FRACTAL DIMENSION ESTIMATION AND ITS APPLICATION TO IMAGE SEGMEMTATION

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ABSTRACT:
The fractal-based approaches have been used in a variety of applications. In this paper, A new method (SAVR) to estimating the fractal dimension (FD) of image data is proposed. A Comparative study of the properties of current methods for the FD estimation is carried out and the experiments results show that the SAVR method and the BLANKET method are suitable for image processing. Furthermore, from the viewpoint of application, we discuss how the FD can be used as an important measure for image segmentation. Finally, the experiments on feature extraction and texture segmentation are presented, and some meaningful conclutions are obtained.

KEYWORDS: Image analysis, Fratal dimension estimation, Image segmentation, Texture analysis

1. INTRODUCTION

Both a description and a mathematical model for many of the seemingly complex forms found in nature can be provided by fractal models developed by Mandelbrot (Mandelbrot, 1982). The defining characteristic of a fractal is fractal dimension (FD), which has been used as a measure of spatial complexity. Recently, the fractal- based methods have been applied to many areas of digital image processing, such as, image synthesis, image compression and image analysis (Fournier, 1982, Barnsley, 1988, Pentland, 1984). however, there are two main problems which limit the application. The first one is that the image data are not pure fractals, that means, the FD value is not a constant accross a range of scales. The second problem is the different values of FD may be estimated by different FD estimation methods, the experiments results detailed in section 3 of this paper will show this fact clearly.

In order to overcome the above problems the following research objectives are determined. Since the FD is a key characteristic in fractal-based analysis of digital image, the FD values should be calculated correctly and reliably, which implies the FD values must be calculated within its certain scale limit. But in fact, the scale limit could not identified easily and using different FD estimation methods, the length of scale limit may be different. Therefore, first of all, many current methods of estimating the FD are evaluated in such aspects: correction of the FD estimation, scale limits, characteristic with multiresolution, window size and directions of the FD. The purpose of this work is that, which of these methods is more suitable for image analysis and image processing and which of them is more effective in certain above aspect with the needs of application. Secondly, we use the FD as a measure of complexity of image intensity surfaces in small areas to apply to image segmentation, instead of using the FD as a mathematic model to represent image surfaces. It can be seen that we have obtained many satisfying experiments results which seem difficult to acquire by other image processing techniques.

2. FRACTAL DIMENSION ESTIMATION FROM IMAGERY

2.1 BLANKET method

According to the fractal geometry, the relation between the area A(r) and the scale r is:

$$A(r) \propto r^{2-D} \qquad \ldots\ldots (1)$$

where D is the FD value. Now the problem is how to estimate the A(r) from a image. In the BLANKET method developed by Peleg (Peleg, 1984), the image intensity surface is covered with a 2r thick "blanket". thus the A(r) can be calculated by the blanket volume divided by 2r. The blanket volume is considered by its upper and lower surfaces u and b:

$$V(r)=\Sigma_{i,j}[u(r,i,j)-b(r,i,j)] \quad \ldots\ldots (2)$$

where the blanket surfaces are defined as following:

$$U(r,i,j)=\max\{u(r-1,i,j)+1,\max_{(m,n)\in s}[u(r-1,m,n)]\}$$
$$b(r,i,j)=\min\{b(r-1,i,j)-1,\min_{(m,n)\in s}[b(r-1,m,n)]\}$$
$$u(0,i,j)=b(0,i,j)=G(i,j) \quad \ldots\ldots (3)$$

in which $s=\{(m,n) \mid \|(m,n)-(i,j)\|=1\}$ hence we have obtained the measured A(r) at a distance r:

$$A(r)=V(r)/2r \quad \ldots\ldots (4a)$$
$$A(r)=[V(r)-V(r-1)]/2 \quad \ldots\ldots (4b)$$

Peleg considered that the Eqs. (4b) provides reasonable results for both fractal and quasi -fractal surfaces. however, it has been shown from our computation results that when the Eqs. (4b) is used, the scale limit is so small that the reliability of the FD values will be influenced. Fig. 1 gives the illustration of the BLANKET method in one dimension.

