# ANALYSIS OF GEOGRAPHICAL DATA AND VISUALISATION OF THEIR QUALITY

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**Abstract:**

Geographical information such as elevations or immission, is most often represented either as vectors, e.g. isolines, or as raster, e.g. density slicing. Here, the precision of the representation is dealt with. Special attention is paid to the accuracy of secondary forms of information such as terrain slopes as derived from DEMs. For a mathematical analysis, synthetic surfaces are employed. As a conclusion, new ways are shown to visualize the quality of geographical data.

**Key words:** Accuracy, Data Quality, DEM, Theory, Visualisation

## 1. PRELIMINARY REMARKS

This paper is dealing only with geographical data such to be represented by some function of land coordinates. In other words, the value of the function, Z, is a function of X and Y, i.e. Z(XY). The mathematical form of function Z(XY) is not regarded here. The function must be continuous. The first derivatives $Z'_x(XY)$ and $Z'_y(XY)$ may, however, show discontinuity. This means that the surface represented by Z(XY) contains break lines.

Function Z(XY) may describe immissions of some polluting substance, terrain elevations, etc. Without limiting generality, let us consider the special case of function Z(XY) describing the terrain surface.

In our time, geographical data are stored in geographical information systems (GIS). Until recently, little attention has been paid to the quality of data and of derived products. A first step in the right direction is to store, alongside with the function values, their accuracy - i.e. introducing into the GIS standard deviations $\sigma_z$.

Many GISs possess the capability to derive isolines of the function Z(XY), i.e. Z(XY) = const. Isolines are then intersected with other data such as cadastral boundaries. Sometimes, isolines of different functions are intersected with each other. In these cases, accuracy characteristics in form of $\sigma_z$ are of no use; what is needed is rather the accuracy of isolines in the XY-plane, i.e. $\sigma_{zp}$[1]. This accuracy can be obtained as:

$$\sigma_{zp} = \sigma_z / \tan\alpha = \sigma_z * \Delta Z_p / \Delta Z \qquad (1)$$

| | | |
|---|---|---|
| $\tan\alpha$ | ... | Maximum terrain slope at the point in question |
| $\Delta Z$ | ... | Isoline interval (usually constant over the entire area of interest) |
| $\Delta Z_p$ | ... | Distance to the neighbouring isolines in the XY-plane ($\Delta Z_p$ varies along the isoline). |

We have to give careful consideration to the distance $\Delta Z_p$ of neighbouring isolines. It may become very large. As a consequence, the standard deviation $\sigma_{zp}$ of isolines may become very large, as well. And, therefore, processes of intersection with isolines may yield bad or even unacceptable results. It has to be emphasized at this point that the above conclusion is valid for applying isoline information, i.e. Z(XY) = const, in any form, both vector or raster (a way similar to density slicing).

The above problem will be examined here from various points of view. We start in chapter 2 with considering the behaviour of distances between neighbouring isolines of analytical surfaces. In this, dealing with derived surfaces is of special interest. In chapter 3, function Z(XY) and its derivatives become stochastic variables; this means taking into account accidental errors $\sigma_z$, and creating in GIS accuracy models in parallel to the corresponding functional ones. Visualising the quality of geographical data is prevalent in chapter 3.

[1]Applying such accuracy characteristics it becomes possible to derive the accuracy of areas as deduced, using formulae in (Prisley et al., 1989).

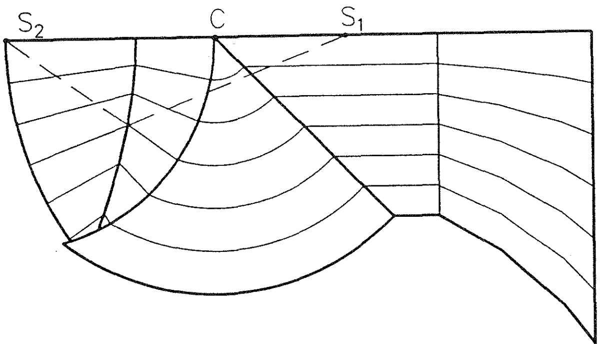


Figure 1: A set of analytic surfaces with the corresponding isolines.

