OPERATIONAL RULES AND ACCURACY MODELS FOR GPS-AEROTRIANGULATION

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Abstract

Airborne kinematic camera positioning by GPS has reached a mature state of development. The paper reviews some remaining practical problems, like signal discontinuities, cycle slips, drift errors, datum transformation and derives recommendations which make GPS application to aerial triangulation operationally secure. In the second part the theoretical accuracy of GPS blocks is analyzed for various cases of overlap and control, resulting in simple accuracy models which can be used for planning GPS supported aerial triangulation.

1 Present status of kinematic camera positioning

1.1 State of development

Airborne kinematic camera positioning, for aerial triangulation purposes, by differential GPS carrier wave phase observations has recently been thoroughly studied and also experimentally tested. The modern GPS receivers and appropriate software have been developed to a point that successful practical application is imminent. In fact, it has started already, and GPS-supported blockadjustment is expected to become the standard case of aerial triangulation soon.

1.2 Accuracy performance

In this paper we refer only to high precision relative kinematic camera positioning for aerial triangulation by differential carrier wave phase observations. Relative positioning eliminates almost all systematic positioning errors directly and also reduces greatly the effects of signal degradation by 'selective availability' (SA). For operational reasons there are always and only two GPS receivers involved, one stationary on a known point in or near the photo-mission area, the other in the aircraft. Both receivers record simultaneously carrier wave (L1 and preferably also L2) phase measurements to at least 4 identical GPS satellites, as well as C/A-code (or P-code) pseudo ranges, at measuring rates of ≤ 1 sec.

The internal r.m.s. precision of ranging by phase observations has been established empirically to be in the order of 1 mm - 2 mm. This gives a positioning precision in the stationary mode of 1 cm - 2 cm, depending on the satellite constellation (PDOP < 10, preferably < 6).Controlled test flights have shown that relative kinematic camera positioning in-flight comes rather close to that accuracy. R.m.s. coordinate accuracies of < 5 cm (< 3 cm) have been empirically confirmed, except for some systematic errors wich will be discussed below. Thus, kinematic GPS camera positioning is of greatest interest to aerial triangulation practically for all conventional photo scales. The computation of the sequence of GPS antenna positions, which represent the flight trajectory, is a standard, straight forward adjustment procedure which does not present particular problems.

There are, however, some practical problems which have to be given attention in order to receive high precision camera postions by GPS. They are reviewed next.

2 Some practical problems

2.1 Time off-set and spatial off-set

The kinematic GPS positioning gives directly the positions of the GPS antenna on the aircraft, the time sequence being given by the GPS measuring rate. The positions have to be interpolated onto the times of camera exposure and reduced for the spatial off-set between GPS antenna and the perspective centre of the camera (outer node of the lens).

The time interpolation is usually based on a signal from the camera with which the mid-time of exposure is recorded on the GPS time scale. All modern cameras are equipped for emitting such a pulse. Old cameras can be equipped with a light-detector serving the same purpose. The time accuracy of the pulse must be about 1 msec as it corresponds to a forward movement of the aircraft of 6 cm at a ground speed of

200 km/h. The interpolation of the GPS position is usually linear between the 2 neighbour positions. In 1 sec the aircraft moves about 55 m, at 200 km/h. Hence the GPS measuring rate should preferably be shorter than 1 sec, in order to keep the deviation between the actual flight path and the linearly interpolated positions small (< 10 cm). Also a more sophisticated interpolation procedure is recommended.

There is a second approach by which the time interpolation can be avoided. If the signal for camera exposure is given by the GPS system, it can be arranged that the camera exposure coincides nearly with a GPS observation. In that case an interpolation is not required at all or it can be kept simple and safe if it has to bridge only 0.1 sec or less.

The second kind of correction reduces the (interpolated) GPS antenna position onto the perspective centre of the camera. For that purpose the off-set coordinate components must be known. They can be measured directly at the airplane, by tacheometric ground survey, for instance. The off-set components should refer to the axis system of the aircraft.

The computational reduction of the GPS antenna position to the camera position is wanted with regard to the coordinate system of the kinematic GPS positioning. For that purpose the attitude parameters of the aircraft must be known. They may be measured directly by INS. In connection with aerial triangulation there is another solution, as the attitude parameters of the photographs can be derived from the first iterations of the combined blockadjustment. In that case, however, the zero-settings of the camera have to be considered, especially the crab setting, which is to be kept constant during a flight strip and should be manually or automatically recorded during the flight.

The off-set corrections do not require precise attitude data. The reduction is particularly insensitive if the GPS antenna is mounted more or less directly above the camera. In that case the horizontal off-set component is small and may be negligible altogether. The z-correction is then reduced to a constant and the remaining x- and y-corrections amount to only 3.5 cm per degree tilt, for a 2 m vertical off-set. Thus tilt corrections can be neglected in most cases, except for the high precision demands of large scale photography.

2.2 Ambiguity solution, signal interruption, drift errors

There is a second group of problems, related to the ambiguity solution of phase observations and the risk of cycle slips and signal interruptions.

Carrier wave phase observations measure only the phase shift within one cycle. The total integer number of cycles, the signal has travelled through from the satellite to the receiver, remains unknown. Those initial unknown phase ambiguities are to be solved before the kinematic positioning can start. In the case of relative positioning by one stationary receiver on the ground and one receiver in the aircraft the problem can be solved by stationary recordings of both receivers before take-off. There are two cases: Either start from a known base-line (both receivers at known GPS points), or determine an initial base-line from the known stationary receiver position to the unknown position of the also stationary aircraft receiver. The simultaneous stationary recordings had to continue, until recently, for about one hour, in order to solve safely for all initial phase ambiguities. Recently fast ambiguity solutions have been developed which reduce the stationary recording time to a few minutes. Once the initial phase ambiguities are solved the receivers stay locked on the satellites' carrier waves during the flight, until an interruption would occur.

Unfortunately, there are several effects which can cause signal interruptions during the flight. They are known as cycle slips, signal obstruction by body and wings of a turning aircraft, and changes of the number and constellation of recorded satellites. There is no need to go into any details here, as to the causes of such disturbancies. It suffice here to state that signal disruptions do occur during the flight missions and are not likely to be completely avoided.