In fact, the calculation of u and b can be implemented by erosion and dilation transformation in mathematic morphology:

$$u(r)=G\oplus rk=\max_{\substack{m,n\in k\\ i-m,j-n\in H}}\{G(i-m,j-n)+r\cdot k(m,n)\}$$
$$b(r)=G\ominus rk=\min_{\substack{m,n\in k\\ i-m,j-n\in H}}\{G(i+m,j+n)-r\cdot k(m,n)\} \quad \ldots\ldots (5)$$

where k is the unit structure element with a sphere shape; K, H is the project sets onto the plane from G and k (Haralick, 1987). Essentically, Eqs. (5) is consistent with Eqs. (3).

2.2 SAVR method

The relation between the superficial area and volume has been given by Mandelbrot (Mandelbrot, 1982):

$$S^{1/D} \propto V^{1/3} \quad \ldots\ldots (6)$$

But Mandelbrot has not explained the meaning of the above expression explicitly, so that many papers have quoted the relation in a simple way: $S^{1/D}=K\cdot V^{1/3}$. Dong (Dong, 1991) pointes out the above equation between S and V is not correct and derives the concrete equation from (6) written as following:

$$S^{1/D_{n-1}}=k\cdot r^{(n-1-D_{n-1})/D_{n-1}}\cdot V^{1/n} \quad \ldots\ldots (7)$$

where n denotes the Euclidean dimension which is generally greater than the FD. The above equation (7) has been proved theoretically, however, it has not been so far used in application, since both the S and V can not be calculated easily.

In this paper, a new method called SAVR method (Superficial Area-Volume Relation method). is proposed. Fig. 2 illustrates the SAVR method in a profile of image. In a r×r area centered by an arbitrary point of a image window, $u_a(r)$, $b_a(r)$, $V_a(r)$ and $A_a(r)$ can be computed according to Eqs. (2), Eqs. (3) and Eqs. (4a). Then we can estimate the supperficial area S and volume V by a virtual way:

$$S(r)=2n\cdot A_a(r)$$
$$V(r)=n\cdot V_a(r)-(n-1)(2n+1)^3 \quad \ldots\ldots (8)$$

If we select point every certain distance in the image window, the average $\overline{S}(r)$ and $\overline{V}(r)$ are obtained for the whole image window. To estimating the FD value of this image window, Eqs. (7) can be written as following (n=3 in this case):

$$\overline{S}(r)^{1/D}=k\cdot r^{(2-D)/D}\cdot \overline{V}(r)^{1/3} \quad \ldots\ldots (9)$$

the logarithm of both sides of (9) are taken to yield:

$$\log\overline{S}(r)-2\log r=D\log k+(1/3)D\log\overline{V}(r)-D\log r \ldots (10)$$

where D and k are both constants, so we have:

$$\log(\overline{S}(r)/r^2)=D\log(\overline{V}(r)^{1/3}/r)+C \quad \ldots\ldots (11)$$

which is in the form of a linear equation. This question can be used as the basis for a linear regression, taken at different r. The parameter D is the slope of the best-fit line within its range of scales. The difference between the BLANKET method and the SAVR method is, the former method provides the FD value of the whole image window, in which only the thick of "blanket" is varied with different r, the latter method takes the variation of r in all directions into account, that is, not only the thick but also the size of area are all related to r. The comparative studies which descriped in the next section show the SAVR method has many good properties for image analysis. It should be noted that if the average processing step is omitted, the SAVR method can provide the FD value related to a single point of image, called single point SAVR (PSAVR) method and if n of Eqs. (7) is equal to 2, the SAVR method can provide the FD value of a profile of image in such case.

2.3 The other methods

Fractional Brownian Increase Random Field (FBIRF) can be applied to model the surface of image (Petland, 1984), FBIRF based methods, we called FBM method and FBV method, can be written as following:

$$\log E[\,|G(i,j)-G(k,l)|\,] = H\log r + C \quad \ldots\ldots (12)$$
$$\|(i,j)-(k,l)\| = r$$
$$\log Var[\,|G(i,j)-G(k,l)|\,] = 2H\log r + C \quad \ldots\ldots (13)$$
$$\|(i,j)-(k,l)\| = r$$

where D=n-H, In Eqs. (12) the mean values are used (FBM) and the variance values are applied in Eqs. (13) (FBV). the FBM method has described and applied in many papers (Petland, 1984).

Box measuring method (BOXM) developed by Mandelbrot and Voss (Peitgen, 1988) has been given a detail description in Keller's paper (Keller, 1989). We just adopted the original method not modified by Keller to use for the comparative experiments.