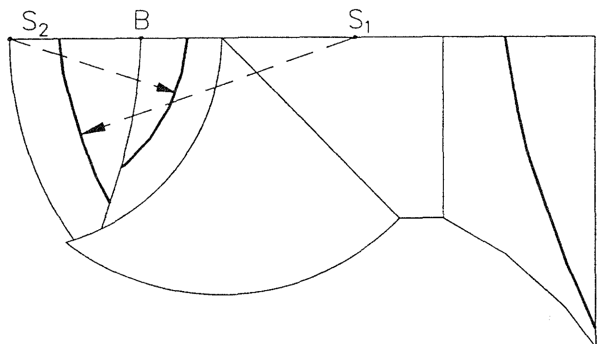


Figure 2: Lines of equal slope on the surfaces as defined in Figure 1.

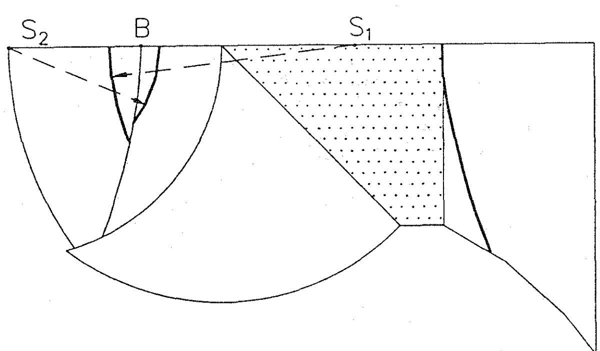


Figure 3: Lines and area (plane) of equal slope on the surfaces as defined in Figure 1.

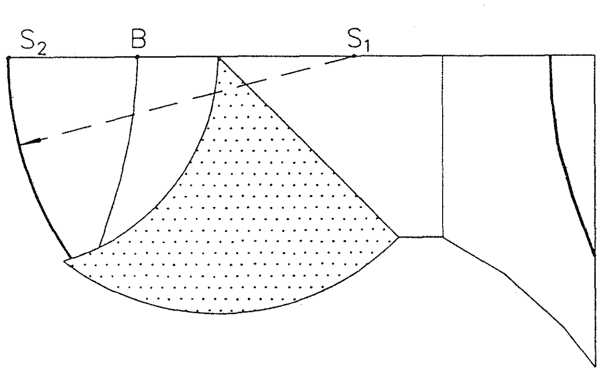


Figure 4: Lines and area (cone of revolution) of equal slope on the surfaces as defined in Figure 1.

## 2. ISOLINES OF ANALYTIC SURFACES

Figure 1 shows a set of analytic surfaces with isolines. To the left there are two helicoidal surfaces; straight isolines of the first surface intersect in point $S_1$, and those of the second surface in point $S_2$. In the middle of the figure there is shown a cone of revolution with center point C of the isolines. In the oblique plane intersecting the cone from the right, isolines are parallel equidistant straight lines. The surface still further to the right is a hyperbolic paraboloid; its isolines are hyperbolas.

The distance $\Delta Z_p$ between neighbouring isolines is inversely proportional to the (terrain) slope tan $\alpha$. In a horizontal terrain the distance $\Delta Z_p$ of neighbouring contour lines becomes infinitely large. A horizontal area of the terrain with an elevation unequal to that of any contour line is irrelevant to constructing such lines. In the opposit case, however, of a horizontal area with an elevation equal to that of some contour line, the corresponding line widenes to cover the entire area, i.e. as a line it is undefined. In other words, in case of surfaces incorporating horizontal areas, one has to deal with both lines and areas of equal (terrain) elevation.