As direct effect of a signal disruption the ambiguity solutions are lost. In other words, the common system reference is lost and the continuity of the trajectory is interrupted. Recently, sophisticated software development has succeeded to bridge such gaps or jumps by applying prediction and filter techniques. In many cases the interruptions do not affect all signals, some satellites continue to be recorded with the help of which the lost signals can be reconnected. Software programs become available which are capable of bridging signal interruptions and of reassessing and updating the phase ambiguity solutions.

There are cases of quite serious signal interruptions which may extend over 10 sec or more. In such cases it is possible that the ambiguity solution can be restored only approximately. It is well established that approximate ambiguity solutions result in GPS drift errors which are, however, linear in first approximation. This brings us to the general problem of GPS drift errors. Practically all experimental tests on kinematic GPS positioning have shown some systematic GPS drift errors, the typical magnitudes being in the order of 10 cm to 50 cm per hour. It is a matter of controversy amongst experts what are the causes of systematic GPS drift errors and whether they can be avoided completely.

From an operational point of view it has to be accepted as a fact, for the time being, that signal discontinuities may occur during a flight mission, especially during flight turns. It has equally to be accepted that there may be small GPS drift errors, possibly as a result of incomplete phase ambiguity solutions, or for other reasons. Considering on the other hand that linear GPS drift errors can be assessed and corrected subsequently, during combined blockadjustment, it can be concluded that no particular efforts need to be made to avoid drift errors. They can be just accepted and dealt with during the blockadjustment. This is an operational consideration which holds only in connection with aerial triangulation. It has, however, convenient operational consequences. If we cannot rule out signal interruptions during the flight with the consequence of reassessment of phase ambiguities there is no point in determining the initial phase ambiguities by stationary recordings before take-off. It can be recommended, therefore, referring to GPS aerial triangulation flights, not to attempt any stationary base-line determination before take-off, i.e. to start flying and carry out the photo flight mission in the usual way, without particular care about GPS continuity. The GPS recordings (at both receivers) need only be switched on a few minutes before the mission area is reached. In case of signal interruptions during flight turns (within a strip no serious interruptions are expected) the phase ambiguity solutions are redetermined in the post-processing by using the C/A-code or P-code pseudo-range positioning, and by considering a dynamic modelling of the aircraft movement. The solution may leave some systematic errors for the following stretch of the GPS trajectory. Such drift errors are practically linear, according to available experience. They may change after each major signal interruption. In the extreme case each strip may have its own linear drift error. It still needs to be investigated how large distances (several hundred km) between stationary receiver and mission area and long flight missions (several hours) will affect the GPS drift behaviour. Linear drift errors can be assessed and compensated in combination with aerial triangulation, by including additional parameters into the combined block adjustment, as will be discussed in chapter 3.

2.3 The datum problem, ground control

GPS positioning refers generally to the earth-centered rectangular GPS coordinate system WGS 84. The same is true, in principle, for relative positioning, although a transformed coordinate system may be locally tied to the reference point at the stationary receiver. It is to be stated clearly that aerial triangulation with combined blockadjustment can be carried out in that case, without any ground control point, provided the GPS trajectory is not interrupted over the complete flight mission. The result would refer to the original or a local transformation of the GPS reference system WGS 84.

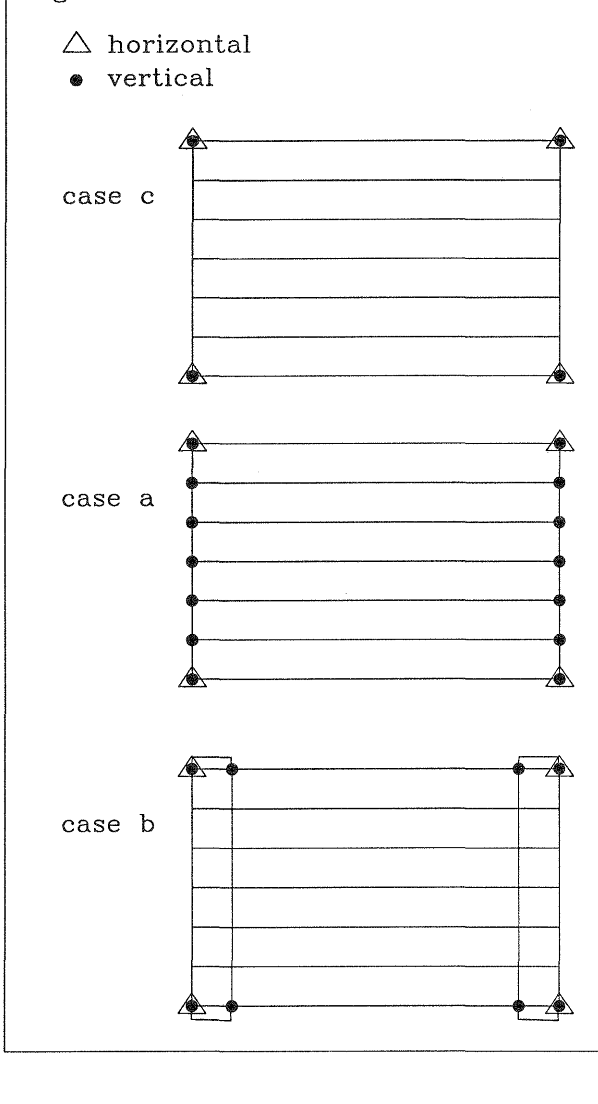
Normally, however, the results of aerial triangulation (and of mapping) are wanted in a national horizontal and vertical reference system. There are no absolute geodetic transformation formulae available, at present, which would precisely enough link the WGS 84 to a national reference system. Hence, the datum transformation must be provided in the traditional photogrammetric way, i.e. by some ground control points which would preferably be given in both coordinate systems. The standard recommendation is, for aerial triangulation, to use 4 XYZ-ground control points, located at about the corners of a photo-block. They are sufficient to provide the datum transformation (in GPS blocks control points have no accuracy function any more), provided the GPS trajectory is continuous. They are even capable of correcting for overall drift errors. It is only the geoid reference which is not completely solved by 4 ground control points alone. If the geoid is known its undulations can be superimposed in addition, or additional vertical control points could introduce the geoid indirectly. There have been discussions about using only one control point. It allows the determination of the shift parameters of a datum transformation. All other parameters must be derived from the known geographical position. The result can only be an approximate solution which may, however, be sufficient for low accuracy requirements.