Density Correlation Function based method (DCF) developed by Tao (Tao, 1992) to apply to estimating the FD from grey level image. The FD values are estimated according to relation between the density correlation function C and r:

$$C(r) = k \cdot r^{n-D} \quad \ldots\ldots (14)$$

where, C(r) is obtained by box covering technique which has been applied to BOXM method.

3. COMPARATIVE STUDIES ON FRACTAL DIMENSION ESTIMATION METHODS

In this section, the above FD estimation methods first are compared in many aspects. For the purpose of comparing the methods correctly, the simulated fractal images with kown FD values are generated by recursive sub-division approach (Amanatides, 1987).

3.1 Correction of FD estimation

The above six methods are all tested on the simulated images (FD ranging from 2.1 to 2.9). It is shown from Tab. 1 that SAVR, BLANKET and FBM three methods acquired good results, which behave in such two cases: first, the estimates of FD are approximated to the original FD values, second, the linear relation between the estimates and original of FD values is explicit.

3.2 Scale limits

The FD values can be estimated correctly only within its scale limits. In application, the problem is that the scale limit is difficult to determined. To overcoming the problem, we should select the method which is not sensitive to scale limits, that is, the scale limit of this method is relative long, since the different methods used in FD estimation may produce different scale limits. Fig. 3 illustrates the experiments results of testing the six methods mentioned above on simultated images whose FD is 2.5. As can be seen, BLANKET method and SAVR method behave good straight linearities and have long scale limits.

3.3 Characteristic with multiresulotion

A theoretical fractal object is self-similar under all magnification and the FD is stable with changing resolution, however, it is obvious that this property will change with using different methods. To test the characteristics of multiresolution of different methods, the four levels multiresolution images of the simulated fractal images are genernted by averaging processing. Three methods are carried out to test their characteristics of multiresolution and the results are shown in Tab. 2. We can find that both SAVR method and BLANKET method have good results, especially, the former is more stable in FD estimates. FBM method behaves not well in this aspect.

level of resolution	original F D	FBM	BLANKET	SAVR
Level 1	2.2	2.24	2.51	2.41
	2.5	2.40	2.65	2.64
	2.8	2.92	2.77	2.84
Level 2	2.2	2.00	2.53	2.43
	2.5	2.54	2.67	2.64
	2.8	2.70	2.80	2,79
Level 3	2.2	1.64	2.40	2.38
	2.5	1.86	2.54	2.54
	2.8	2.01	2.71	2.75
Level 4	2.2	1.96	2.52	2.26
	2.5	2.68	2.65	2.57
	2.8	2.97	2.70	2.74
window size		21×21	8×8	21×21

Table: 2 the FD values estimated from multiresolution images

3.4 Window size and direction

As the generated test fractal images are simulated, the size of window can affect the FD estimates. It is shown in Tab. 1 and Tab. 2 that besides the BLANKET method of which window size is small (8 × 8) , the other methods have used large window with size 21× 21. By comparision the BLANKET method can obtain good FD values in very small window of size 4×4 and thus its time consumption of FD estimation is lower than others.

Direction is one of the important properties of the FD. By changing window size and window shape, we can estimate the FD values in a certain direction. It should be mentioned that some of the above methods can provide the FD values of a single profile of images, namely, FBM, FBV, DCF and SAVR methods.

4. FRACTAL FEATURE BASED IMAGE SEGMENTATION

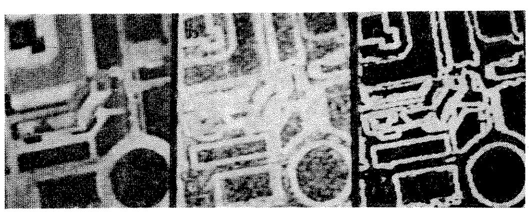
The fractal-based approaches have been applied in several different types of image analysis application (Peleg, 1984, Petland, 1984, Keller, 1989, Stein, 1987). Stein has made use of the fact that man- made objects are not fractal structures and, accordingly, will not provide reasonable fits to fractal models. In reality, this kind of techniques which use the differences between the FD of natural phenomena and that of man-made objects to dectect objects is not always reliable and effective, because of the problems discussed in section 1. However, the FD of subsets of an image can be taken as a useful texture measure which can be used to discriminate features in an image.