Conclusions become more interesting if considering lines and, being more cautious now, areas of equal (terrain) slope. Figure 2 shows lines of equal slope. The slope value for the isoline is chosen to be larger than that of the oblique plane, and smaller than that of the cone of revolution. We obtain three line sections. The one in the hyperbolic paraboloid is part of an ellipse [2]. Line sections in both helicoidal surfaces on the left of the figure are circles with center points $S_1$ and $S_2$. At break line B there is a displacement. This phenomenon can be generalised as: non-symmetry of contour lines on both sides of a break line shows up as displacement of lines of equal (terrain) slope at the break line.

In figure 3, isolines of (terrain) slope are shown being equal to the slope of the oblique plane. In figure 4, isolines of the (terrain) slope are shown equal to the slope of the cone of revolution. Figures 3 and 4 facilitate the following generalisations:

- In surfaces of constant maximum slope (planes, cones of revolution, ...) the distance $\Delta Z_p$ between neighbouring lines of equal (terrain) slope becomes infinitely large.
- Should a surface of constant maximum slope have a slope equal to that represented by some isoline of slope, then the line of equal slope is undefined within the area of this surface, and it widenes to an area of equal slope. Consequently, in surfaces incorporating areas with surfaces of maximum slope, one has to deal with lines and areas of equal (terrain) slope.

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[2] I am grateful for this information to Prof. Dr. Paukowitsch, of the Institute of Geometry at The Vienna University of Technology. He called my attention to the work of Burmester (1871) on "isophots" in analytic surfaces. Isophots correspond to lines of equal slope for the case of vertical lighting. In his publication, Burmester has shown that isophots of hyperbolic paraboloids are ellipses.

## 3. ISOLINES AFFECTED BY ACCIDENTAL ERRORS

Knowing the behaviour of isolines in analytic surfaces helps greately the interpretation of their behaviour in natural surfaces. Isolines in natural surfaces are affected by accidental errors originating, on the one hand, in the roughness of the terrain, and on the other, in errors of data acquisition. Next, contour lines are going to be analyzed, followed by an analysis of lines of equal slope.

### 3.1 Contour Lines Affected by Accidental Errors

In figure 5, contour lines with an interval of 10 m are shown, as derived from a digital elevation model (DEM). This area does not contain any horizontal parts, and therefore the derivation of contour lines is without problems.

There are numerous ways to express the height accuracy $\sigma_z$ of such contours (e.g. Li, 1982, Tempfli, 1980). As we are in this paper primarily interested not in the height accuracy of the contours but rather in their accuracy in the XY plane, we are going to apply a rather simple formula to express height accuracy. This simple formula has proven to be quite adequate for describing the accuracy of contour lines derived from a DEM (e.g. Ackermann, 1978, Kraus, 1987):

$$\sigma_z = a + b * Z' \qquad (2)$$

a ... 0.15 thousandth of the flying height
b ... 150 $\mu$m at image scale
$Z' = \tan \alpha$ ... maximum terrain slope

Considering formulea (1) and (2), the accuracy of contour lines in the XY plane becomes:

$$\sigma_{zp} = a / Z' + b \qquad (3)$$

For contour lines in figure 5, estimates based on the flying height and the focal length of the camera yield:

$$\sigma_{zp}[m] = (0.4 / Z') + 3 \qquad (4)$$

The accuracy in the XY plane, $\sigma_{zp}$, becomes in areas with terrain slope of 100% 3.4 m, and in areas with 20% 5.0 m. For the entire area of interest, $\sigma_{zp}$ is visualised as pixel graphics in figure 6. The error in the XY plane is in inverse ratio to the terrain slope. The values of the terrain slope Z' for each pixel have been inquired from the digital slope model (DSM, to be treated below); the values Z' represent the inverse value of the distance $\Delta Z_p$ between neighbouring contour lines.