Ground control points can be used in two ways in GPS supported aerial triangulation. If the GPS trajectory is continuous the GPS-block may be adjusted without control or by using them as GPS control only. The subsequent datum transformation (if necessary in 2 steps, or non linear because of map projection and spherical vertical reference) would then use the control point coordinates in the national system. It is preferable, however, and in case of different drift errors necessary, to include the control points directly in the combined block-adjustment.

In the approach just described the linear datum transformation parameters can be treated as additional unknowns and solved for in the combined blockadjustment. That solution would include the correction for overall linear drift errors. In fact, both effects cannot be separated completely from each other. Drift correction is identical with a datum correction, although it may include different parameters.

In section 2.2 it was discussed that there may arise independent drift errors, even per strip. It means that there can be a datum problem for sections of the GPS trajectory, even per strip. To solve in that case for all unknown drift parameters in combined blockadjustment additional information is required, in order to prevent singularities. The additional in-

Fig.1 Control scenarios for GPS blocks



formation can come from additional control points or from additional photo overlap. In case of standard overlap (parallel strips with 20% side overlap) drift parameters per strip can be solved, if 2 chains of vertical control points, running across the block at both front ends, are given, in addition to the standard 4 XYZ control points. If a block has double stereo coverage (parallel strips with 60% side overlap, or double coverage with crossed flight directions), stripwise drift corrections are possible with only 4 ground control points. There is a simpler overlap case, however, with which stripwise drift parameters can be solved for during the combined adjustment. It is sufficient to run 2 cross-strips, across a standard block (20% side overlap) at either end, the cross-strips replacing the 2 chains of vertical control points. In that case the adjustment can be based on 4 XYZ ground control points alone, although it is suggested to add one vertical control point at each corner of the block. That cross-strip version is highly recommended as the standard case for GPS aerial triangulation. Cross-strips may not be needed, if there are no serious signal interruptions. But drift errors can be corrected with their help, in case necessary. In that sense cross-strips are an operational precaution, in order to solve for singularities in the combined blockadjustment, in case needed.

3 Combined blockadjustment

The introduction of GPS camera position data into aerial triangulation constitutes a certain extension of conventional block adjustment. It is assumed that the aerial triangulation as such is done in the same way as usual, as far as tie-points, point transfer, measurement of image- or of model-coordinates, data reduction etc. are concerned. Only the number of ground control points is considerably less, in general.

The GPS camera station coordinates, as obtained from the kinematic GPS processing and possibly transformed approximately into the national coordinate system, are treated as additional observations. They are introduced into the combined blockadjustment appropriately weighted. Treating additional observations is very well known in blockadjustment and does not present particular problems. Especially the matrix structure of normal equations is not altered at all.

It is only the unknown drift parameters which require some attention. Linear drift parameters are treated as unknown parameters in the combined adjustment. They will extend the well known matrix structure of observation- and normal equations, but the standard numerical solution techniques can still be applied, for instance by reduction to banded-bordered matrices.

With regard to unknown drift parameters there are 3 cases to be distinguished which a GPS-blockadjustment program should have as options: (1) no drift corrections at all, (2) one set of linear drift correction parameters for the whole block, (3) several independent sets of parameters for certain subdivisions of the GPS trajectory, up to independent corrections per strip, as the case may demand. Case (1) and (2) can be handled with the four standard XYZ ground control points, provided there are no interruptions in the GPS trajectory. Case (3) takes care of possible interruptions. But it has to rely on the 2 additional chains of vertical control, or on 2 cross-strips, as described in section 2.3, in order to prevent numerical singularities or near-singularities in the combined adjustment.

Such computer programs for combined blockadjustment with additional datum parameters have been developed and are being applied, for both the bundle method and the independent model method of adjustment.

4 Accuracy of adjusted GPS blocks

4.1 Theoretical investigations

The accuracy features of adjusted GPS blocks are expected to be highly favourable. The GPS camera stations act essentially as if the camera air stations were 'ground' control points. It can be anticipated, therefore, that GPS blocks are generally very well controlled, even if free drift parameters will weaken the geometrical strength of a block somewhat. It can also be anticipated that there is very little propagation of errors in a block, and that the accuracy of blocks is little dependent on block size. It is further intuitively evident that conventional ground control points are not required any more for stabilizing the accuracy of a block, but only for providing the datum reference. The conventional accuracy function of ground control points is taken over by the GPS camera station positions. This has been confirmed by early computer simulations.

That general picture sets the scene for a more detailed investigation into the accuracy properties of GPS blocks. A comprehensive investigation into the effects of various parameters can only be done theoretically, in view of the multitude of combinations. The theoretical accuracy of a great number of cases has been worked out, by data simulation and inversion of the respective normal equation matrices. The most urgent questions concern the overall accuracy features of GPS blocks, and the effects of ground control and GPS camera positioning accuracy on the blocks, in combination with block size and the various cases of drift corrections. The main results are here demonstrated and summarized.

Theoretical accuracy studies usually make idealized assumptions. Also here flat terrain is assumed, zero-tilts and ideally regular photo-overlap, 9 tie-points per photograph in the standard positions.

All image coordinates are assumed to be uncorrelated and have equal accuracy, expressed in the variance factor σ_0^2. Systematic image errors are not considered, they are assumed to be sufficiently corrected before or during the blockadjustment. Ground control points as well as GPS camera station coordinates are also treated as uncorrelated observations. All cases refer to combined bundle-blockadjustment, with GPS camera station coordinates as additional observations.

