IMAGE ANALYSIS BASED ON MATHEMATICAL MORPHOLOY*

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ABSTRACT

Basic information of objects (regions) in digital image is obtained by image segmentation. More precise information about the object (regions) is extracted based on image analysis, includeing edge extracting , thinning , configuration fitting and shape decompositing, which are mainly based on mathematical morphology. The primitives of structural features can be produced by means of the methods. At last, the polygons and the primitives of structure features can be acquired for further image matching or understanding.

key words: Image segmentation, Mathematical morphology, Edge extraction, Thinning, Region decomposition.

1.INTRODUCTION

The reliability of image matching and the image interpretation are the problems that many photogrammetrists and informatics scientists are studying. Solving these problems should be based on image processing in higher level than in grey level. Vision is a complex procedure of imformation processing. The task of elementary vision is constructing the proper description of local geometric structure on the image from the variation of grey levels. To this end, the primitives of objects ought to be organized in different level, in order to acquire the structure features and carry out the structure matching and shape recognition.

An important sort of the primitive employed in structure matching and shape recognition is based on the surfaces of objecs. Its acquisition can be by two ways. One is from the edges. The other is from the regions. In this paper, the information of edges and regions are obtained by image segmentation. Then, more precise information about the objects (regions) is extracted based on image analyses, including edge thinning, configuration fitting and region decomposition with method of mathematical morpholog .

2.IMAGE SEGMENTATION

The purpose of segmentation is to partition the image space into meaningful regions with certain consistency of grey level, texture, color, gradient or other properties. For given image Image

$$Image = \{X=f(i,j) \mid i=0,1,2,...,M-1, j=0,1,2,...,N-1\}$$

and consistency measure P(), the segmentation of Image is a decomposition (X1, X2, ..., Xn) of Image satified

(1) $X_i \; != \phi$, where "!=" means "Be not equal"

(2) $X_i \cap X_j = \phi, \; i \;!= j$

(3) X_i is connected

(4) $P(X_i)$ = True and $P(X_i U X_j U...)$ = False

In this section the thresholding clustering and separation-merger algorithm are introduced.

2.1 preprocessing

For eliminating the noise degradation, the image smoothing is performed, and image enhancement is also completed in order to sharping the edges.

2.2 Thresholding algorithm of image segmentation

2.2.1 Method of searching valley. The threshold is obtained by simply searching the valley along the distribution curve in the histogram, which is smoothed with 3-order spline or moving average.

2.2.2 Polynomial threshold. The intensity on a image is not even sometimes. In this case, only one threshold on entire image is not suitable. The threshold should be the function of the position. For the simplest example, it is a 1-order polynomial.

$$T(x,y) = ax + by + c$$

The coefficients a,b,c, can be computed by surface fitting with least squares method.

2.3 Clustering algorithm

Image segmentation is a classification of pixels. There are n measures at each pixel instead of one, the grey level, in thresholding algorithm. Common measures are

(1) Average $x = \Sigma xi/m$
(2) Mean square deviation $sqrt(\Sigma(xi-x)^2/m)$
(3) Contrast $|max\{xi\}-min\{xi\}|$

The clustering algorithms of n-dimensional feature space include k-average, ISODATA based on k-average and ASP based on ISODATA.

2.4 Region growing

Region growing starts at the known pixel or a group of pixels and appends all neighboring pixels untill the measure of consistency is false. A typical example of region growing is separation-merger algorithm.

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2.4.1 Measures

(1) Mean grey level

$$Max|xi-x|<T0$$

where T0 is a threshold.

(2) Texture

$$sqrt(\Sigma(xi-x)^2/m)<T1, \text{ or}$$
$$H = -\Sigma p_{ij} ln p_{ij} < T2$$
$$G = \Sigma(i-j)^2 p_{ij} <T3$$

where H is the entropy, G is the contrast and p is the properbility.

2.4.2 Data structure----doubletree. Doubletree, which node consists of the regions from dividing the image equally and alternately in x-direction and y-direction, is simpler than quadtree.