### 3.2 Lines of Equal Slope Affected by Accidental Errors

In a DEM it is possible to derive the components of the normal vector to the terrain surface in each point of the raster, and at every intersection of a break line with the raster. The component of the normal vector along the maximum slope is used as the function value of the DSM. From the DSM lines of equal slope can be derived. Figure 7 shows the lines of equal terrain slope with an interval of 10% and, by thick lines, the break lines of the DEM. A comparision with the contour lines (figure 5) enables us to note:

- At break lines there occur displacements of varying size in the lines of equal terrain slope.
- The distance $\Delta S_p$ of neighbouring lines of equal terrain slope becomes very large in areas with regular terrain slope.

The area of the terrain with elevations below some 1450 m shows a fairly regular slope of 70%. In this area the slope lines are hardly defined. The position of the isolines of slope in this area is more or less arbitrary[3]. Eyes trained by contents of chapter 2 can easily detect a high degree of uncertainty in these lines. By the way, a raster image with the threshold 70% would yield in this area a so-called "salt-and-pepper" pattern.

The uncertainty in lines of equal terrain slope can be well visualised applying slope zones. Figure 8 is such an image. It represents the slope zones 19-21%, 29-31%, 39-41%, etc. Overlaying these zones and the vector graphics (figure 7) is of great advantage to GIS users. This accuracy overlay can be made even more impressive, as shown in figure 9. In it, the level of probability of the position of the lines in the XY plane is represented in corespondence with the Gaussian distribution.

There are few theoretical investigations on the accuracy of terrain slope values as derived from DEMs. The following is based upon formula (2):

$$\sigma_s = c + d * Z'' \qquad (5)$$

c ... 1.5% as derived from empirical studies (Kraus, 1991)
d ... To derive this value, there is an empirical study in process (for purposes of this paper, this parameter is of little importance).
Z" ... Maximum slope of the surface as defined by the lines of equal slope.

Considering formula (1) the accuracy of lines of equal slope in the XY plane can be expressed as:

$$\sigma_{sp} = c / Z'' + d \qquad (6)$$

This value for the lines of equal terrain slope in figure 7 can be estimated as:

$$\sigma_{sp}[m] = 1.5 / Z'' \qquad (7)$$

E.g. when $Z'' = 0.1[m^{-1}]$, the distance $\Delta S_p$ between neighbouring lines of equal terrain slope is 100 m and their accuracy in the XY plane $\sigma_{sp}$ is 1.5 / 0.1 = 15 m. For the entire area of interest, figure 10 contains a visualisation of $\sigma_{sp}$ as pixel graphics. In areas with very little curvature of the surface the corresponding errors exceed even the accuracy limit of 15 m represented on the graphics in black.

## 4. CONCLUSION

GISs have to be extended so to contain, in addition to models representing geographical data, corresponding models to describe their accuracy. Inquiries into the information system should yield both the value of the function and its accuracy. Accessing multiple models is characteristic for deriving complex results. Much work has yet to be done to enable the simultaneous derivation of the corresponding accuracies for such derived products. GIS users have to be informed about the accuracy of all direct and derived products of the system. Information on the accuracy model has to be given attractive visualisation. This way the misuse of geographical data yielded by GIS can be considerably reduced.

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[3]In the paper (Killian, Kraus, 1992) this topic will be treated in detail.