The actual investigations concern wide-angle photo-blocks of photo-scale 1 : 30000, h = 4500 m, and extend to different block size and different control cases, in relation to different assumptions about drift corrections. The image coordinate accuracy is generally assumed to be $\sigma_0 = 10\mu m$ in photo scale, which corresponds to $\bar{\sigma}_0$ in terrain units. The direct results of the theoretical investigation are standard errors of adjusted tie-point terrain coordinates, which are summarized to r.m.s. errors $\mu_{x,y}$ for horizontal coordinates and μ_z for vertical coordinates. Those r.m.s. values represent the accuracy of the adjusted blocks.

It is recalled that the actual magnitudes of the standard errors do not represent any restrictions, as only the weight relations act in least squares adjustment. The results can therefore be transferred to other error magnitudes and to other photo scales by expressing all standard and r.m.s. errors in units of σ_0.

4.2 Some examples

Let us look first at the accuracy distribution within some adjusted GPS blocks in detail. In fig. 2-4 the standard errors of adjusted tie point coordinates are shown, referring to two block sizes with 20% side overlap and one case with 60% side overlap. In all cases 4 ground control points in the corners of a block are assumed, and one set of unknown linear correction parameters for datum transformation or overall drift correction has been applied in the blockadjustment. It means that here no unbridged signal discontinuities during the flight are considered. The photogrammetric image coordinate accuracy is set to $\sigma_0 = 10\ \mu$m, and the ground control point coordinates as well as the GPS camera air station coordinates are given the moderate accuracy of $\sigma_{CP} = 30$ cm and $\sigma_{GPS} = 30$ cm, respectively, which would be sufficient for 1 : 10000 scale mapping from 1 : 30 000 scale photographs. Those assumed standard errors correspond to $\bar{\sigma}_0$, i.e. to the precision $\sigma_0 = 10\ \mu$m of image coordinates in the photo scale 1 : 30000, projected onto the ground. The tie-point distribution is six points per model, giving 3 rows of tie-points along each strip, the rows in the common lateral overlap between strips coinciding through identical points. The figures 2-4 show only the upper left quarter of a block, for reasons of symmetry.

The first example (fig. 2) refers to a block of 6 strips with 21 photographs each. The figures represent theoretical standard errors, after combined blockadjustment, of the X, Y, Z coordinates of all tie-points, arranged in the regular row and column array. Within a strip always 3 x 2 points belong to a stereo-model. It is immediately evident that the standard errors are quite evenly distributed over the block. Only the border points of the block show generally, as usual, the largest errors. If we disregard them, for the moment, all other standard errors lie within a narrow band of variation: σ_x between 38 cm and 50 cm, σ_y between 45 cm and 55 cm, σ_z between 48 cm and 81 cm. In the central parts of the block the standard errors are even closer together. The accuracy of the perimeter points is generally lower, pushing the maximum standard errors in X, Y, Z to 59 cm, 75 cm, 94 cm , respectively. The overall r.m.s. coordinate accuracy of all points of the adjusted block amounts to 46 cm, 55 cm, 68 cm, which corresponds to 1.5 $\bar{\sigma}_0$, 1.8 $\bar{\sigma}_0$, 2.3 $\bar{\sigma}_0$, respectively.

Fig. 3 concerns a small block of 4 x 13 photographs. The distribution of standard errors in the block displays the same overall picture as in the previous case. The magnitudes of the standard errors are in general a little larger, by about 3% in X, Y, and by about 6% in Z.

Fig.2 Standard errors [cm] of adjusted tie points

block size 6*21, 1:30000
$\sigma_0 = 10\,\mu m \;\hat{=}\; 30$cm
4 control points for drift correction (datum transformation)

rows/ columns of points		2	3	4	5	6	7	8	9	10	11	12	strip
1	X	28	40	48	55	58	58	56	55	54	54	54	
1	Y	29	61	70	74	75	74	73	72	72	72	72	
1	Z	29	82	92	92	89	86	84	84	85	86	86	
2	X	47	38	43	47	50	50	49	48	47	47	47	
2	Y	46	45	50	54	55	55	55	55	55	55	55	1
2	Z	91	75	81	81	78	75	74	74	74	75	75	
3	X	55	44	41	42	43	43	43	43	42	42	42	
3	Y	50	47	50	52	53	53	53	52	52	52	52	
3	Z	81	70	67	64	61	59	57	57	57	57	57	
4	X	55	44	43	43	44	44	44	44	43	43	43	
4	Y	65	53	52	51	52	52	52	52	52	52	52	2
4	Z	94	69	67	66	65	64	63	63	63	63	63	
5	X	59	48	43	42	41	41	41	41	41	41	41	
5	Y	61	52	50	50	50	50	50	50	50	50	50	
5	Z	73	61	56	54	53	52	51	51	50	50	50	
6	X	57	46	44	43	43	43	43	43	43	43	43	
6	Y	65	55	52	52	51	51	51	51	51	51	51	3
6	Z	90	63	62	62	61	61	60	60	60	60	60	
7	X	59	48	43	42	41	41	41	41	41	41	41	
7	Y	62	54	51	51	50	50	50	50	50	50	50	
7	Z	70	57	52	51	50	49	49	49	49	48	48 oooo	
oooo		oooo										oooo	

R.m.s. accuracy of block

[cm]			$[\bar{\sigma}_0]$		
μ_x	μ_y	μ_z	μ_x	μ_y	μ_z
46	55	68	1.5	1.8	2.3
51		68	1.7		2.3

The comparable results of a large block of 12 x 41 photographs are not displayed here. They would show that the distribution of the standard errors within the block is even more regular than in smaller blocks. The overall r.m.s. accuracy of the block in X, Y, Z coordinates amounts to 44 cm, 53 cm, 63 cm, respectively, which is equivalent to 1.5 $\bar{\sigma}_0$, 1.8 $\bar{\sigma}_0$, and 2.1 $\bar{\sigma}_0$. Compared with the medium size block (6x21) there is an overall improvement of theoretical accuracy of 4% in X and Y, and of 7% in Z. Those examples confirme the expectation that the accuracy of GPS blocks is little dependent on block size.