The encoding criteria are

left : 0
right : 1
up : 0
down : 1

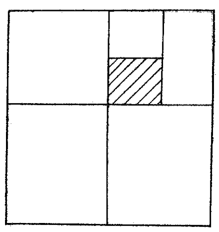

Fig. 0

The code of the region on Fig.0 is 1001

2.4.3. Separation-merger algorithm.

(1) separation. For each node, if the measure of the consistency is false it is divided into left and right (or up and down) parts, untill all leaves represnt a consistent region.

(2) merger. For each region, if the consistent measure of the region and its neighbor region is true, then the neighbor region is merged into it.

2.4.4 Labling algorithm of neighbor regions. The connectivity of a region is considered in the separation-merger algorithm. So, it can be a labling algorithm of neighbor regions. In this case, the consistent measure is true, if all pixels in the region are 1, and the region, in which no pixel is 1, do not store in the tree.

As a special exmaple, the unconnected curves can be separated by separation-merger algorithm, so that the line following will be simple.

The result of image segmentation is a binary or multiple value image. It can be used in image analysis.

3. IMAGE ANALYSIS BASED ON MATHEMATICAL MORPHOLOGY

The human vision is concerned in not only the images or objects, but also human thought, knowledge and new perception.

On the basis of this idea, the structuring elements with different size and shape can more easily be designed to adapt to our task, while the mathematical morphology is used in image analysis. The morphological filtering with the structuring elements is applied in the extraction of the useful imformation and the restraint of the uninterested imformation.

3.1. Background of mathematical morphology[Matheron 1975], [Serra, 1982]

The operations of mathematical morphology can be divided into set operations and function operations. A binary image is a set in which the objects are its subsets. A grey-level image is a function on a set.

If X is a binary image on a plane, it is equivalent to a binary function f(x,y), where $(x,y)\in X$ and $x,y\in R$, $\in$ means belong to.

Let A ,$K\in 2^{R*R}$. K called structuring element is a limited set. $z=(x0,y0)\in R^2$.

Difinition 1: the Translate of f(x,y) or A by z is defined as

$$Trans(f,z) = f(x+x0,y+y0)=fz$$
$$Trans(A,z) = \{a+z: a\in A\} =Az$$

Difinition 2: the Reflection of K is defined by

$$\check{K} = \{-k: k\in K \}$$

Difinition 3: the Dilation of A and f by K is

$$A\oplus K = \{z| \check{K}z \ \ A != \phi \}$$
$$f\oplus k(x)= \inf_{z\in K}\{f(x+z) - k(z)\}$$

Difinition 4: the Erosion

$$A\ominus K = \{z| Kz\subseteq A \}$$
$$f\ominus k(x) = \sup_{\substack{z\in k \\ x-z\in F}}\{f(x-z) + k(z)\}$$

Difinition 5: Opening

$$AoK = (A\ominus K)\oplus K = \bigcup_{Ky\in A} Ky$$

Difinition 6: Closing

$$A\cdot K = (A\oplus K)\ominus K = \bigcap_{\{y|\check{K}y\cap A \,!= \phi \}} \check{K}y^c$$

where $K^c=\{x|x\in 2^{R*R}, x \notin K \}$

Difinition 7: Let X be image and T=(T1,T2), where $T1,T2\in 2^{R*R}$ are structuring elements.

$$Hitmiss(X,T) = (X\ominus T1) / (X\oplus \check{T}2) = X\otimes T$$

where / is the subtract of sets.

$$X\otimes T = (X\ominus T1)\cap(X^c\ominus T2).$$

3.2 Analysis of Edge

3.2.1. Edge. The edge extraction with mathematical morphology is simple for the binary image. The method 1 is that the dilation of an object f subtracts the object itself. The other method is that the object substracts its erosion.

$$dg(f)=(f\oplus B)-f$$

or

$$dg(f)=f-(f\ominus B)$$

where B is the structuring element.

For the grey level f, there is the summability. Suppose

$$fg(m,n)=\begin{cases}1, & f(m,n)>=g,\\ 0, & f(m,n)<g\end{cases}$$

where grey level g=0,1,2,...,N (usually N=225). Thus,

$$f(m,n)=\sum_{g=1}^{N} fg(m,n) = \max\{g\,|\,fg(m,n)=1\}$$

and,

$$eg(f)=eg\Big(\sum_{g=1}^{N} fg(m,n)\Big)=\sum_{g=1}^{N} eg(fg(m,n))$$

It is interest that the region can be obtained by the inverse procedure. That is the region filling can be completed with mathematical morphology. If X is edges, and P is a point of a region R. Repeat

$$Sn=(p+nB)\cap X^{c}$$

until the result is the same as previous one.