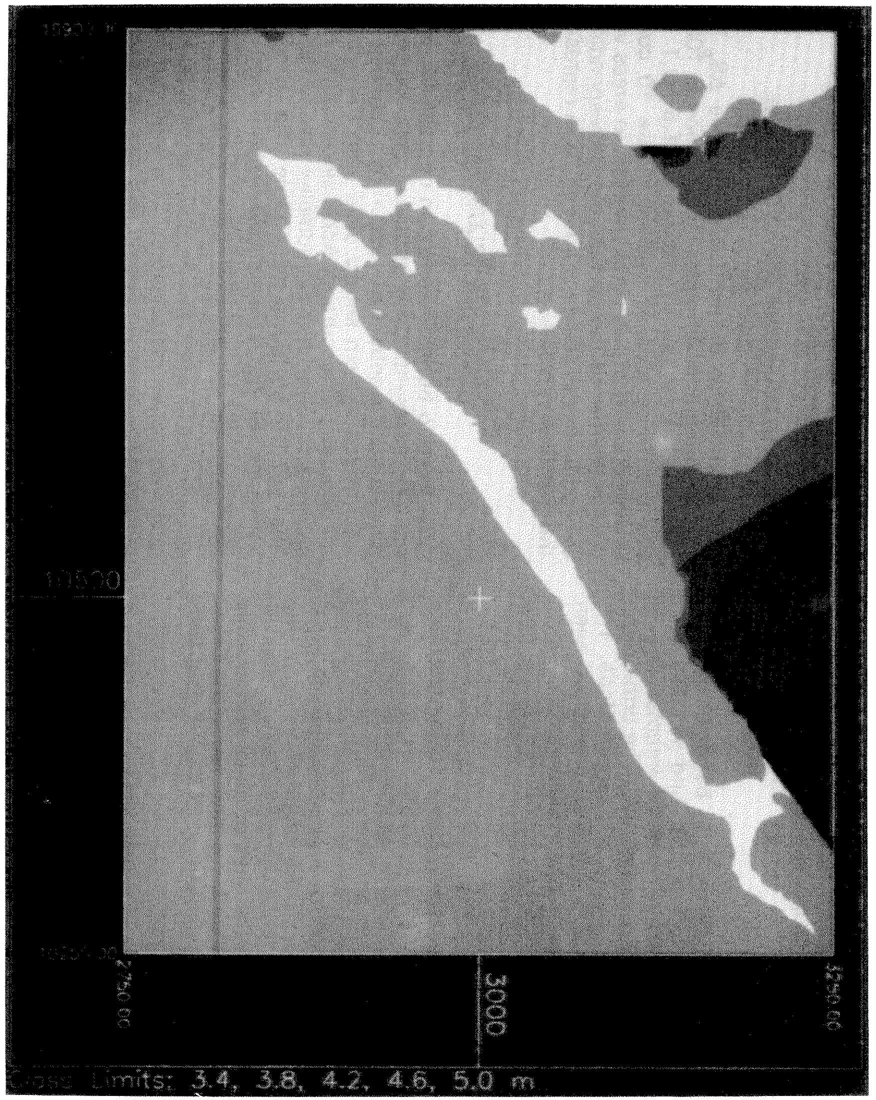


Figure 6: Accuracy of contour lines in the XY plane.