Fig.3 Standard errors [cm] of adjusted tie points

block size 4*13, 1:30000
$\sigma_0 = 10\mu m \hat{=} 30cm$
4 control points for drift correction
(datum transformation)

rows/ columns of points		2	3	4	5	6	7	8	strip
1	X	28	40	48	55	58	58	58	
1	Y	29	61	71	75	75	74	74	
1	Z	29	83	94	94	89	84	82	
2	X	48	39	43	47	50	50	50	
2	Y	47	46	51	54	56	56	56	①
2	Z	91	75	82	80	77	73	72	
3	X	56	45	41	42	43	43	44	
3	Y	51	48	50	52	53	53	54	
3	Z	80	69	66	63	60	57	56	
4	X	55	44	42	43	44	44	44	
4	Y	66	54	52	52	52	52	52	②
4	Z	92	68	67	65	64	62	62	
5	X	59	47	42	41	42	42	42	
5	Y	62	52	50	50	51	51	51	
5	Z	72	61	57	55	53	52	51 ...	
⋮		⋮						⋮	

R.m.s. accuracy of block

[cm]			[$\bar{\sigma}_0$]		
μ_x	μ_y	μ_z	μ_x	μ_y	μ_z
47	57	72	1.6	1.9	2.4
52		72	1.7		2.4

Fig.4 Standard errors [cm] of adjusted tie points

block size 7*13
q=60% side overlap
4 control points

rows/ columns of points		2	3	4	5	6	7	8	strip
1	X	28	37	41	45	47	47	47	
1	Y	28	55	62	62	61	60	60	①
1	Z	29	76	82	81	78	76	75	
2	X	39	33	34	37	38	39	39	
2	Y	41	40	44	45	45	44	44	②
2	Z	65	59	61	60	58	56	56	
3	X	40	34	33	34	35	35	35	
3	Y	41	39	40	41	41	41	41	③
3	Z	58	53	53	51	49	48	48	
4	X	41	35	34	34	34	34	34	
4	Y	44	39	39	40	40	40	40	④
4	Z	54	50	48	47	45	44	44	
5	X	42	36	34	33	34	34	34	
5	Y	45	40	39	40	40	40	40	
5	Z	53	48	47	45	44	43	43 ...	
⋮		⋮						⋮	

R.m.s. accuracy of block

[cm]			[$\bar{\sigma}_0$]		
μ_x	μ_y	μ_z	μ_x	μ_y	μ_z
37	45	58	1.2	1.5	1.9
41		58	1.4		1.9

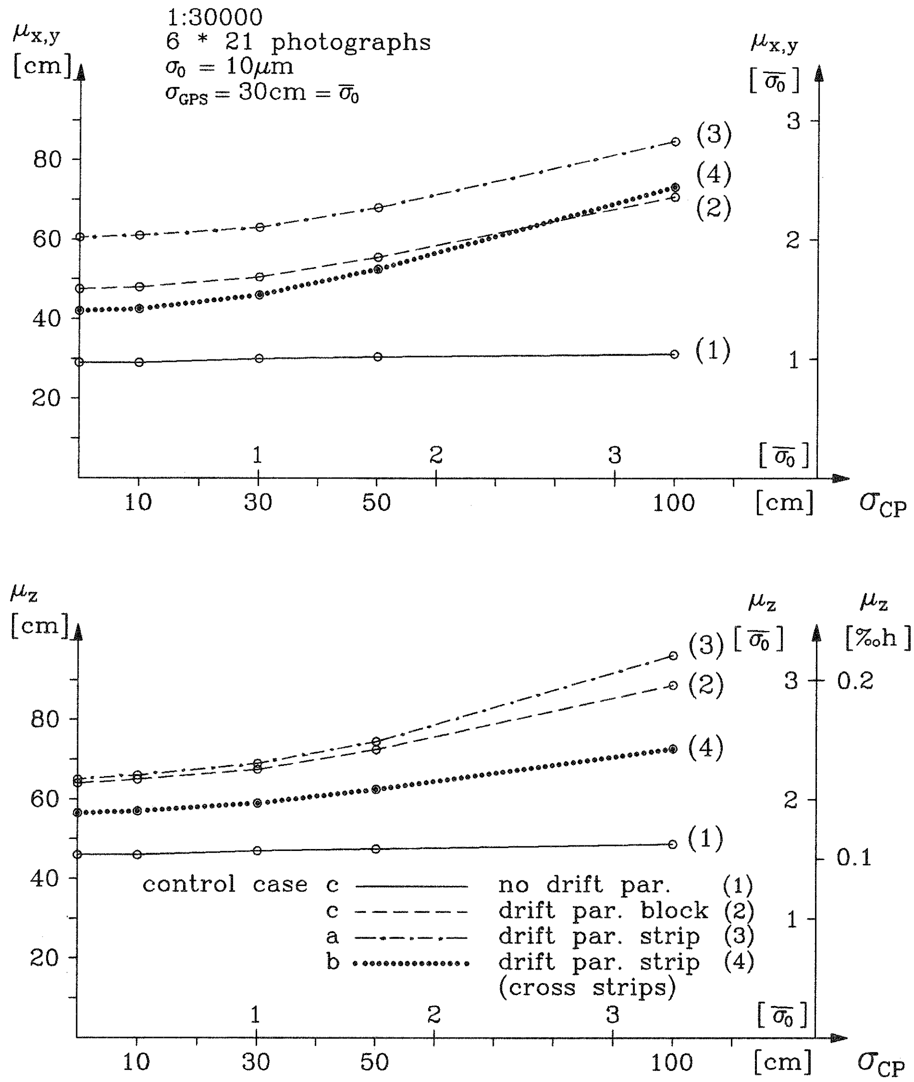
Fig. 4 finally shows an example of a block with 60% side overlap. It contains 7 x 13 photographs which cover the same area as the example of fig. 3. The distribution of standard errors shows again high regularity, comparable to the other cases. The double overlap improves the overall accuracy by the factor 1.25, as compared with the example of fig. 3. That improvement is not as high as might have been expected. The reason is mainly, in this case, the magnitude (30 cm) of the standard errors of ground control coordinates.

