PHOTOGRAMMETRIC FEATURE INTERSECTION

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ABSTRACT:

Photogrammetric space intersection requires image coordinates of a point in space to be digitized on two or more photos. In digital photogrammetry or in single photo digitizing, it is difficult to locate the matching image of poorly defined points on different photos. A mathematical technique has been developed that requires only that lines or features be digitized on different photos without the need to digitize the same image points to perform the intersection. This method is very useful in single photo digitizing and 3D robot vision.

KEY WORDS: Aerotriangulation, Algorithm, Photogrammetry, Robot Vision, 3D

1. INTRODUCTION

In digital photogrammetry, edge detection algorithms result in discrete points. In digital stereopairs, the discrete points of an edge on one image do not match the same discrete points of the other stereopair. This prohibits using traditional photogrammetric space intersection of a point in space which requires at least two photos of known interior and exterior orientation and the image coordinates of the same point on all the photos. This problem is obvious in single photo digitizing of feature lines on multiple photographs.

In this paper a modified photogrammetric space intersection technique will be presented that does not require the same point to be digitized on multiple photos. The new approach requires only that the feature lines be digitized on multiple photos (minimum two photos).

In the next section, the reader will be presented with some background on photogrammetric space intersection. In the third section the new modified model of photogrammetric space intersection of feature lines will be presented. Finally a report about results obtained with this method is presented, followed by conclusions.

2. BACKGROUND

The mathematical relationship between an image point, the camera orientation (exterior and interior), and the object space coordinates of a point can be expressed using the collinearity equation:

$$x_j = x_{pj} - c_j \frac{m_{11j}(X-X_j) + m_{12j}(Y-Y_j) + m_{13j}(Z-Z_j)}{m_{31j}(X-X_j) + m_{32j}(Y-Y_j) + m_{33j}(Z-Z_j)}$$

$$y_j = y_{pj} - c_j \frac{m_{21j}(X-X_j) + m_{22j}(Y-Y_j) + m_{23j}(Z-Z_j)}{m_{31j}(X-X_j) + m_{32j}(Y-Y_j) + m_{33j}(Z-Z_j)} \qquad (1)$$

or

$x_j = FX_j\ (X, Y, Z)$

$y_j = FY_j\ (X, Y, Z)$

where:

x_j, y_j	photo coordinates of the point on photo j
x_{pj}, y_{pj}	principal point coordinates of photo j
c_j	principal distance of photo j
X_j, Y_j, Z_j	object space coordinates of photo j
$m_{11j} \ldots m_{33j}$	orthogonal orientation matrix elements for photo j
X, Y, Z	object space coordinates of the point.

Equation 1, is a non-linear equation with 3 unknowns (X, Y, Z). For n photos, the collinearity equation will result in 2n equations. Obviously a minimum of two photos will be required to solve for the object space coordinates of the point. The linearized observation equation for Eq. 1 for one photo is:

$$V + B\Delta = f$$

where:

$$V = \begin{Vmatrix} v_x \\ v_y \end{Vmatrix}$$

$$\Delta = \begin{vmatrix} \delta X \\ \delta Y \\ \delta Z \end{vmatrix} \qquad (2)$$

$$B = \begin{vmatrix} \frac{\partial fx}{\partial X} & \frac{\partial fx}{\partial Y} & \frac{\partial fx}{\partial Z} \\ \frac{\partial fy}{\partial X} & \frac{\partial fy}{\partial Y} & \frac{\partial fy}{\partial Z} \end{vmatrix}$$

The normal equation is:

$$(B^T W B)\Delta = B^T W f \qquad (3)$$

where W is a weight matrix.

Equation 1 can be reformulated to produce a linear model in the following form:

$$((x - x_{pj})\, m_{31j} + c_j\, m_{11j})\ X + ((x - x_{pj})\, m_{32j} + c_j\, m_{12j})\ Y + ((x - x_{pj})\, m_{33j} + c_j\, m_{13j})\ Z =$$

$$((x - x_{pj})\, m_{31j} + c_j\, m_{11j})\ X_j + ((x - x_{pj})\, m_{32j} + c_j\, m_{12j})\ Y_j + ((x - x_{pj})\, m_{33j} + c_j\, m_{13j})\ Z_j$$

(4)

$$((y - y_{pj})\, m_{31j} + c_j\, m_{21j})\ X + ((y - y_{pj})\, m_{32j} + c_j\, m_{22j})\ Y + ((y - y_{pj})\, m_{33j} + c_j\, m_{23j})\ Z =$$

$$((y - y_{pj})\, m_{31j} + c_j\, m_{21j})\ X_j + ((y - y_{pj})\, m_{32j} + c_j\, m_{22j})\ Y_j + ((y - y_{pj})\, m_{33j} + c_j\, m_{23j})\ Z_j$$

Equation 4 will form a pseudo least squares solution.