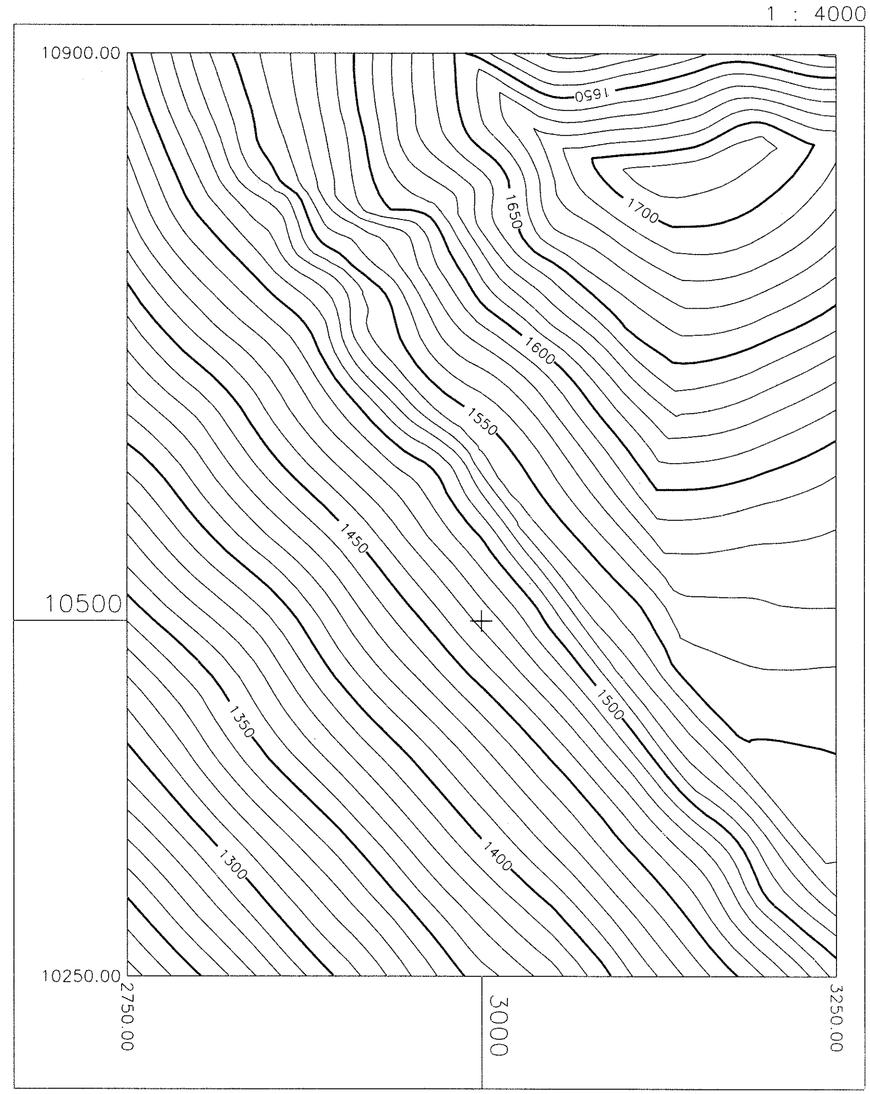


Figure 5: Contour lines with interval 10 m.