4.1 Feature extraction

The SAVR method and BLANKET method can provide the FD estimates from both fractal and nonfractal image data, and their straight linearities are quite well accross the long ranges of scales. Therefore both methods can be applied in feature extraction and image segmentation on real image data. Fig. 4 and Fig. 5 illustrate the feature extraction by BLANKET method in which the maximun of r is 13 pixel and the window size is 3×3. The FD values of points located in the center of the window are calculated by using the sliding window in the original images, thus, we obtained the processed images in which the grey value of each point denotes the transformed FD value which has converted from the value ranging in 2.0-3.0 into the value

(a) (b) (c)

Fig. 4 Fractal-based feature extraction
(a) original image
(b) image convoluted by Sobel operator
(c) D-feature image



(a) (b) (c)

Fig. 5 Fractal-based image segmentation
(a) original image
(b) D-feature image
(c) binary image

ranging in 0-255. We called this image D -feature image displayed in Fig. 4c and Fig. 5b. As compared with the image of Fig. 4b which is convoluted by Sobel operator on the original image, D-feature image Fig. 4c can give us some detailed features which can not be detected by the traditional feature extraction operators. The image Fig. 5c resulted from thresholding on D-feature image Fig. 5b shows us that the FD computering techniques can be regarded as image processing operators to apply to extracting edge feactures.

4.2 Texture segmentation

Coarseness and directions are two main features in texture analysis. The FD values can be suitable well for representing the two features. The image of Fig. 6a is a composite image with two different textures subimages in coarseness sand- bank and broadloaf selected from the aerophoto samples. The image Fig. 6b is a D-feature image which is transformed from image Fig. 6a by using the PSAVR method (The maximum r is 5). For the histogram of image Fig. 6b has clear two modes and that of image Fig. 6a has not, the image of Fig. 6c is obtained easily by thresholding the image Fig. 6b. The image of Fig. 6d which has been filtered by 5×5 median filter shows us a good result for segmenting two kinds of textures in different coarseness. As can be seen, the FD measure is very useful for discriminating different textures in coarseness.

The FD also has a directional property which has been discussed in sction 3. 4 . Some meaningful image features can be constructed by the directional FD values in different directions. Let Di denote the FD value in i direction. We have:

$$MEAN = 1/4 \sum_{i=0}^{3} D_i$$

$$VAR = 1/4 \sum_{i=0}^{3} (D_i - MEAN)^2$$

$$MIN = \min_{o < i < 3} \{ D_i \} \qquad \ldots\ldots\ (15)$$

where i=0 (east-west), 1 (northeast-southwest), 2 (north-south), 3 (northwest-southeast). The feature MEAN indicates the degree of texture coarseness, VAR indicates intensity of texture direction, MIN indicates that i direction is a trendency of direction of texture variation. We can derive more features through the composite of D_i in application. The image of Fig. 7a composed of two ridge subimages in two directions of texture variation is used for experiments. We obtained the D-feature image Fig. 7b from image of Fig. 7a by using PSAVR method to estimate the FD values D_1 and D_3 (in this

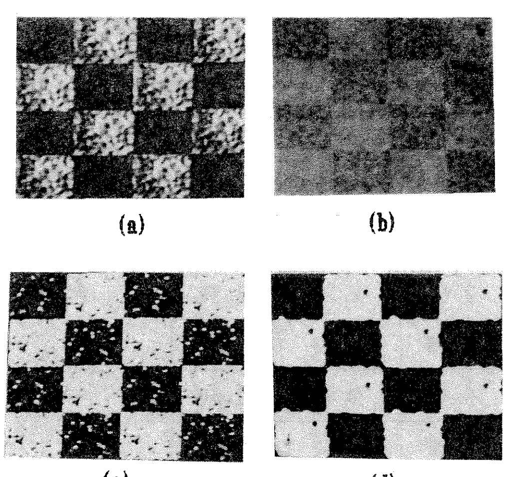


Fig. 6 Texture segmentation in coarseness
(a) original image
(b) D-feature image
(c) binary image
(d) image convoluted by median filter