3.2.2 Thinning. The edge extracted by above processing is not one pixel thinkness, even though it is skeletized progressively. How can the connected edge with one pixel thinkness be captured? the feasible way is that the pixels in out layer are removed gradually on the condition of connectedness, until no pixel can be removed.

The thinning operator is difined by Hitmiss operation as XOT=X/X⊗T where X is the edge, and T is the structure element. For structure element sequence D={D1, D2, D3, D4 }, the m thinning is

$$\{XOD\}_m=(...((((XOD1)OD2)OD3)OD4)...)$$
for m times

Algorithm 1:

(1). $X'=\{XOD\}_m$ where D={D1,D2,D3,D4},

$$D1=\begin{array}{ccc}0&0&\cdot\\0&1&1\\\cdot&1&\cdot\end{array}\quad D2=\begin{array}{ccc}\cdot&0&0\\1&1&0\\\cdot&1&\cdot\end{array}\quad D3=\begin{array}{ccc}\cdot&0&0\\1&1&0\\\cdot&0&0\end{array}\quad D4=\begin{array}{ccc}\cdot&1&\cdot\\0&1&1\\0&0&\cdot\end{array}$$

(2). $X''=\{X'OE\}_n$, where E={E1,E2,E3,E4}

$$E1=\begin{array}{ccc}\cdot&0&\cdot\\1&1&1\\\cdot&1&\cdot\end{array}\quad E2=\begin{array}{ccc}\cdot&1&\cdot\\1&1&0\\\cdot&1&\cdot\end{array}\quad E3=\begin{array}{ccc}\cdot&1&\cdot\\1&1&1\\\cdot&0&\cdot\end{array}\quad E4=\begin{array}{ccc}\cdot&1&\cdot\\0&1&1\\\cdot&1&\cdot\end{array}$$

(3). X=X" and repeat.

If the orders of structure elements are different, the results are different also. The improved algorithm is

Algorithm 2:

$$X=\{(XODi)\ (XODi+1)\ (XOEi)\}^{m}$$

where

i=1,2,3,4 and i=i(mod4) when i>4.

The result with the constant length of the branch includes some noisy branch which should be cut off. The corrected method is

Algorithm 3:

(1) Before (or after) each iteration

$$X=XOGi,\ i=1,2,...,8$$

where

$$G1=\begin{array}{ccc}\cdot&1&\cdot\\0&1&0\\0&0&0\end{array}\quad G2=\begin{array}{ccc}0&0&1\\0&1&0\\0&0&0\end{array}\quad G3=\begin{array}{ccc}0&0&\cdot\\0&1&1\\0&0&\cdot\end{array}\quad G4=\begin{array}{ccc}0&0&0\\0&1&0\\0&0&1\end{array}$$

$$G5=\begin{array}{ccc}0&0&0\\0&1&0\\\cdot&1&\cdot\end{array}\quad G6=\begin{array}{ccc}0&0&0\\0&1&0\\1&0&0\end{array}\quad G7=\begin{array}{ccc}\cdot&0&0\\1&1&0\\\cdot&0&0\end{array}\quad G8=\begin{array}{ccc}1&0&0\\0&1&0\\0&0&0\end{array}$$

(2) $X1=(XOD)OE,\ X2=\{XOG\}^{2},\ X3=\bigcup_{i=1}^{8}(XOGi)$

$$X=X2\cup\{(X3\oplus 2M)\cap X1\}$$

The result can obviously be improved with the possibility of one pixel less in length.