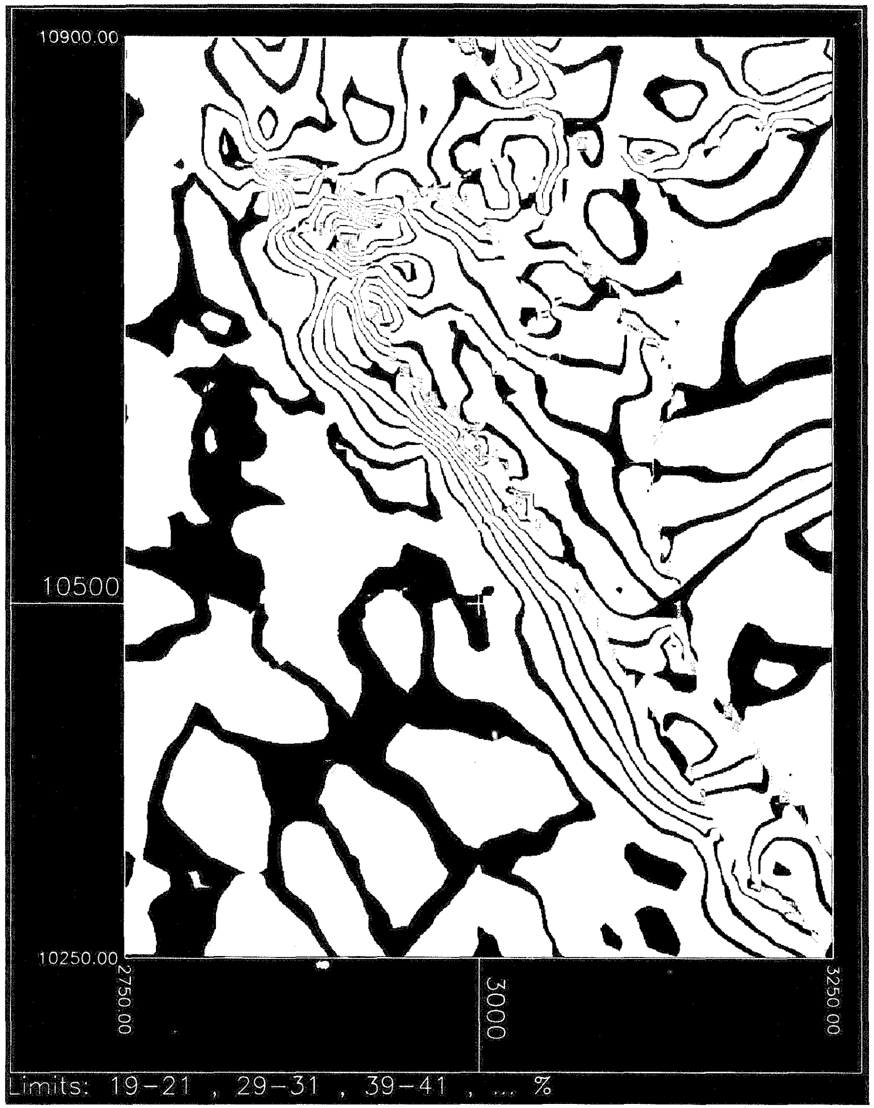


Figure 8: Zones of equal slope.

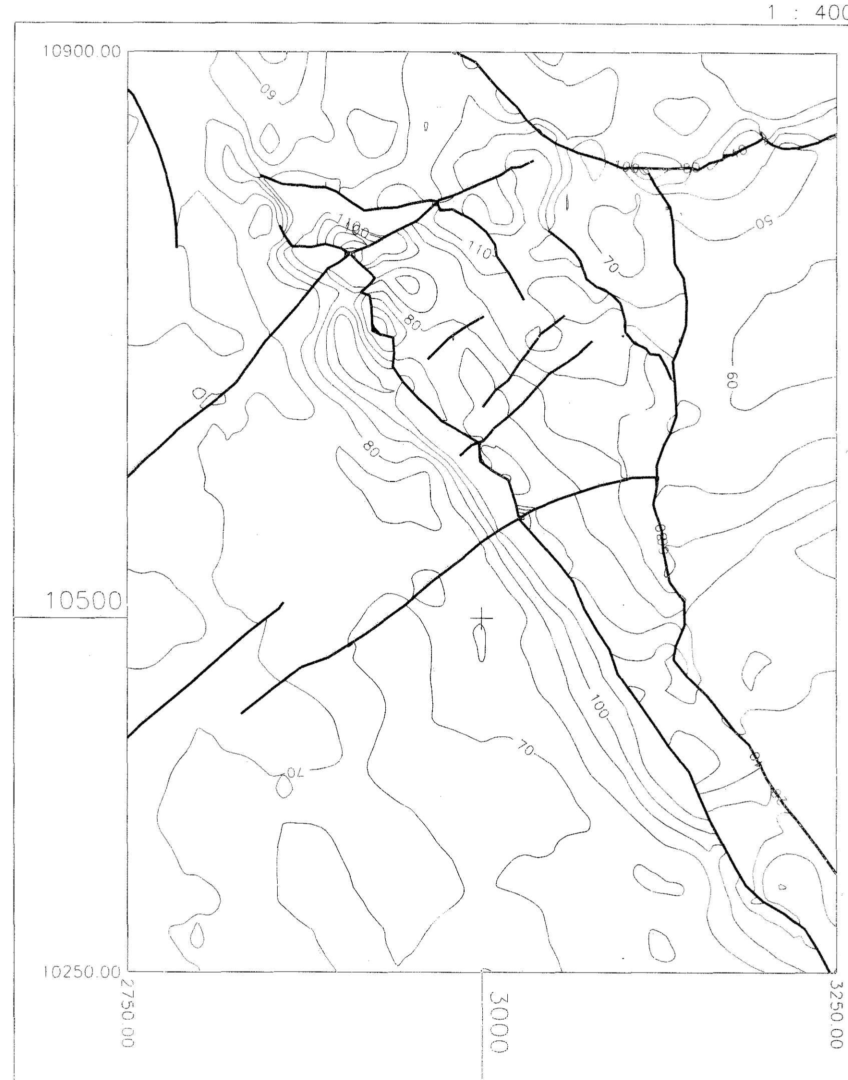


Figure 7: Lines of equal terrain slope, interval 10 %


Figure 10: Accuracy in the XY plane of lines of equal terrain slope.

