A Research of Boundary Extraction Based on Zero Crossing of Second Directional Derivatives

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ABSTRACT:

At present, discrete edge features can be extracted by many edge extraction methods. Because these edge features are not exact boundary, it is difficult to use in the image analysis and classification.

In this paper, a new boundary extraction approach is introduced based on zero crossing of second directional derivatives, heuristic searching of artifical intelligence and manual editing.

Using this approach, the boundary on image can be extracted accurately and extraction quality of boundary can be greatly improved.

KEY WORDS: Edge extraction, Zero crossing, Derivative, Artifical intelligence

1. Introduction

Edge or boundary, generally corresponds to great change of geometry or physical property of scene, it has been widely used as the important features in two and three dimension computer vision. Edge extraction has become an important research subject in image processing for many years.

Differential operator is a powerful means for exploring the features of function change, many kinds of operator have been proposed in recent than 20 years. In order to improve the accuracy and speed many improvement methods have been proposed also [1][2][3].

The present research results show that most edge extraction algorithms exist the following problems, for some algorithms edge feature points along the edge could be extracted but it is not real edge.

For some algorithms real edge points can be extracted, but it is discrete. Besides image matching, it is difficult to use in other area. In order to improve the quality of edge extraction, a research on boundary extraction based on zero crossing of second derivatives has been introduced. A test on remote sensing image has been executed, and the test results indicate that this approach is successful.

2. The principle of Edge Extraction Using Zero Crossing of Second Directional Derivatives

In digital image, edge generally means that brightness value has great change or the derivatives of brightness value has partical extreme value. More precisely, a pixel called edge must has the following condition: within the area around pixel, zero crossing of second directional derivatives exist on gradient direction.

Digital image grey generally is discrete, for determining edge accuratelly, the discrete grey value should be represented by a fitting function. Orthogonal basis has been selected. There are following relationship for discrete orthogonal polynomial.

$$\sum_{r \in R} P_K(r)\,(r_n + a_{n-1}r^{n-1} + ... + a_1 r + a_0) = 0$$

This is a linear equation, after solving the front 4 polynomial function formulas are:

$$P_0(r) = 1, \qquad P_1(r) = r$$

$$P_2(r) = r^2 - \mu_2/\mu_0, \qquad P_3(3) = r^3 - (\mu_4/\mu_2)r$$

where: $\mu_K = \sum_{s \in R} S^K$

For two dimensional discrete orthogonal polynomial, it can be constituted by two one dimension orthogonal polynomial using tensor product.

Suppose R and C are two dimension fitting interval, let $\{P_0(r), ... , P_N(r)\}$ is a set of discrete polynomial on R, $\{Q_0(c), ... , Q_M(c)\}$ is a set of discrete polynomial on C, then, $\{P_0(r) \cdot Q_0(c), ... , P_N(r) \cdot Q_M(c)\}$ is a set of discrete orthogonal polynomial on $R \times C$.

Using this relations, a fitting formula of two dimensional image can be derived and first derivatives, second derivatives can be found.

1. Fitting using discrete othogonal polynomial

Suppose R is fitting interval in row direction and has symmetrical features and n elements, C is in column direction with fitting interval of symmetrical features and also has n elements, using tensor product, two dimension discrete or thogonal Pm (r, c) can constituted.

After derivation, coefficient for fitting is:

$$a_m = \frac{\sum_{r \in R} \sum_{c \in C} P_m(r,c) \cdot d(r,c)}{\sum_{S_r \in R} \sum_{S_c \in C} P_m^2(S_r, s_c)} \qquad (1)$$

Fitting polynomial Q (r, c) can be expressed by the following formula:

$$Q(r,c) = \sum_{m=0}^{K} a_m \cdot P_m(r,c) \qquad (2)$$

2. Method of Edge Determination using Directional Derivatives

According to the definition of edge derivative, it can be written into the following form:

$$f_{\alpha}(r,c) = \lim_{h\to 0} \frac{f(r+h\sin\alpha, c+h\cos\alpha) - f(r,c)}{h}$$

second directional derivatives is:

$$f_{\alpha}(r,c) = \frac{\partial f}{\partial r}(r,c)\sin\alpha + \frac{\partial f}{\partial c}(r,c)\cos\alpha$$

If taking f as cubic polynomial of point (r, c), that is

$$f(r,c) = K_1 + K_2r + K_3c + K_4r^2 + K_5rc + K_6c^2 + K_7r^3 + K_8r^2c + K_9rc^2 + K_{10}c^3$$