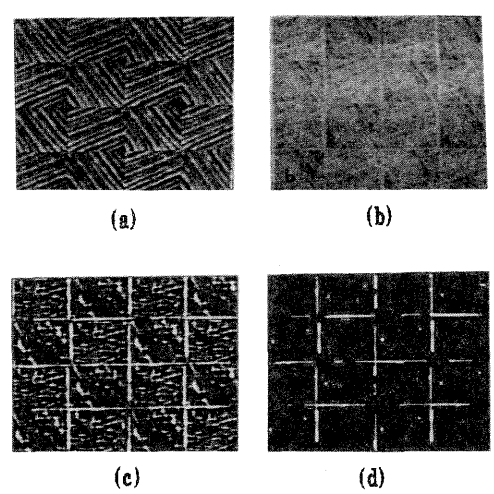


Fig. 7 Directional texture segmentation
(a) original image
(b) D-feature image
(c) line detection from (b)
(d) binary image

case , the maximum of r is 4, n is 2) . The values of points in D- feature image are computed according to:

$$G(i, j) = f(1/2D_1 + 1/2D_3) \quad \ldots\ldots (16)$$

f(·) indicates the linear transformation from range 2.0-3.0 into range 0-255. At the boundary of two neightbouring subimages, the intensities of D_1 and D_3 are both strong and the output G is higher than that of the other locations. Using Nevatia operator to do line detection on the D-feature image and then median filter to do convoluting with 5×5 mask, the image Fig.7c is obtained. After thresholding image Fig.7c, we have extracted the texture boundary displayed in Fig.7d. As a result, the directional FD is one of important measures for image texture segmentation.

5. CONCLUTION

The fractal-based technique is a new approach in image analysis and image segmentation. The method (SAVR) based on the ralation between the superficial area and volume is developed for estimating the FD. From evaluation on six FD estimation methods in many aspects, the SAVR method and the BLANKET method both behave good properties for image processing. It is difficult to apply fractal model to represent the image surfaces, but we can take the FD as a useful texture measure which can be used to characterize features of images.

REFERENCE

M.F.Barnsley, 1988. Application of recurrent iterated function system to image, Visual Communication and Image processing, vol.10001, SPIE.

B.B.Mandelbrot, 1982. The Fractal Geometry of Nature, W.H.Freeman, New York.

L. K. Dong, 1991. Fractal theory and its application, Liaoning science and technology press (in chinese).

A.Fournier, et al., 1982. Computer Rendering of Stochastic Models, Communications of the ACM, Graphics and Image processing Vol.25, No.6.

R.M.Haralick, et al., 1987. Image Analysis Using Mathematical Morphology, IEEE PAMI, NO.4.

J.Amanatides, 1987. Realism in Computer Graphics: A Surrey, IEEE Computer Graphics and Applications, Vol.7, No.1.

J.M.Keller, S.chen, 1989. Texture Description and Segmentation through Fractal Geometry, CVGIP, 45.

C.Tao, Z.J.Lin, 1992. Image Fractal Dimension Estimation. Acta Geodetica et Cartographica Sinica (to be presented).

H.O.Peitgen, D.Saupe, 1988. The Science of Fractal Images, Springer.

S. Peleg, J. Naor, 1984. Multiple Resolution Texture Analysis and Classificaton, IEEE PAMI, VOL.6, NO.4.

A.P.Pentland, 1984. Fractal-Based Description of Natural Scenes, IEEE PAMI-6, NO.6.

M.C.Stein, 1987. Fractal Image models and object detection, Visual Communication and Image processing II, SPIE, vol.845.

N.Yokoya, et al., 1989. Fractal-Based Analysis and Interpolation of 3D Nature Surface Shapes and Their Application to Terrain Modeling, CVGIP vol.46.

original FD	FBM	FBV	BOXM	DCF	BLANDET	SAVR
2.1	2.16	2.18	2.18	2.40	2.15	2.19
2.2	2.18	2.19	2.26	2.41	2.26	2.26
2.3	2.31	2, 24	2.38	2.45	2.33	2.36
2.4	2.48	2.39	2.42	2.49	2.45	2.50
2.5	2.54	2.42	2.44	2.56	2.53	2.59
2.6	2.61	2.51	2.49	2.62	2.63	2.69
2.7	2.67	2.59	2.51	2.68	2.74	2.75
2.8	2.73	2.65	2.51	2.72	2.81	2.81
2.9	2.75	2.71	2.52	2.87	2.87	2.87
window size	21×21	21×21	21×21	21×21	8×8	21×21