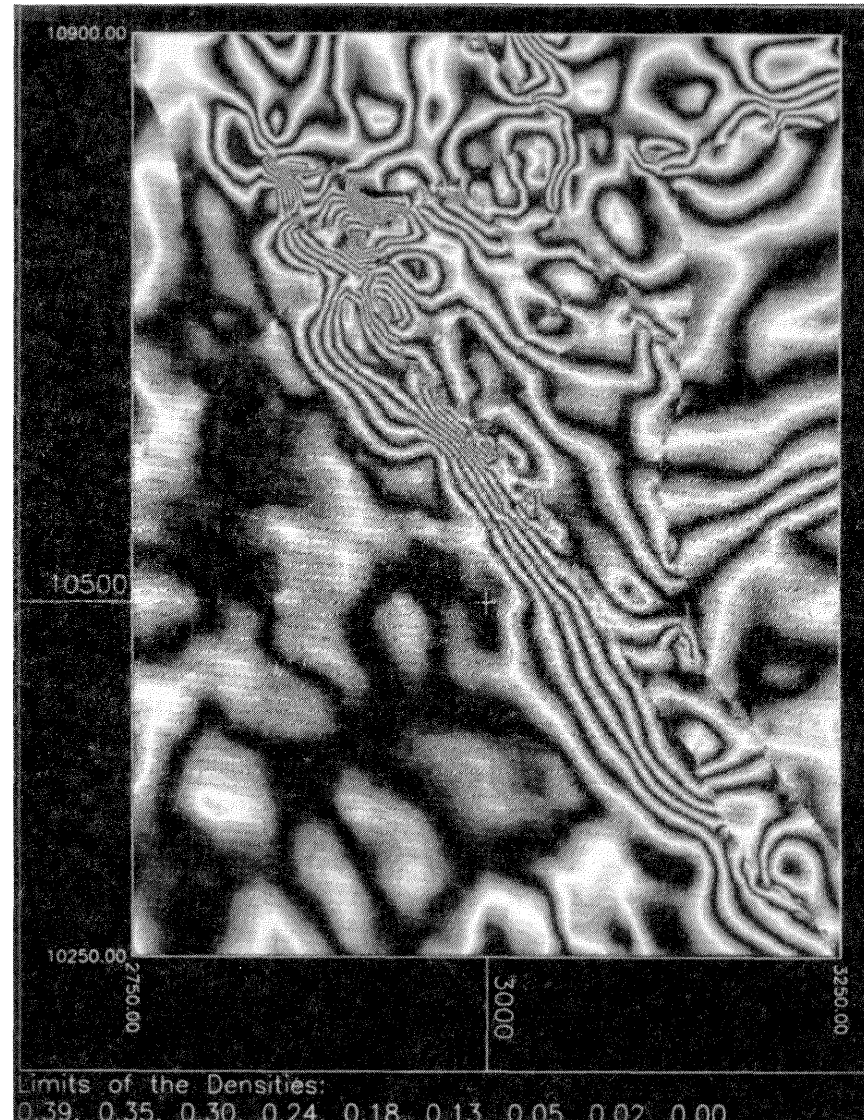


Figure 9: Gaussian distribution for lines of equal slope.

**Remarks:**

Examples for this paper have been created by Dr. K. Pyka, Institute of Geodesy, Technical University Cracov, Poland, in the time of his visiting research at the Institute of Photogrammetry and Remote Sensing of The Vienna University of Technology. He compiled the examples using SCOP - a program package for DEMs. With its new modeles SCOP.INTERSECT (Sigle, 1991) and SCOP.PIXEL (Ecker, 1991), SCOP meets the requirements as mentioned in chapter 4 to a considerable extent. Development is started to yield a new edition of SCOP (Molnar, 1992); it is to facilitate such capabilities, and to enable easy integration with other (host) software systems.

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