The angle can be calculated by the following formula:

$$\sin\alpha = \frac{K_2}{\sqrt{K_2^2 + K_3^2}}$$

$$\cos\alpha = \frac{K_3}{\sqrt{K_2^2 + K_3^2}}$$

Second derivatives for any point (r, c) on direction is

$$f_{\alpha}''(r,c) = (6K_7\sin^2\alpha + 4K_8\sin\alpha\cos\alpha + 2K_9\cos^2\alpha)r + (6K_10\cos^2\alpha + 4K_9\sin\alpha\cos\alpha + 2K_8\sin^2\alpha)c + (2K_4\sin\alpha^2 + 2K_5\sin\alpha\cos\alpha + 2K_6\cos\alpha)$$

$$\because r = \rho\sin\alpha, \quad c = \rho\cos\alpha, \text{ than}$$

$$f_{\alpha}''(r,c) = 6\,(K_7\sin^3\alpha + K_8\sin^2\alpha\cos\alpha + K_9\sin\alpha\cos^2\alpha + K_10\cos^3\alpha)\rho + 2(K_4\sin^2 + K_5\sin\alpha\cos\alpha + K_6\cos^2\alpha) = A\rho + B \qquad (3)$$

When $f_{\alpha}''(\rho) = 0$ and $f_{\alpha}'(\rho) \neq 0$, i. e. when first derivatives is not zero and second directional derivatives is equal to zero, then, this point is edge point.

3. Edge Extraction for Two Dimension Image Using Zero Crossing of Second Directional Derivatives

It can be known from the front derived formula that to extract edge using zero crossing of second directional derivatives not only is related with the order number of fitting polynomial, but also is related with the window size at the same time. The larger the window, the longer the calculation time. In order to beadapted for micro—computer processing, 5 × 5 window size is selected, fitting polynomial comes up order.

According to weight coefficient and window grey value, the coefficient of polynomial can be solved, thus first and second derivatives can be further calculated.

The approximate value of first derivative is

$$f'(r,c) = K_2\sin\alpha + K_3\cos\alpha \qquad (4)$$

Second derivative is

$$f''(r,c) = A\rho + B \qquad (5)$$

When $\rho = 0$, that is center point of window, If symbles of second derivatives changed among $\rho = 0$ and $\rho = 1$ and first derivatives is not equal to 0 and larger than threshold, the center point of window is considered as edge point.

For remote sensing image, its grey change is extremely complex. Therefore, two problems similar to the method existed in actual processing, one is the determination of threshold, the other is edge which can not be continued.

For different kind of edge, its threshold should be different too. For an image, a threshold is obviously not suitable. In order to solve this problem, a method of manual assistant has been used.

Great difficulties of image analysis will cause from discrete edges. For reasons of edge extraction method to be practical, a method of heuristic search of artifical intelligence has been used, discrete edge turn to continuous edge.

Finally, using editing method, the real satisfied edge results can be achieved.

The main points of edge connection method will be described in the following.

1. Threshold Selection

In edge extraction using zero crossing of second directional derivatives, as the point meets the requirements of second directional derivatives and large then initial threshold, then first derivatives of this point should be reserved.

After processing, all the first derivatives of corresponding edge points can be obtained, then, the original image will be shown on screen (e. g. 512×512 pixels), and subimage on the screen will be selected, for example, 100×100 pixels, selecting first derivatives threshold, the points whose first derivatives is larger than the threshold should be superimposed on theoriginal image, using visual to check the edge whether is suitable, if it is not, then to adjust threshold until optimum results can be got.

Using this method, threshold can be selected flexibly, so as to meet the needs of partial region of image, to achieve the optimum goal of complete image edge extraction.

2. Automatic Connection of Discrete Edge

Owing to the edge change for remote sensing image is extremely complicated, after edge extraction, discrete point often happens, themethod of heuristic search of artificial intelligence has been used for automatic connection.

A algorithm has been used. The formula of A* algorithm is:

$$f^*(n) = g^*(n) + h^*(n)$$

where, $g^* = K(s,n)$, the real cost of an optimum path n from node s to node n. $h^*(n)$, the cost of an optimum path from node n to object node.

$f^*(n)$, the cost of an optimum path started from s through node n.