Table: 1 Comparison on the FD values estimated by 6 methods

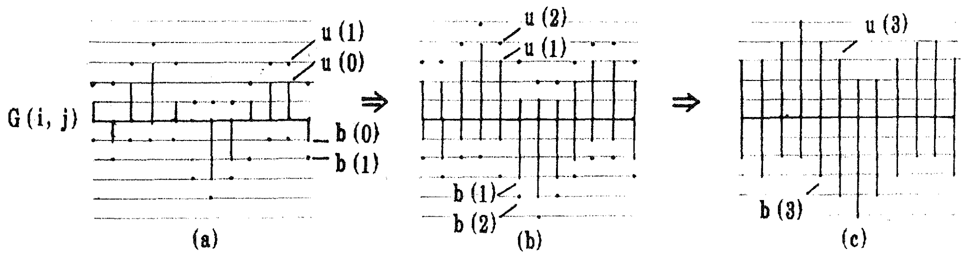


Fig. 1 Illustration of BLANKET method in a profile of image

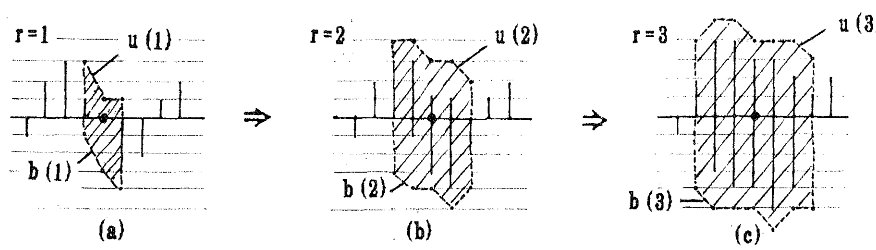


Fig. 2 Illustration of SAVR method in a profile of image

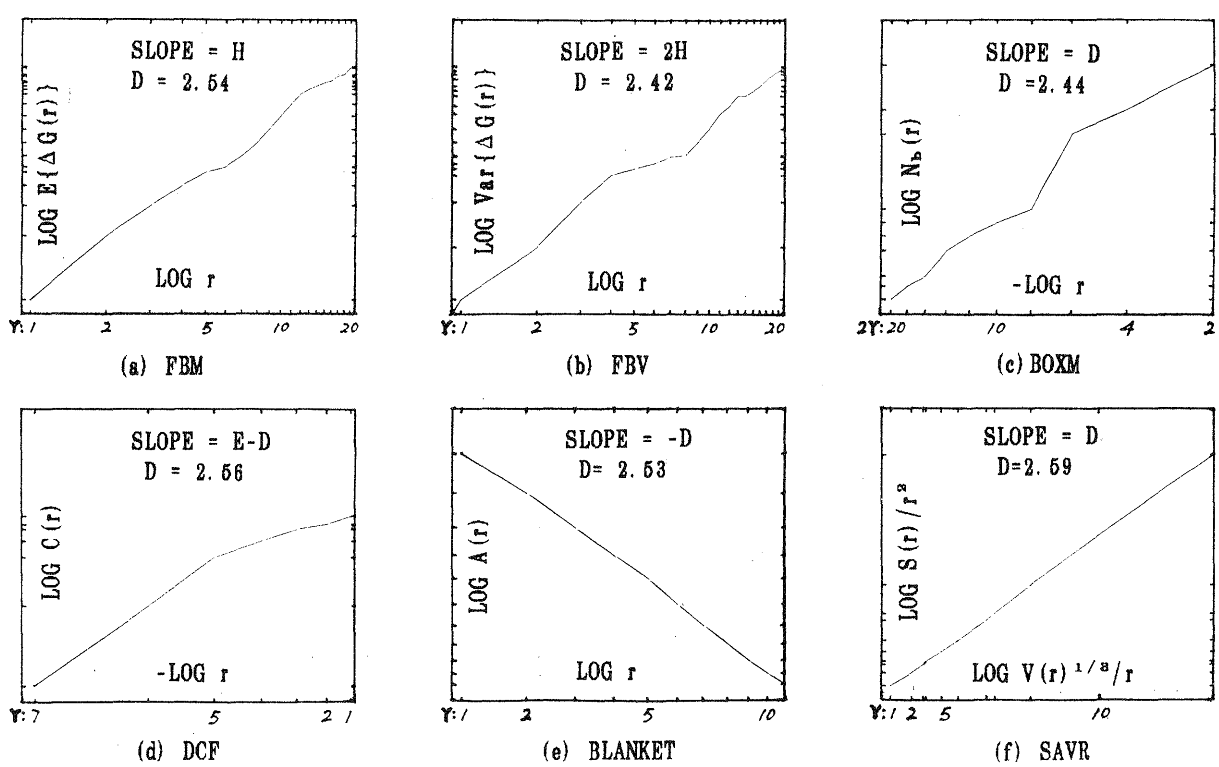


Fig. 3 Fractal plots of 6 methods