3.2.3 Node detection

(1) End-point set

$$end(x)=\bigcup_{i=1}^{8}(X\otimes Gi)$$

(2) 3-intersection set

$$cross3(X)=\bigcup_{i=1}^{4}(X\otimes Ti)\ \cup\ \bigcup_{i=1}^{4}(x\otimes Fi)\ \cup\ \bigcup_{i=1}^{4}(X\otimes Bi)$$

where

$$T1=\begin{array}{ccc}\cdot&0&1\\0&1&0\\1&0&1\end{array}\quad T2=\begin{array}{ccc}1&0&1\\0&1&0\\1&0&\cdot\end{array}\quad T3=\begin{array}{ccc}1&0&1\\0&1&0\\\cdot&0&1\end{array}\quad T4=\begin{array}{ccc}1&0&\cdot\\0&1&0\\1&0&1\end{array}$$

$$F1=\begin{array}{ccc}1&\cdot&1\\0&1&0\\\cdot&1&\cdot\end{array}\quad F2=\begin{array}{ccc}\cdot&0&1\\1&1&\cdot\\\cdot&0&1\end{array}\quad F3=\begin{array}{ccc}\cdot&1&\cdot\\0&1&0\\1&\cdot&1\end{array}\quad F4=\begin{array}{ccc}1&0&\cdot\\\cdot&1&1\\1&0&\cdot\end{array}$$

$$B1=\begin{array}{ccc}1&0&\cdot\\0&1&1\\\cdot&1&\cdot\end{array}\quad B2=\begin{array}{ccc}\cdot&0&1\\1&1&0\\\cdot&1&\cdot\end{array}\quad B3=\begin{array}{ccc}\cdot&1&\cdot\\1&1&0\\\cdot&0&1\end{array}\quad B4=\begin{array}{ccc}\cdot&1&\cdot\\0&1&1\\1&0&\cdot\end{array}$$

(3) 4-intersection set

$$cross4(X)=(X\otimes M1)(X\otimes M2)$$

where

$$M1=\begin{array}{ccc}1&0&1\\0&1&0\\1&0&1\end{array}\quad M2=\begin{array}{ccc}0&1&0\\1&1&1\\0&1&0\end{array}$$

3.2.4. Straight line fitting of outline of polygon. For each curve between neighbor nodes, fitting a straight line is carried out with the correlation coefficient

$$r=lxy/\ sqrt(lxx\cdot lyy)$$

where

$$lxy=\Sigma(x_i-\bar{x})(y_i-\bar{y})$$
$$lxx=\Sigma(x_i-\bar{x})(x_i-\bar{x})$$
$$lyy=\Sigma(y_i-\bar{y})(y_i-\bar{y})$$

and the maximal distance dmax between edge points and the straight line. When r<rt or dmax>dt (rt and dt are thresholds),the curve will be divided into two parts according to the golden section, and the procedure will be repeated. After that, the polygons are determined.

3.3 Region decomposition

Region decomposition means: X is the set represented objects(regions). If a group of subsets X1, X2,...,Xn satisfies:

$$X=\bigcup_{i=1}^{n} X_i$$

then {X1,X2,...Xn} is a decomposition of X. Usually the decomposition should be

(1) concise

(2) invariant in shift, rotation and scale transformation

(3) represent ative of the object

(4) unique

3.3.1 Algorithm 1 Selecting sole structure element B with symmetry such as a squre, rhombus or disk, the processing

$$X_i= ((X-X'(i-1))\ominus n_iB) \oplus n_iB$$
$$X'(i)= \bigcup_{0<j<=i} X_j$$
$$X'0= \phi$$

where ni is the maximum size of niB included in X-X'(i-1) in step i, is repeated untill (X-X'(i))ΘB=ϕ Then X1,X2, ... is a decomposition of X. In this way, X is decomposed as more parts unconnected.

3.3.2 Algorithm 2. There ara several structure elements B1, B2, ... B_m. Supose

$$D_{n,i}=(X o nB)/[X o(n+1)B],\ 0<n<N_i$$

where Sn,i is the n-skeleton subset of X by Bi.

Step 1: Remove some Dn,i overlaped by other Dn,i

Step 2: In remained dn,i, find the point p satisfied

$$\bigcup_{i=1}^{M} \bigcup_{n=0}^{N_i} \bigcup_{p\in S_{n,i}} (p+nB_i)=X$$

according to Dn,i ⊆ Sn,i⊕nBi, and the number of p is the least. In this way, the computer load is astonishing.