The examples of fig. 2-4 give a first impression of the accuracy results of GPS blocks, based on minimum ground control. The main features are quite regular accuracy distribution within a block, a generally high level of accuracy, as compared with the photogrammetric measuring accuracy (σ_0) of image coordinates, and weak dependence on block size. In the following it will be more thoroughly investigated how especially the accuracy of the ground control points and of the GPS camera air stations influence the resulting accuracy of the adjusted blocks. Hereafter we consider only the overall horizontal ($\mu_{x,y}$) and vertical (μ_z) r.m.s. accuracy of the blocks.

4.3 Effects of ground control

We want to investigate how the accuracy relevant features influence the accuracy of adjusted GPS blocks. There is mainly the accuracy influence of ground control points and of GPS camera station positioning to be considered, in combination with the various scenarios of ground control and datum transformation resp. drift error correction. In this section we study the influence of the accuracy ground control coordinates. The GPS accuracy is kept constant at σ_{GPS} = 30 cm = $\bar{\sigma}_0$, which can be considered a representative value. Also the block size is kept constant at 6 x 21 photographs. Smaller and larger blocks would give almost identical results. It is also assumed that the precision of measuring the image coordi-

Fig.5 Influence of ground control accuracy (σ_{CP}) and of drift parameters on the accuracy of adjusted GPS blocks



nates of ground control image points is the same ($\sigma_{CP'} = \sigma_0$) as for image tie-points. A few cases have been run in which the image coordinate accuracy of ground control points has been assumed to be $\sigma_{CP'} = 3\mu$m, as could be with signalized control points. The effects have turned out to be very minor, and that case is not pursued here any further.

In fig. 5 the results are displayed which demonstrate the effects of variation of ground control accuracy on the adjusted blocks, for the described specifications where especially the GPS camera position accuracy is kept fixed to $\sigma_{GPS} = \bar{\sigma}_0$. Four different cases of ground control and datum transformations resp. drift corrections are distinguished. The cases are depicted in fig. 7 and 8.

The graphs show directly that the curves (1) behave unlike the others. That case refers to blocks with 4 control points only and no datum transformation applied. The result is evidently determined by the internal block accuracy and by the absolute GPS postioning. The errors of the 4 ground control points have practically no additional effect, even if they are large, if no free drift or datum parameters are applied.

In all other cases, where the block is fitted via free parameters onto the control points, the error level is considerably higher, even if the control points have no errors at all. It is the GPS errors and the internal block accuracy which raise the error level immediately, although the different control scenarios (c, a, b) react differently. The influence of ground control errors is superimposed on that basic behaviour of blocks. They represent an additional error component which can gain considerable effect if the errors of ground control points increase to large values. The relations of fig. 5 indicate, however, that the effects of ground control errors are very minor (in most cases < 5%) as long as the control points are adequately precise, i.e. as long as $\sigma_{CP} < \bar{\sigma}_0$.

It is pointed out, finally, that the case of blocks with cross-strips, which is here generally favoured, behaves best if compared with the other cases which allow free parameters. It is closest to the ideal case in which GPS camera positioning gives direct absolute positioning, with respect to the national reference system.

4.4 Influence of GPS camera station accuracy and of datum transformations

The previous investigations have shown that it is sufficient to concentrate on medium block size (6 x 21). They have also shown that the variation of ground control accuracy has no very significant influence on the blockadjustment results, at least not within realistic conditions. We assume, therefore, in the following investigations for the ground control coordinates standard errors of $\sigma_{CP} = 30$ cm. That corresponds, for the photo scale 1 : 30000, to the photogrammetric measuring accuracy, projected into ground units ($\sigma_{CP} = \bar{\sigma}_0$). That assumption is not particularly restrictive. Such control point accuracy is the least which must be asked in standard photogrammetric practice for aerial triangulation (unless larger errors are tolerable for secondary reasons). It is further assumed that the measuring precision of ground control point image coordinates in the photographs is the same as for image tie-points ($\sigma_{CP'} = \sigma_0$).

On that basis of standardized assumptions we can now investigate the influence of GPS camera station positioning accuracy on the results of combined blockadjustment, in combination with the main cases of ground control and drift corrections. It is the central part of the theoretical investigation into the accuracy of GPS supported aerial triangulation.

We distinguish in particular the 3 cases of ground control, as defined in fig. 1 (c : 4 XYZ control points; a : as c, + 2 chains of vertical control; b : as c, + 4 vertical control points + 2 cross-strips). They are combined with 3 according cases of drift corrections, namely (1) with no correction at all (case c), (2) with drift correction per block (case c), and (3), (4) with drift corrections per strip (case a, b). From a practical point of view the case b (block with 2 cross-strips) is the most interesting one.

For a number of blocks the resulting theoretical accuracy has been computed (by inversion of the normal equation matrices). The results are summarized in fig. 6 and fig. 7 which show the horizontal and vertical r.m.s. values $\mu_{X,Y}$ and μ_Z of the horizontal and vertical standard coordinate errors, of all adjusted tie-points.

A discussion and interpretation of the results may distinguish between the cases of precise and less precise GPS camera positioning. Let us look first at the lower left parts of the functions in fig. 6 and 7, as specified by $\sigma_{GPS} \leq 30$ cm resp. $\sigma_{GPS} \leq \bar{\sigma}_0$. That parts of the curves show that, starting from $\sigma_{GPS} = 0$, the GPS errors are quadratically added, in some cases at a reduced rate. The main result is, however, that the different cases of control and drift corrections determine the results.

The ideal case is represented by the curves (1). They refer to the case that no drift corrections are applied at all. The results are determined by the internal block accuracy, based on σ_0, and by the absolute GPS positioning. Case c includes 4 ground control points, but their effect is negligible, as here no free drift or datum parameters are applied. The curves (1) refer to consistent absolute GPS positioning, as long as there are no datum of drift effects at all. In that case there is very little propagation of errors in the block, and the resulting r.m.s. accuracy of $\leq 1.0\bar{\sigma}_0$ and $\leq 1.6\bar{\sigma}_0$ for horizontal and vertical coordinates, respectively, is practically determined by the mere intersection errors of rays and the minor effects of the tilt errors of the blockadjustment.