3. Feature Space Intersection

The photo image of a feature in space can be represented by discrete points on each photo. The discrete image points on the second photo can be represented by a single continuous function or a group of finite elements. The modified form of the collinearity equation for a point seen on two photos will be as follows:

$$x_1 = x_{p1} - c_1 \frac{m_{11}(X-X_1) + m_{12}(Y-Y_1) + m_{13}(Z-Z_1)}{m_{31}(X-X_1) + m_{32}(Y-Y_1) + m_{33}(Z-Z_1)}$$

$$y_1 = y_{p1} - c_1 \frac{m_{21}(X-X_1) + m_{22}(Y-Y_1) + m_{23}(Z-Z_1)}{m_{31}(X-X_1) + m_{32}(Y-Y_1) + m_{33}(Z-Z_1)}$$

$$x_2 = x_{p2} - c_2 \frac{r_{11}(X-X_2) + r_{12}(Y-Y_2) + r_{13}(Z-Z_2)}{r_{31}(X-X_2) + r_{32}(Y-Y_2) + r_{33}(Z-Z_2)}$$

$$y_2 = y_{p2} - c_2 \frac{r_{21}(X-X_2) + r_{22}(Y-Y_2) + r_{23}(Z-Z_2)}{r_{31}(X-X_2) + r_{32}(Y-Y_2) + r_{33}(Z-Z_2)}$$

$$y_2 = f(x_2) \qquad (5)$$

or

$x_1 = FX_1\ (X, Y, Z)$

$y_1 = FY_1\ (X, Y, Z)$

$x_2 = FX_2\ (X, Y, Z)$

$f(x_2) = FY_2\ (X, Y, Z)$

where:

x_1, y_1	photo coordinates of the point on photo 1
X_1, Y_1, Z_1	object space coordinates of photo 1
$m_{11} \ldots m_{33}$	orthogonal orientation matrix elements for photo 1
X_2, Y_2, Z_2	object space coordinates of photo 2
$r_{11} \ldots r_{33}$	orthogonal orientation matrix elements for photo 2
X, Y, Z	object space coordinates of the point.

The unknowns in Eq. (5) are (X, Y, Z, and x_2). This mathematical model will provide a unique solution for the problem. Each additional photograph will provide three equations and one additional unknown.

The function $f(x_2)$ can be linear, polynomial or a group of finite elements. This function should be defined from the discrete points that defines the feature on the second photograph. The function $f(x_2)$ should exist. If it does not exist, we can use $x_2 = h(y_2)$. If the feature line slope changes between 0 and 360 degrees, then we can use the following equations:

$x_2 = h_1(z)$

$y_2 = h_2(z)$

where the unknowns will be (X, Y, Z, x_2, y_2, and z).

The linearized form of equation 5 will be:

$$B\Delta = f$$

$$B = \begin{Vmatrix} \frac{\partial FX1}{\partial X} & \frac{\partial FX1}{\partial Y} & \frac{\partial FX1}{\partial Z} & 0 \\ \frac{\partial FY1}{\partial X} & \frac{\partial FY1}{\partial Y} & \frac{\partial FY1}{\partial Z} & 0 \\ \frac{\partial FX2}{\partial X} & \frac{\partial FX2}{\partial Y} & \frac{\partial FX2}{\partial Z} & 0 \\ \frac{\partial FY2}{\partial X} & \frac{\partial FY2}{\partial Y} & \frac{\partial FY2}{\partial Z} & \frac{\partial f}{\partial x_2} \end{Vmatrix}$$

$$\Delta = \begin{Vmatrix} \delta X \\ \delta Y \\ \delta Z \\ \delta x_2 \end{Vmatrix}$$

3. RESULTS

To test the new mathematical model, ground features from two aerial photos, taken from a flying height of 1500.00 ft above the terrain with a camera whose focal length was 152.00 mm, were mapped using the KERN DSR-14 analytical plotter. The same features were digitized using the KERN MK-2 mono-comparator using the same photographs. The average rms errors between the two methods were 0.06 ft in planimetric position and 0.181 ft in elevation.

The ground features were a combination of straight and curved lines that defined the edges of a highway. The rms error in modeling the image coordinates on the second photograph was in the order of 0.006 mm.

4. CONCLUSIONS

Feature space intersection has been tested and can be used in digital photogrammetry, 3D robot vision, and single photo digitizing. In digital photogrammetry, the number of discrete points that define the features will allow more accurate mathematical representation of the feature in Eq. 5.

The new formulation of space intersection gives a unique solution for the problem, and least squares adjustment is not needed. In this approach, error will not be easily identified. To obtain the redundancy that was available in Eq. 1, the function $y_1 = f(x_1)$ can be added to Eq. (5) and the least squares adjustment will be used to solve the problem. Also the new solution is non-linear and Eq. (4) can not be used.

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