3.3.3 Algorithm 3. Given pattern Bi, i=1,2,...,m

(1) For each connected subset X' of X,

$$P(n,i)=PS_{X'}(n,B_i)/A(X'),\ 1<=i<=m,\ 0<=n<=N_i$$
$$R(n,i)=H((X' o nB_i)/B_i)$$
$$A(n,i)=A(nB_i)$$

where

$$N_i=\max\{n|X' o nB_i \,!\!=\emptyset\}$$
$$PS_{X'}(n,B_i)=A[X' o nB_i/X' o(n+1)B_i]$$

is the pattern spectrum of X' by Bi, A(X') is the area of X'.

$$H((X' o nB_i)/B_i) = \ln A(x' o nB_i) - (1/A(x' o nB_i))\cdot \sum_{n<j<=N_i} PS_{x'}(n,B_i)\cdot \ln[PS_{x'}(n,B_i)]$$

is the average roughness of X'By Bi.

(2) Selecting the suitable n,i satisfies

$$P(n,i)=\max$$
$$A(n,i)!=0$$
$$R(n,i) \text{ is small.}$$

(3) Supose S'n,i is skelton subset corresponding to X', p∈S'n,i. Then $\bigcup_{p\in S'_{n,i}} (p+nB_i)$ is a decomposition of X.

The running time is less than that in algorithm 2, and the result is better than that in algorithm 1.

4. EXPERIMENTAL RESULTS

The partial resalts of image segmentation are shown in Fig.1. a) shows the result with multi-thresholding. b) shows the result with clustering. c) shows the result of region growing. The thninng results are shown in Fig.2 where a) is the result of the algorithm 1, b) is the result of the algorithm 2 and c) is the result of the algorithm 3 Fig.3 shows the results of 3-intersection extraction. The extracted polygons are shown in Fig.4.

5. CONCLUSION

The meaningful region can be separated from image by thresholding, clustering and separation-merger algorithm. Based on preprocessed image, the variable threshold cna be employed in the segmentation. The separation-merger is batter labling algorithm.

The edge can be extracted on binary image or grey level image with mathematical morphology. The basic thinning method is improved, and after boundary fitting, the polygon is obtained.The primitives of object shape are acquired from region decomposition.

The information extracted by mathematical morphology can be applied in model discription of object and structure matching and image interpretation.

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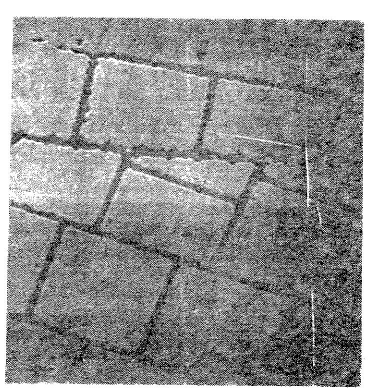