As soon as one set of free datum or drift parameters is applied (curves (2), case c) the errors of the 4 control points are added to the basic block accuracy of the previous case. There is practically no adjustment effect, with 4 control points only. The block is transformed onto the ground control points practically as a unit. Their errors are almost independently superimposed. That basic situation remains valid in all cases, in which the datum transformations are determined via ground control points.

If independent drift parameters are applied per strip, the geometry of the block is weakened further, as is evident from the curves (3) which refer to the case a of additional vertical control points. The blocks with 2 cross-strips, which are back to minimum control (except for 4 additional vertical control points), are subject to the same principle, that additional parameters weaken the geometry. Fortunately, however, the cross-strips counteract effectively to the extent that amongst all cases which apply free datum or drift corrections, blocks with 2 cross-strips give best results, even if drift corrections are applied per strip. This strengthens the previous recommendation for cross-strips also from the accuracy point of view.

The results, as far as they refer to precise GPS camera positioning ($\sigma_{GPS} \leq \bar{\sigma}_0$) can be condensed in very simple rules which can serve for the planning of GPS aerial triangulations. They are summarized in table 1, together with 2 more cases from an earlier investigation. In case GPS camera positioning is precise to $\sigma_{GPS} \leq 0.3\bar{\sigma}_0$, as can easily be reached in combination with medium scale photography, the values of table 1 can be reduced by about 10%. The derived relations suggest that GPS camera positioning is effectively applicable also for large scale aerial triangulation and for large scale mapping.

If we now look at the main parts of the relationships in fig. 6 and 7 it can be seen how the accuracy of GPS blocks reacts to poorer accuracy of GPS camera positioning, i.e. to larger values of σ_{GPS}. All relations increase monotonously with σ_{GPS} at roughly similar rates, except for the curves (2) and (4) in fig. 7, which react more sensitive to GPS positioning errors. The overall remarkable feature is, however, that the r.m.s. errors of blocks increase at considerable slower rates than the GPS positioning errors. If the GPS camera position accuracy is as poor as $\sigma_{GPS} = 3$ m, for instance, which is for 1 : 30000 photo scale equivalent to $\sigma_{GPS} = 10\ \bar{\sigma}_0$, the accuracy of the adjusted blocks is still about 1 m (3.5 $\bar{\sigma}_0$) or better horizontally and about 1.5 m (5 $\bar{\sigma}_0$) or better vertically. The explanation is given by the well known averaging effect if a block has many control points. It can be concluded that rather large GPS camera positioning errors can be tolerated if the required block accuracy needs to be, for instance, only 2.5 $\bar{\sigma}_0$ in x, y or 3 $\bar{\sigma}_0$ or 0,2‰ h in z.

Fig.6 Influence of GPS camera positioning accuracy (σ_{GPS}) and of drift parameters on the horizontal accuracy ($\mu_{x,y}$) of adjusted blocks (combined bundle block adjustment)

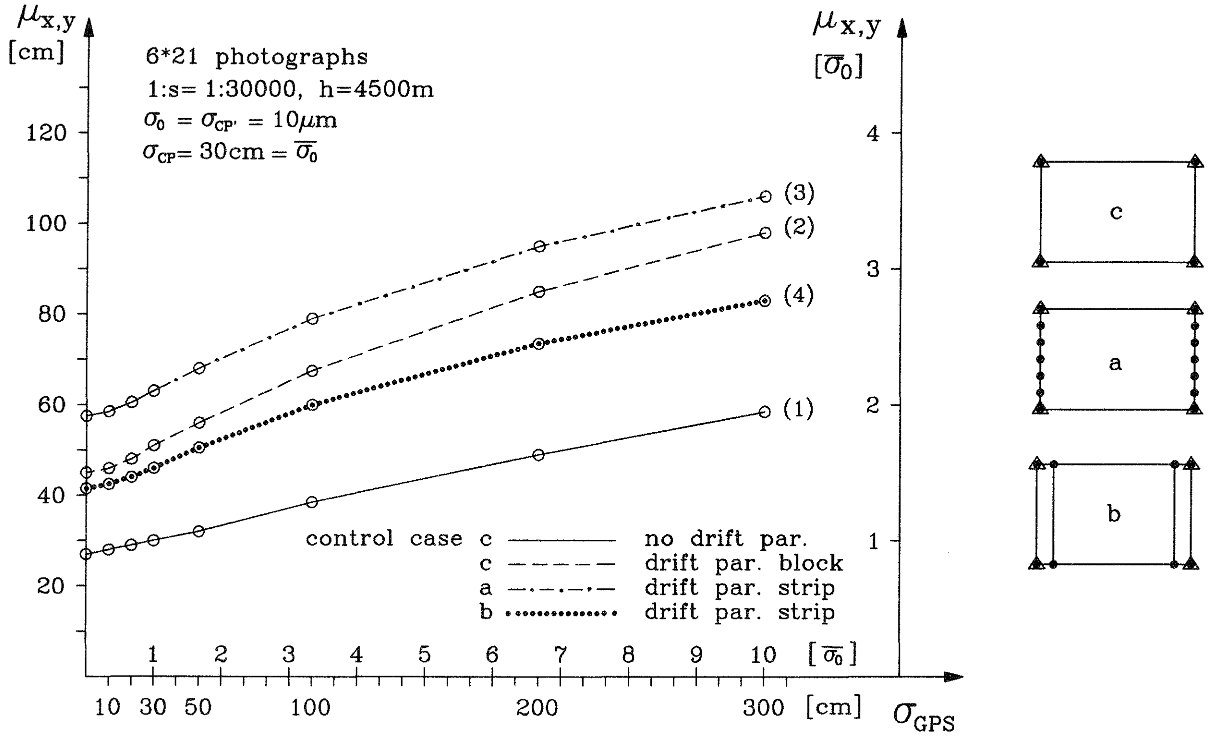


Fig.7 Influence of GPS camera positioning accuracy (σ_{GPS}) and of drift parameters on the vertical accuracy (μ_z) of adjusted blocks (combined bundle block adjustment)