A algorithm is used for automatic connection of discrete edge point, its basic principles in connection are:

(1). Given a interval threshold between discrete points, if interval is smaller than threshold, these discrete points will have the possibility to be connected into line, otherwise it can not be connected.

(2). In the selection of equidistance discrete points, cost function should be calculated according to the following formula and the pointwith minimum cost will be considered as connection point.

$$cost = G_n - G_0 + 2\ (GD_n - GD_0)$$

where, G_n, G_0 is the grey value of point n and object point 0, GD_n, GD_0 is the first derivatives of point n and object point 0.

(3). When the number of automatic looking for connection point is small then threshold , then the edge will be removed automatically.

(4). The searched edge points will not be searched again so as to ensure edge point has the limit of search once.

(5). It is prohibited to search in opposite direction in the automatic search so as to avoid mistaken connection.

3. Manual Editing

After automatic connection, the discrete points which can be connected all have been connected into curve, however, as a result of the edge of remotesensing image is very complicated, part of defects still might exist even if many measures have been taken, so it must be through manual editing.

After automatic connection and editing, the image boundary is formed and it is very useful for practical to use.

4. Edge Extraction Test and Results

The test image is TM and obtained in July 1987, located in Ji county of Hebei Province. Band 3 is selected and the image size is 512×512 pixel. Medium filtering is firstly used with 5×5 window to eliminate the affects of noise, then first and second derivatives are determined also using 5×5window, the point of which first derivatives is larger than 5 and second derivatives has zero crossing is taken as edge point. In terms of thestatistics, first derivatives are from 6 to 65. It can be seen that the change range for first derivatives of corresponding edge point is more wide, so it is not suitable to use a threshold to control an image. Image should be divided into subimage with 110×110 pixel size and the threshold of each subimage should be adjusted by visual monitor method. After the adjustment, the threshold change of each subimage is from 8—25.

Then initial editing will start, this step is mainly to delete evident mistake.

After the initial editing, discrete point will be connected using automatical method. The maximum search region is 11×11 pixel. Then the final editing will be conducted, through deleting, addition and modification, the results of image edge extraction can come upto the optimum standard and is shown in Fig 1. From the processing results it can be seen that image has been divided into reasonable blockette and reached the goal of image segmentation.

In order to for comparison, Gauss — Laplace method has been used, the result is shown in Fig 2, in the test, window size is 11×11 pixel, standard deviation is 1. 4, threshold is 5. From the the comparison, the new method introduced in this paper is better than Gauss—Laplace operator.

5. Conclusion

Research on edge extraction has been conducted for many years, the edge feature points can be extracted for many methods. These feature points can be used for image matching, but there are some difficulties in image analysis. For the requirement of image analysis in the feature, it isbetter to provide continuous boundary. For this reason, a research of boundary extraction based on zero crossing of second directional derivatives has been carried out.

This paper describes the principles of edge extraction using zero crossing of second directional derivatives, the formula for fitting of two dimension image, first and second derivatives have been derived. In order to overcome the difficulties of dynamic threshold in edge extraction, threshold will be determined using manual interaction. For reasons of edge extraction to be continued, curve tracking can be conducted using artificial intelligence search method and discrete point will be connected automatically, and it has flexible editing function, which is able to correct the defects in edge extraction conveniently. The test proves that the edge extracted by this method is better than Gauss — Laplace operator.

REFERENCE

1. Lu Yan, A New Algorithm of One Dimension Edge Extraction using Sequence Analysis, symposium of Digital Photogrammetry, Wuhan, China, 1988.
2. L. Vliet & I. young, A Non — Linear Laplace Operator as Edge Detector in Noisy Image, Computer Vision, Graphics and Image Processing, 1989.
3. Zheng Zhaobao, Image Matching Method via Dynamic Programming, Acta Geodetica et Cartographica Sinica, Vol 28, No. 2, 1989.

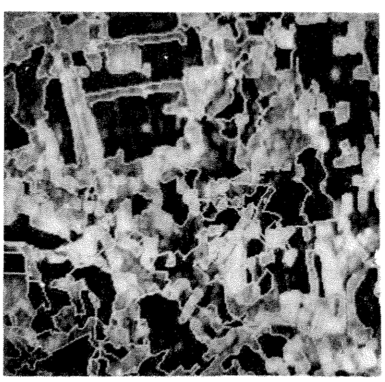

Fig. 1 The Result of New Method

Fig. 2 The Result of The Gauss—Laplace Operator