Fig.1 a).Image segmentation with thresholding

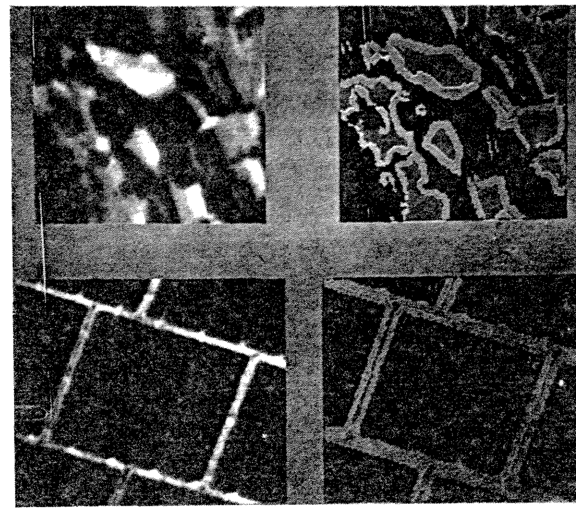


Fig.1 b).Image segmentaion with clusturing

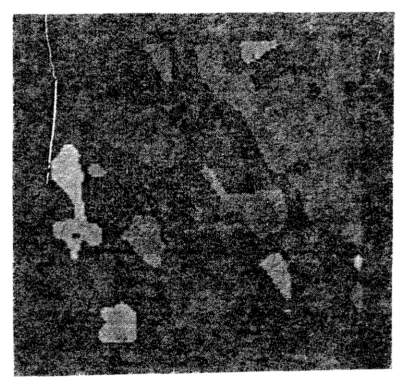


Fig.1 c).Image segmentation with region growing

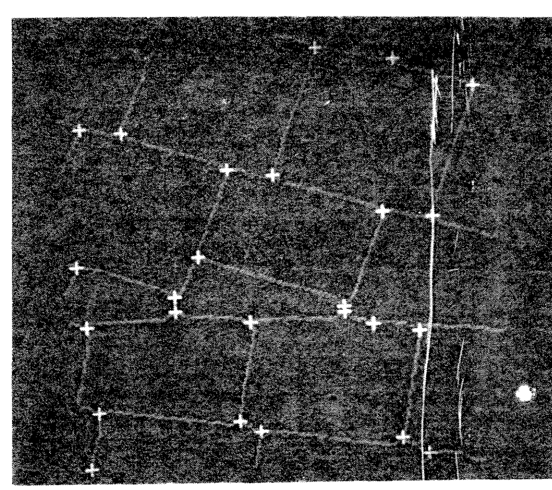
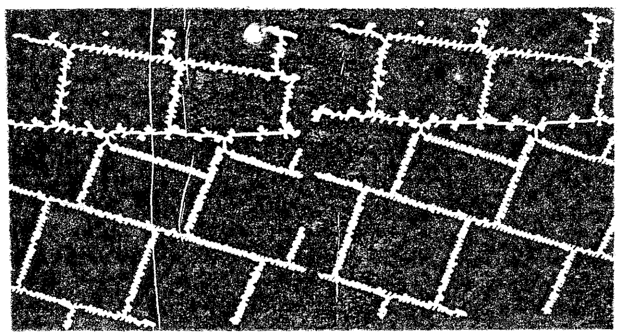
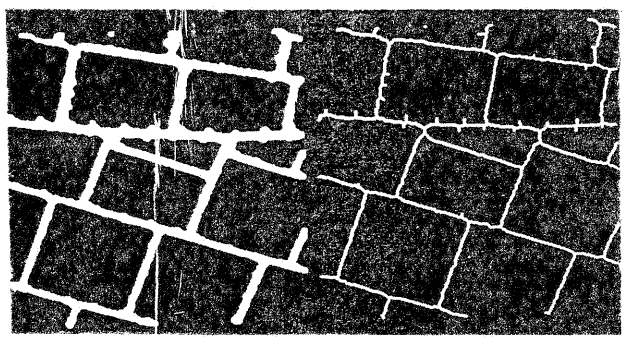


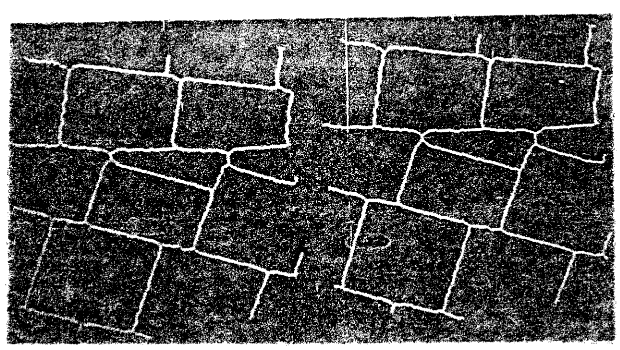
Fig.3. 3-intresection points



a).Algorithm 1



b).Algorithm 2



c).Algorithm 3

Fig.2. Thinning Results

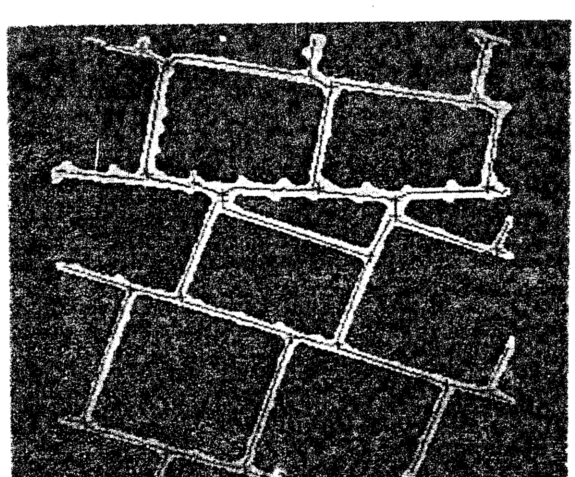


Fig.4. Polygons