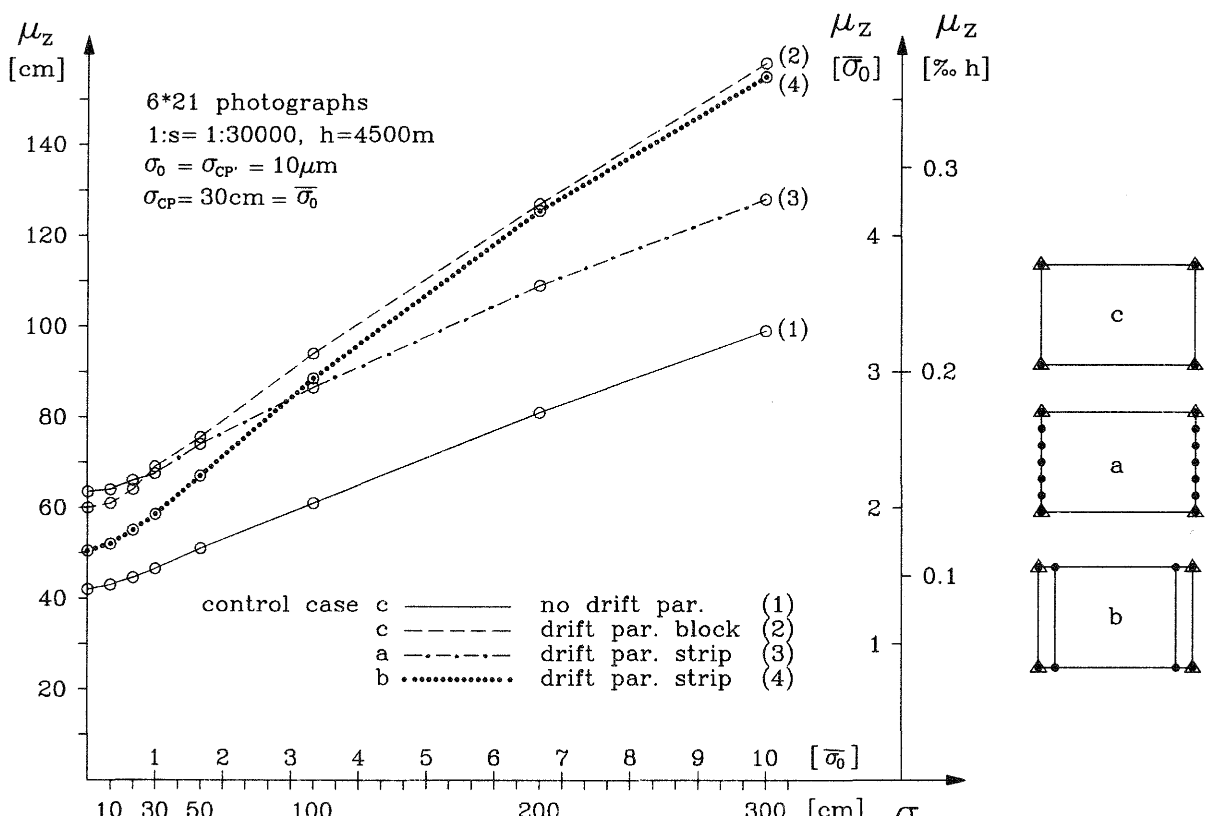


Table 1. Accuracy of adjusted GPS–bundle–blocks

Control cases: c (4 CP)
a (4 CP + 2 chains of vertical CP)
b (4 CP + 4 vertical CP + 2 cross–strips)

If

$$\sigma_{CP'} = \sigma_0 \ , \ \sigma_{CP} \leqq \overline{\sigma}_0 = \sigma_0 \cdot s \ , \ \sigma_{GPS} \leqq \overline{\sigma}_0 = \sigma_0 \cdot s \quad (s = \text{photo scale number})$$

then

$\mu_{x,y} \leqq 1.0\ \overline{\sigma}_0$	$\mu_z \leqq 1.6\ \overline{\sigma}_0$	(c, no drift parameters)
$1.0\ \overline{\sigma}_0$	$1.6\ \overline{\sigma}_0$	(a, no drift parameters)
$1.7\ \overline{\sigma}_0$	$2.3\ \overline{\sigma}_0$	(c, drift parameters per block)
$1.7\ \overline{\sigma}_0$	$1.7\ \overline{\sigma}_0$	(a, drift parameters per block)
$2.1\ \overline{\sigma}_0$	$2.3\ \overline{\sigma}_0$	(a, drift parameters per strip)
$1.5\ \overline{\sigma}_0$	$2.0\ \overline{\sigma}_0$	(b, drift parameters per strip)

block–size: 6 * 21 photographs

Considering the high accuracy which kinematic GPS camera positioning is capable of providing it can be concluded that it is more than sufficient for aerial triangulation / mapping with medium or small scale photography. It is not suggested to deteriorate intentionally the GPS camera positioning accuracy as there would hardly be any economic advantage. It only means that the accuracy requirements for GPS camera positioning are not critical at all in those applications. Hence, long flight missions and large distances to the stationary receiver, which may be located as convenient as possible, seem feasible.

The theoretical accuracy studies lead to the conclusion that GPS aerial triangulation has highly favourable accuracy features and that the practical application is of greatest operational and economic interest. It is recalled that theoretical accuracy investigations are always somewhat schematic and idealized. However, the results are becoming confirmed by the few empirical tests which have been carried out, so far. There is no doubt that the method is applicable and highly effective over the full scale range of photogrammetric aerial triangulation for mapping. Its main effect is, in all cases, that ground control points are only required for datum transformations. Thus their number can be greatly reduced, down to very few points per block. In conclusion, the application in practice of GPS supported aerial triangulation is highly recommended.

Ceterum censeo disponibilitatem delectam (SA) esse delendam.