise sequence affecting the region border. Leaf nodes record the adjacent relationship with other triangles out of the affected region. When logic edge presents, there will be no any record of the adjacent relationship in the leaf node, so the two triangles of the two adjacent leaf nodes are of contiguity each other with their own corresponding physical edge as public edge.

According to the important property mentioned above, it is easy to get the triangles and their spatial adjacent relationship after region being divided in the spatial division tree.

6. TIME COMPLEXITY ANALYSIS OF ALGORITHM

Here gives the comparison of time complexity between the radial spatial division algorithm and the spatial division algorithm based on distance.

Formula (5) is the function used to calculate the distance from the point to the line.

$$d_i(x_i, y_i) = \left|Ax_i + By_i + C\right| \qquad (5)$$

where:

$$A = \frac{y_2 - y_1}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}}$$

$$B = -\frac{x_2 - x_1}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}}$$

$$C = \frac{x_2 y_2 - x_1 y_1}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}}$$

Formula (6) is the function used to calculate the Qi length (see Qi and Liu, 1996, Qi, 1997 and Qi, Li and Zhu, 2003 for details).

$$Qi(x_i, y_i) = \begin{cases} \Delta y_i/\Delta x_i & (\Delta x_i > 0) \wedge (\Delta y_i \geq 0) \wedge (\Delta x_i > \Delta y_i) \\ 2 - \Delta x_i/\Delta y_i & (\Delta x_i > 0) \wedge (\Delta y_i > 0) \wedge (\Delta x_i \leq \Delta y_i) \\ 2 - \Delta x_i/\Delta y_i & (\Delta x_i \leq 0) \wedge (\Delta y_i > 0) \wedge (\Delta y_i > -\Delta x_i) \\ 4 + \Delta y_i/\Delta x_i & (\Delta x_i < 0) \wedge (\Delta y_i > 0) \wedge (\Delta y_i \leq -\Delta x_i) \\ 4 + \Delta y_i/\Delta x_i & (\Delta x_i < 0) \wedge (\Delta y_i \leq 0) \wedge (\Delta x_i \leq \Delta y_i) \\ 6 - \Delta x_i/\Delta y_i & (\Delta x_i \leq 0) \wedge (\Delta y_i < 0) \wedge (\Delta x_i > \Delta y_i) \\ 6 - \Delta x_i/\Delta y_i & (\Delta x_i > 0) \wedge (\Delta y_i < 0) \wedge (\Delta x_i \leq -\Delta y_i) \\ 8 + \Delta y_i/\Delta x_i & (\Delta x_i > 0) \wedge (\Delta y_i < 0) \wedge (\Delta x_i > -\Delta y_i) \end{cases} \qquad (6)$$

Table 1 shows the comparison of the basic operation between $d_i(x_i, y_i)$ and $Qi(x_i, y_i)$.

Seen from Table 1, when the problem scale is n, if 'judgment' is taken as the same level as '+/-', and 'evolution operation' is taken as the same level as '×/÷', the frequency count of calculating distance $d_i(x_i, y_i)$ to implement '×/÷' is 10n, while the frequency count of calculating $Qi(x_i, y_i)$ to implement '×/÷' is n. Take '×/÷' as the basic operation to measure the difference of the progressive time complexity between the two algorithms. The result is that the progressive time complexity of calculating $Qi(x_i, y_i)$ is 10O(n) less than that of calculating distance $d_i(x_i, y_i)$.

Basic operation	Implementation times	
	$d_i(x_i, y_i)$	$Qi(x_i, y_i)$
Judgement	0	3
+/-	6	2
×/÷	9	1
$\sqrt{x}$	1	0
$\|x\|$	1	0

Table.1 Comparison of basic operation between $d_i(x_i, y_i)$ and $Qi(x_i, y_i)$

7. CONCLUSION

It is very necessary to improve the efficiency of the algorithm for mass data processing. This paper discusses the theory and method of restrained edge mosaic in constructed TIN and describes our algorithm, the radial spatial division algorithm, in detail. The following conclusions can be obtained.

(1) This paper proposes radial spatial division method and relative Formulates based on $Qi(x_i, y_i)$, and proves that this division method can assure that expanded triangle is in the independent sub-zone theoretically;

(2) Relative algorithm of restrained edge mosaic in constructed TIN with radial spatial division method based on $Qi(x_i, y_i)$ is proposed. The analysis shows that the frequency count of executing this algorithm's basic operation , '×/÷', is just one tenth of that of spatial division algorithm based on distance;

(3) The conception of spatial division tree is also proposed in this paper. It is shown that this tree will be great benefit for the reconstruction of triangles and their topology in region affected by restrained edge after spatial division.

REFERENCE

M. Renslow, 2001. White paper: technology assessment of remote sensing applications: LIDAR, U. S. Department of Transportation · National Consortia on Remote Sensing in Transportation. http://riker.unm.edu/DASH_new/pdf/White%20Papers/LIDAR.pdf (accessed 31 Dec. 2002)

Zhu Q. and Chen C., 1998. Quick generation of TIN and is dynamic updating, *Journal of Wuhan Technical University of Surveying and Mapping*, 23(1), pp. 204-207.

Lu Z., Wu C. and Lu X., 1994. A generalized algorithm of optimal triangulation of simple polygon, *ACAT ELECTRONICA SINCA*, Vol. 22，No. 1, pp. 86-89.

Lu C. and Wu C., 1997. Optimal Triangulation of Data Points Scattered in arbitrary Polygon With Characteristic Constrained, *J. CAD & CG*, Vol. 9, No. 4, pp. 302-308.

Li L. and Tan J., 1999. Multiple Diagonal Exchanging Algorithm for Inserting Constrained Boundary in Constrained Delaunay Triangulation. *CHINESE J. COMPUTERS*, Vol. 22, No. 10, pp. 1114-1118.

Wang J., 2001. *Principles for Spatial Information System*. Publishing House of Science, Beijing, pp. 246-248.

Zhou X. and Liu S., 1996. A robust algorithm for constrained Delaunay triangulation [J], *CHINESE J. COMPUTERS*, Vol. 19，No. 8, pp. 615-624.

L. D. Floriani and E. Puppo, 1988. Constrained Delaunay Triangulation for Multiresolution Surface Description. *9th Inter. Conf. On Pattern Recognition*, Rome, Nov., pp. 566-569.

Qi H. and Liu W., 1996. Qi algorithm for arc-arc topological relationship built on nodes. *ACTA GEODAETICA et CARTOGRAPHICA SINICA*, 25(3), pp. 233-235.

Qi H., 1997. Optimization and improvement for the algorithm to automatically establish polygon topological relationship. *ACTA GEODAETICA et CARTOGRAPHICA SINICA*, 26(3), pp. 254-260.

Qi H., Li D. and Zhu Q., 2003. Time complexity analysis for two non-angle algorithms to determine the radial spatial adjacent relationship, Advances in Spatial Analysis and Decision Making (Zhilin Li, Qiming Zhou and Wolfgang Kainz, Editors), A.A. Balkema Publishers, Lisse, pp. 45-52.

Guo R., 2001. *Spatial Analysis*, Higher Education Publishing House, Beijing.